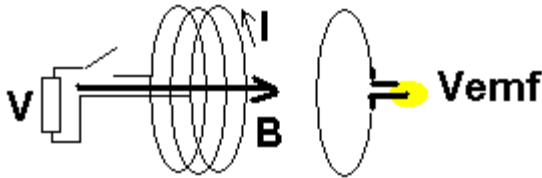


## ECE 3300 Maxwell's Equations (Time-Varying) / Faraday's Law

APPLICATION: Motors, Generators, Transformers

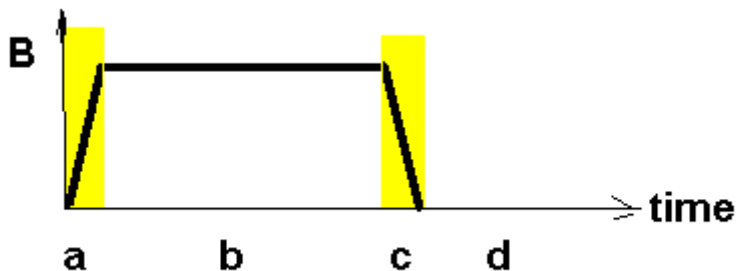
### Faraday's Experiment:



Recall that a current  $I$  produces a magnetic ( $B$ ) field. (We can find this using Ampere's law, which works for both static and dynamic (time varying)  $B$  fields.)

The direction of the  $B$  field is defined by the right hand rule.

Assume the switch is open. When it is closed, the  $B$  field suddenly rises (a in the graph below) and then stabilizes to a constant value (b in the graph below). When the switch is opened, the  $B$  field suddenly falls (c) and then stabilizes to zero (d).



Faraday observed that the light bulb turned on for a very short time (yellow areas) when the switch was first closed ( $a$ ) or opened ( $c$ ). The light bulb was OFF whenever the  $B$  field was stable ( $b$  and  $d$ ).

### How does this work?

The light bulb is a small resistor. It lights when a current is produced through its resistor. When the  $B$  field changes, it creates an electric field. The electric field is in the direction perpendicular to the  $B$  field. It is along the direction of the loop. This electric field creates a current in the loop and a voltage difference across the resistive light bulb. This voltage difference is called the electromotive force or  $V_{emf}$ . The law that predicts this is called Faraday's law. The law that predicts the polarity (positive/negative) of the electric field is called Lenz's Law.

### Faraday's Law:

For the time-varying case,  $\mathbf{E}$  and  $\mathbf{B}$  are coupled (inter-dependent)  
Time-varying  $\mathbf{B}$  generates  $\mathbf{E}$  (and remember that  $\mathbf{B} = \mu\mathbf{H}$ ).

Point Form:

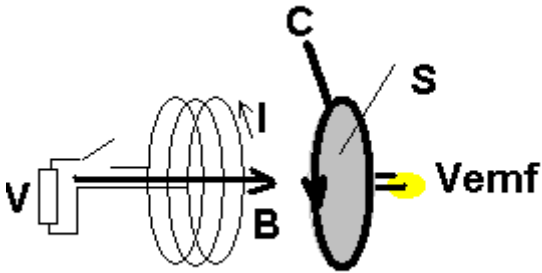
$$\nabla \times \bar{\mathbf{E}} = -\frac{\partial \bar{\mathbf{B}}}{\partial t}$$

OR Integral form

$$\oint_C \bar{\mathbf{E}} \cdot d\bar{\ell} = -\int_S \frac{\partial \bar{\mathbf{B}}}{\partial t} \cdot d\bar{\mathbf{S}}$$

Here is how contour C and surface S are defined:

Put your right thumb in the direction of  $\mathbf{B}$ . Your fingers will curl in the direction of C.  
The area that is enclosed by contour C is surface S.



The electric field  $\mathbf{E}$  is in the direction of the contour.<sup>1</sup> The total magnetic flux passing through the contour is:

$$\Phi = \int_S \bar{\mathbf{B}} \cdot d\bar{\mathbf{S}}$$

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<sup>1</sup> Note that the contour must be closed. The magnetic flux density  $\mathbf{B}$  is crossing surface S. Note that S is not a closed surface. This means it is not a box and does not enclose a volume.  $\mathbf{B}$  must be changing with time ( $d\mathbf{B}/dt \neq 0$ ).

The time varying magnetic flux produces the electric field and hence the electromotive force ( $V_{emf}$ ). If the contour C has multiple turns (a coil with N turns), the emf is multiplied by N.

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$V_{emf} = -N \frac{d\Phi}{dt}$$

So...

$$V_{emf} = N \left( - \oint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \right)$$

**The current induced in the lightbulb is:**

$$I_{lightbulb} = V_{emf} / R_{lightbulb}$$

**Methods to produce emf:**

- 1) Time-varying magnetic field linking a stationary loop (opening or closing the switch in Faraday's experiment).
- 2) Changing the surface area of the loop. This could be done by constricting the loop, but it is more often done by rotating a loop with a constant area. The rotation makes the area of the loop that is normal to the magnetic field change with time. This is what is usually done in generator.

**Lenz's Law – Tells us the “Direction” of emf:**

Lenz's law states that:

The emf is induced in a direction to **OPPOSE** the change in magnetic field.

Ie. If the magnetic field is decreasing (due to decreasing current), the emf (potential) is positive, and the emf-induced current flow is positive, to produce a magnetic field to counteract the decrease.

This is related to “an object in motion remains in motion” and includes “an electron in motion (current) remains in motion”

