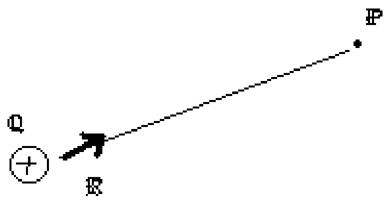
ECE 3300 Electrostatics – Coulomb's Law

COULOMB'S LAW:
Electric charges produce electric fields

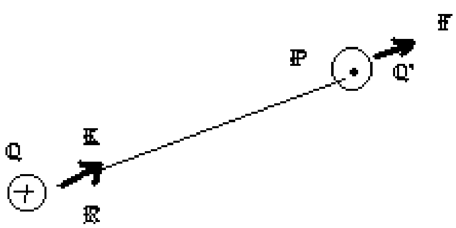


The ELECTRIC FIELD at some point P=

$$\overline{E} = \hat{r}\frac{q}{4\pi\varepsilon R^2} = \overline{R}\frac{q}{4\pi\varepsilon R^3}(V/m)$$

where **r** is a unit vector pointing from +q to P (the point where you want to find E, and **R** is a vector (not a unit vector) from q to P.

The electric field will produce a FORCE on another charge q':
F = q' **E**



The ELECTRIC FLUX DENSITY **D** = ε **E**
the ELECTRIC PERMITTIVITY $\varepsilon = \varepsilon_r \varepsilon_o$ and ε_o = 8.854e-12 F/m

LINEAR MATERIAL: ε does not depend on the MAGNITUDE of **E**
NON-LINEAR MATERIAL: ε does depend on the MAGNITUDE of **E**
as distinguished from true CHANGES in material due heating, etc. (burns, melting, etc.)

ISOTROPIC MATERIAL: ε does not depend on the DIRECTION of **E**
ANISOTROPIC MATERIAL: ε does depend on the DIRECTION of **E**

muscle and long nerves (due to lengthening of cells in one direction), some ceramics, ionosphere

NON-DISPERSIVE: ε does not depend on the FREQUENCY of **E**
DISPERSIVE: ε does depend on the FREQUENCY of **E**

Electric Fields due to multiple point charges

Rp is vector from origin to point where you want to find electric field
Rs1 is vector from origin to the first charge q1
Rs2 is vector from origin to the second charge q2

$$\overline{E}_1 = \frac{q_1(\overline{R}_p - \overline{R_{s1}})}{4\pi\varepsilon \, | \overline{R}_p - \overline{R_{s1}} |^3}$$

$$\overline{E}_2 = \frac{q_2(\overline{R}_p - \overline{R_{s2}})}{4\pi\varepsilon \, | \overline{R}_p - R_{s2} |^3}$$

$$\overline{E} = \overline{E}_1 + \overline{E}_2 = \frac{1}{4\pi\varepsilon}\left[\frac{q_1(\overline{R}_p - \overline{R_{s1}})}{| \overline{R}_p - R_{s1} |^3} + \frac{q_2(\overline{R} - \overline{R_{s2}})}{| \overline{R}_p - \overline{R_{s2}} |^3} \right]$$

$$= \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{q_i(\overline{R}_p - \overline{R_{si}})}{| \overline{R}_p - \overline{R_{si}} |^3}$$

Electric Fields due to a Charge Distribution

(Sorry.. for some reason my vector symbol is not working below.)

1. Define an ORIGIN at a convenient point
2. Write the vector Rs from the ORIGIN to the SOURCE(s) (charge) location
   If you used an origin at the center of the grid (0,0,0):

   $$R_s = x_s \hat{x} + y_s \hat{y} + z_s \hat{z}$$

3. Write the vector $\mathbf{R_p}$ from the ORIGIN to the location where you want to find the FIELD (field point).

   $$R_p = x_p \hat{x} + y_p \hat{y} + z_p \hat{z}$$

4. Apply Coulomb's Law

   a. Write the vector from the SOURCE(s) to the FIELD

$R_{ps} = R_p - R_s$

b. Define the SOURCE (charge) distribution. (Note: This is a scalar.)

$dq = \rho_l \, dl$ (Line charge)

$dl = dx, dy, \text{ or } dz$

$dl = dr, r\,d\phi, dz$

$dl = dR, R\,d\theta, R\sin\theta\,d\phi$

$dq = \rho_s \, ds$ (Surface charge)

$ds = dx\,dy, dy\,dz, \text{ or } dx\,dz$

$ds = r\,d\phi dz, drdz, r\,dr\,d\phi$

$ds = R^2 \sin\theta\,d\theta\,d\phi, R\sin\theta\,dR\,d\phi, R\,dR\,d\theta$

$dq = \rho v \, dv$ (Volume charge)

$dv = dxdy\,dz$

$dv = r\,dr\,d\phi\,dz$

$dv = R^2 \sin\theta\,dR\,d\theta\,d\phi$

c. Write the electric field caused by the charge distribution.

$$dE = \frac{dq}{4\pi\varepsilon |R_{ps}|^2} \hat{R}_{ps} = \frac{dq}{4\pi\varepsilon |R_{ps}|^3} R_{ps}$$

$$\hat{R}_{ps} = \frac{R_{ps}}{|R_{ps}|}$$

To find the magnitude of the vector: Take each vector component, square it, sum them, and take the square root.

$|Rps| = \text{sqrt}\,(R_x^2 + R_y^2 + R_z^2)$

d. Sum or integrate the sources to find the field.

$$E = \int_{startsource}^{endsource} dE = \int_{startsource}^{endsource} \frac{1}{4\pi\varepsilon |R_{ps}|^3} R_{ps}\,dq$$

Note: E, R_{ps}, dE are all vectors.