ECE 3300 **Gauss Law for the Electric Field and Electric Scalar Potential**

DIVERGENCE

Recall that point charges produce electric fields (which start on a + charge and stop on a - charge)
These electric fields are represented by FIELD LINES which are also called FLUX LINES.
An analogy is that we have a stream of water flowing from a + spigot to a - bucket. (Though electric fields do not truly flow, or they flow "instantaneously").

Remember that the electric field varies in 3D space. For a point charge, for instance, it is very strong near the charge, and decreases away from the charge. (Lines are terminted on - charges at infinity)

FLUX DENSITY tells us how strong the field is at a given point in space. (The greater the density, the greater the field strength)

Flux Density = Amount (#) of outward flux (field) lines crossing a unit area ds:

$$FluxDensity\ of\ \overline{E} = \frac{\overline{E} \bullet d\overline{S}}{|d\overline{S}|} = \frac{\overline{E} \bullet \overline{n}\, ds}{ds}$$

where **n** is the OUTWARD-going normal vector (perpendicular) of the surface

The TOTAL FLUX exiting from a closed surface is proportional to TOTAL CHARGE enclosed.

Total Flux (Exiting from **S**) = $\oint_S \overline{E} \bullet d\overline{S} = Q / \varepsilon_o$

$$\int_V \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \int_V \nabla \bullet \vec{E}\, dV$$

This is equal to the net sources in the volume enclosed by S :

The Divergence of **E** :

$$div\vec{E} = \nabla \bullet \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \quad Rectangular\ Coordinates$$

$$= \frac{1}{r}\frac{\partial (E_r r)}{\partial r} + \frac{1}{r}\frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z} \quad Cylindrical \quad Coordinates$$

$$= \frac{1}{R^2}\frac{\partial (E_R R^2)}{\partial R} + \frac{1}{R\sin\theta}\frac{\partial (E_\theta \sin\theta)}{\partial \theta} + \frac{1}{R\sin\theta}\frac{\partial E_\phi}{\partial \phi} \quad Spherical\ Coordinates$$

Divergence Theorem

$$\int_V \nabla \bullet \vec{E}\, dV = \oint_S \vec{E} \bullet d\vec{S}$$

This holds true IFF the vector **E** and its partial derivatives are continuous in the whole volume V.

Gauss Law for the Electric Field

"Point Form" (Differential form – valid at a single point) of Gauss Law:

$$\nabla \bullet \overrightarrow{D} = \rho_v$$

Total Charge enclosed:

$$Q = \int_v \rho_v dv = \int_v \nabla \bullet \overrightarrow{D}\, dv$$

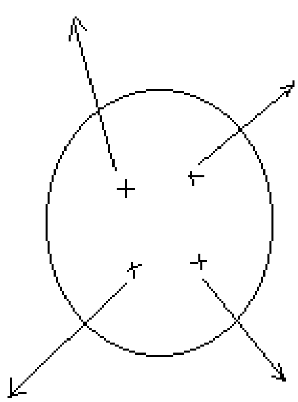
Divergence Theorem:

$$\int_v \nabla \bullet \overrightarrow{D}\, dv = \oint_S \overrightarrow{D} \bullet d\overrightarrow{S}$$

Combining them gives Integral form of Gauss Law;

$$\oint_S \overrightarrow{D} \bullet d\overrightarrow{S} = Q$$

Physical Meaning: Charge produces out-going flux lines for each "charge" enclosed.



Apply this to a single point charge inside a spherical surface:
D = D **R** (single, radially-directed flux line)

$$\oint_S \vec{D} \bullet d\vec{S} = \oint_S D\vec{R} \bullet ds\vec{R} = D(4\pi R^2) = q$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon} = \vec{R}\frac{q}{4\pi R^2} \quad (V/m) \quad Coulomb's\ Law!!$$

Coulomb's Law and Gauss Law for the Electric Field say the same thing:
When you are given the charge distribution and want to find **E**, use Coulomb's Law.
When you are given **E or D** and want to find total charge, use Gauss Law.

OR When you are given a symmetrical charge distribution, use Gauss Law to find **E**.