ECE 3300 Gauss Law for Magnetism and Ampere's Law

Gauss Law for Magnetic Field

Recall Gauss Law for Electric Field (two forms)

$$\nabla \bullet \overline{D} = \rho_v$$

$$\oint_S \overline{D} \bullet d\overline{s} = Q$$

Gauss Law for Magnetic Field (two forms)

$$\nabla \bullet \overline{B} = 0$$

$$\oint_S \overline{B} \bullet d\overline{s} = 0$$

(The 0s are on the RHS because there are no "magnetic charges".)

How do you use this? Unlike Gauss Law for E, Gauss Law for H is rarely used in practice, except to verify the form of a magnetic field.

Ampere's Circuit Law (two forms)
Physical premise: current produces magnetic field

$$\oint_C \overline{H} \bullet d\overline{\ell} = I$$

$$\nabla \times \overline{H} = \overline{J}$$

This is complementary to the **Biot-Savart Law** for H we have been using:
Physical premise: Magnetic field produces current.

$$\overline{H} = \frac{I}{4\pi} \int_\ell \frac{d\overline{\ell} \times \hat{R}}{R^2}$$

E&M Static "Cookbook":

1) Given Q or I find E or H
   a) Identify source type (point, line, surface, volume)
   b) Apply Coulomb's Law or Biot-Savart Law
      i) Coulomb's Law – Integration is over the charge line, surface, or volume; R' is vector from + charge to point where you are calculating E field.

$$\overline{E} = \frac{1}{4\pi\varepsilon}\int \hat{R}' \frac{dq}{(R')^2}$$

      ii) Biot-Savart – Integration is over the current line, in the direction of current flow; R is the vector from the element of current to the point where you are calculating the H field.

$$\overline{H} = \frac{I}{4\pi}\int_{\ell} \frac{d\overline{\ell} \times \hat{R}}{R^2}$$

2) Given E or H, find total charge or current; also given SYMMETRIC Q or I, find E or H.
   a) Apply Gauss Law for E

$$\oint_S \overline{D} \bullet d\overline{s} = Q$$

      i) Find Q given E: Surface integral is any surface that encloses the charge. E must be known on the surface.
      ii) Find E given Q: surface of integration should enclose the charge, and the point(s) where you want to calculate E should be on the surface.
   b) Apply Ampere's Law for H

$$\oint_C \overline{H} \bullet d\overline{\ell} = I$$

      i) To find I given H: Contour integral is closed contour along loop-line of H.
      iii) To find H given I: contour of integration should enclose the current, and the point(s) where you want to calculate H should be on the contour.

3) Relation between V and E:

$$V = -\int \overline{E} \bullet d\overline{\ell}$$

$$\overline{E} = -\nabla V$$

4) Forces

$\mathbf{F}_e = q\mathbf{E}$
$\mathbf{F}_m = \mathbf{I} \times \mathbf{B} = q\mathbf{v} \times \mathbf{B}$