ECE 3300 Reflection and Transmission of EM Waves at Boundaries

Reflection and Transmission of EM Waves

This is similar to transmission line reflection / transmission.

What we want to find:
Suppose a plane wave is normally incident on a layered material (such as air-skin-fat-muscle layers), how much is reflected, and how much is transmitted (into each of the regions)?

What we need to know:

1) How plane waves (TEM waves) transmit and reflect (equations)
2) How to find the Polarization of the transmitted and reflected waves
3) How to find the reflection and transmission coefficients
4) How to find the magnitude of the transmitted and reflected E waves
5) Find the magnitude of H….
6) Now, write this all together in (1).

Here we go……

1) **General solution for a TEM wave: (such as Plane wave)**

Equation for plane wave with forward (incident) and reverse (reflected) traveling waves (note "k" is often used like "β" for plane waves):

$$\overline{E}(z) = \overline{E}^+(z) + \overline{E}^-(z) = \overline{E_0}^+ e^{-j\gamma z} + \overline{E}_0^- e^{j\gamma z}$$

$$= \overline{E_0}^+ e^{-\alpha z} e^{-jkz} + \overline{E}_0^- e^{\alpha z} e^{jkz}$$

$$\overline{H}(z) = \overline{H}^+(z) + \overline{H}^-(z) = \overline{H_0}^+ e^{-j\gamma z} + \overline{H}_0^- e^{j\gamma z}$$

$$= \overline{H_0}^+ e^{-\alpha z} e^{-jkz} + \overline{H}_0^- e^{\alpha z} e^{jkz}$$

2) **How to find the Polarization of the E and H fields.**

There are two types of vectors to deal with here.
The direction of propagation $\hat{k}$ (your book calls it a_p) is represented by wiggly lines in Figure 6.2. This is which way the plane wave is propagating. You can think of it as which direction the power is traveling.

The polarization of the E,H vectors is shown by solid lines in Figure 6.2. The polarization is the direction the E,H vector is pointed.

How to find the polarization of the E,H fields ($\hat{k}$ is the direction of propagation):

$$\overline{H} = \frac{1}{\eta}\hat{k}\times\overline{E}$$

$$\overline{E} = -\eta\hat{k}\times\overline{H}$$

$$\frac{\overline{E}\times\overline{H}}{\left|\overline{E}\right|\left|\overline{H}\right|} = \hat{k}$$

This math can also be done using the right-hand rule. Pointer finger = E. Middle finger = H. Thumb = $\hat{k}$ (direction of propagation).

Look at the picture in your book (figure 6.2) for the polarization of incident and reflected fields. Note that E stays in the same polarization (vector is still pointing up) when it reflects. But H changes polarization (first out of the page, then into the page).

3) **How to find the reflection coefficients**

If you have ONLY two regions:
Here is the derivation that helps us get the reflection and transmission coefficients…
From **the Boundary Conditions** (Source-free Region):

Tangential E equal at boundary

$E_{1t} = E_{2t}$

Normal D equal at boundary

$D_{1n} = D_{2n}$; $\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$

Tangential H equal at boundary

$H_{1t} = H_{2t}$

Normal B equal at boundary

$B_{1n} = B_{2n}$; $\mu_1 H_{1n} = \mu_2 H_{2n}$

Medium 1 (this is the medium the wave is transmitting IN TO)–

$\mathbf{E}_1(z) = \mathbf{E}^i(z) + \mathbf{E}^r(z) = \mathbf{x}(E_o^i e^{-j\,k1\,z} + E_o^r e^{j\,k1\,z})$

$\mathbf{H}_1(z) = \mathbf{H}^i(z) + \mathbf{H}^r(z) = \mathbf{y}(E_o^i e^{-j\,k1\,z} - E_o^r e^{j\,k1\,z})/\eta_1$

Medium 2 – (transmitted wave only)

