ECE 3300 Displacement current

Displacement Current

Time-varying electric field produces magnetic field

$$\frac{\partial \overline{D}}{\partial t} = -\nabla \times \overline{H}$$

To relate this to current:

$$\int_S \frac{\partial \overline{D}}{\partial t} \bullet d\overline{s} = \int_S (\nabla \times \overline{H}) \bullet d\overline{S}$$

$StokesTheorem:$

$$= \oint_C \overline{H} \bullet d\overline{\ell}$$

$Ampere'sLaw:$

$$\oint_C \overline{H} \bullet d\overline{\ell} = I_{enclosed}$$

$WhichGives:$

$$\int_S \frac{\partial \overline{D}}{\partial t} \bullet d\overline{s} = \oint_C \overline{H} \bullet d\overline{\ell} = Displacement\_Current$$

$$\oint_C \overline{H} \bullet d\overline{\ell} = Conduction\_Current + Displacement\_Current$$

$$Total\_current = \sigma \overline{E} + \frac{\partial \overline{D}}{\partial t} = \sigma \overline{E} + \frac{\partial \varepsilon \overline{E}}{\partial t} = (\sigma + \frac{\partial \varepsilon}{\partial t})\overline{E}$$

$For\ e^{-j\omega t}\ time\ dependence$

$$\frac{\partial \overline{E}}{\partial t} = -j\omega \overline{E}$$

$So:$

$$Total\_current = (\sigma + j\omega\varepsilon)\overline{E} = \frac{\partial}{\partial t}\varepsilon^* \overline{E} = j\omega\varepsilon^* \overline{E}$$

$Complex\_\varepsilon:$

$$(\sigma + j\omega\varepsilon) = j\omega\varepsilon^* = j\omega\varepsilon_o(\varepsilon_r - j\sigma / \omega\varepsilon_o)$$

$$\varepsilon^* = \varepsilon_o(\varepsilon_r - j\sigma / \omega\varepsilon_o) = \varepsilon_o(\varepsilon' - \varepsilon'')$$

Why all this???
Conduction Current carries charges.
Displacement current represents displacement (rotationally) of molecules, but no actual motion.

Parallel Plate capacitor example:

Vs(t) = Vo cos(ωt)   (Time-varying voltage)

Conduction current in wire:
I_c = C dV/dt = -CVo ωsin(ωt)
Displacement current in wire = 0 because D=E=0 in metal wire

Electric field between the plates:
E= **y** V_c / d = **y** Vo cos(ωt) / d

Displacement Current between the plates

$$I_d = \int_S \frac{\partial \overline{D}}{\partial t} \bullet d\overline{S} = \int_A \left[\frac{\partial}{\partial t} \left(\hat{y} \frac{\varepsilon V_o}{d} \cos \omega t \right) \right] \bullet (\hat{y} ds)$$

$$= -\frac{\varepsilon A}{d} V_o \omega \sin \omega t = -C V_o \omega \sin \omega t = I_c$$

Conduction Current is flowing down the wire, and becomes displacement current flowing through the plates.