$\mathbf{E}_2(z) = \mathbf{E}^t(z) = \mathbf{x}\,(E_o^t e^{-j\,k2\,z})$

$\mathbf{H}_2(z) = \mathbf{H}^t(z) = \mathbf{y}(E_o^t e^{-j\,k2\,z})/\eta_2$

At Boundary (z=0):

$\mathbf{E}_1(0) = \mathbf{E}_2(0)$ $\quad$ $E_o^i + E_o^r = E_o^t$

$\mathbf{H}_1(0) = \mathbf{H}_2(0)$ $\quad$ $(E_o^i - E_o^r)/\eta_1 = E_o^t/\eta_2$

Solve for reflected and transmitted fields:

$E_o^r = (\eta_2 - \eta_1)/(\eta_2 + \eta_1)\,E_o^i = \Gamma\,E_o^i$

$E_o^t = (2\eta_2)/(\eta_2 + \eta_1)\,E_o^i = \tau\,E_o^i$

Reflection and Transmission Coefficients:

$\Gamma = (\eta_2 - \eta_1) / (\eta_2 + \eta_1)$
$T = (2\eta_2) / (\eta_2 + \eta_1)$

Standing Wave Ratio
$S = |\mathbf{E}_1| \max / |\mathbf{E}_1| \min = (1 + |\Gamma|) / (1 - |\Gamma|)$

Note similarity to transmission lines:

If you have more than two regions: **Smith Chart Solution**

To find reflection coefficients:

a) Define regions origins O1,2,3 as shown in the examples.
b) Start on the right hand side at O3. $Z_3(O_3) = \eta_3$
c) Cross the boundary to O2 using impedance. $Z_2(O_2) = Z_3(O_3)$
d) Normalize $Z_2(O_2) / \eta_2$ and plot.
e) Rotate d2 TWG to –d2.
   IF the problem is lossy, read the $|\Gamma_{lossless}|$ from the smith chart. Find the magnitude of the lossy reflection coefficient $|\Gamma_{lossy}| = |\Gamma_{lossless}|\, e^{-2\alpha(d2)}$. Plot this new magnitude, which should be closer to the center than it was before. Be sure you are measuring from the CENTER of the smith chart when you do this!
   Read $Z_2(-d_2)$ and denormalize it. $Z_2(-d_2) * \eta_2$
f) Cross the boundary to O1 using impedance. $Z_2(O_1) = Z_2(-d_2)$
   If there are more layers Repeat d-f as many times as needed. This gives you reflection coefficients (and impedances) at every point in the model…. Just read them off the smith chart.

   Remember ! Γ has magnitude AND phase, so don't forget to read the angle!

   If the last region is a perfect conductor, $Z_3(O_3) = 0$ (short circuit)

To find transmission coefficients:

$T = 1 + \Gamma$ (or read it off the smith chart using the "transmission coefficient" angles and magnitudes.)

4) Find the Magnitude of the E fields.

The reflected and transmitted E fields in any region are:
$E_o^r = E_o^i = \Gamma(O_1)\, E_o^i$
$Eo^t = T(O_1)\, Eo^i$
For any other region, change O1 to the appropriate origin. For a single region, this is just Γ or T.
There is no reflected field in region 3, only transmitted.

About the transmitted fields…. The transmitted field given here is the transmitted field at $-d_2$. From this you can calculate the incident field (Eo+) in the region using the following equation:

$$\overline{E}^{t}(-d_2) = E^{r}{}_{o} = \overline{E_0}^{+} e^{-j\gamma(-d2)} = \overline{E_0}^{+} e^{-\alpha(-d2)} e^{-jk(-d2)}$$

5) **From E, find magnitude of H** (the polarization was found in (2) above).

This applies to both + and – traveling waves.

$$|H_0| = \frac{|E_0|}{\eta}$$

$$\eta_{lossy} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\eta_{lossless} = \sqrt{\frac{\mu}{\varepsilon}} .... = 377(air)$$