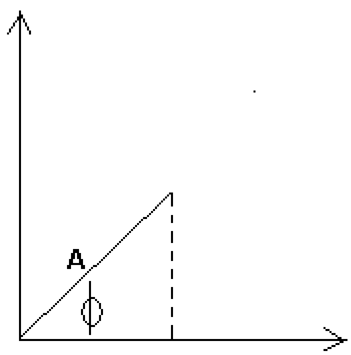
ECE3300 REVIEW OF COMPLEX NUMBERS AND PHASORS

Complex Numbers

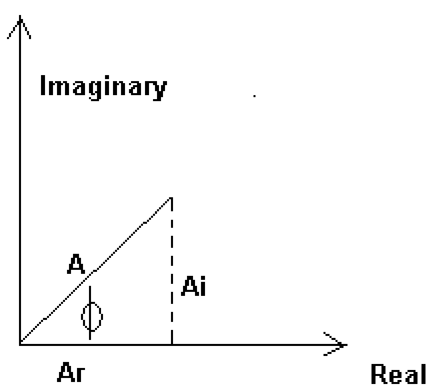
Why they are needed / what they mean to EM:
General wave: $y(x,t) = A\cos(\omega t-\beta x+\varphi)$
Suppose you know the frequency (ω) of a wave, then all you need to define the wave is the MAGNITUDE (A) and PHASE (φ)

This is a 2D quantity:



This can also be represented using the two rectangular axes -- "real" and "imaginary"

From geometry :



A = sqrt(Ai**2 + Ar**2)
φ = arctan (Ai / Ar)

Ai = A sin(φ)
Ar = A cos(φ)

Two ways of writing this number (complex #):

Polar form:
$Z = A \angle\varphi = A \exp(j\varphi)$
$J = \text{sqrt}(-1)$

Rectangular Form:
$Z = \text{Real}(z) + j\,\text{Imag}(Z)$
$= Ar + j\,Ai$

Mathematics of Complex Numbers:

$Z_1 = Ar_1 + j\,Ai_1 = A_1 \exp(j\varphi_1)$
$Z_2 = Ar_2 + j\,Ai_2 = A_2 \exp(j\varphi_2)$

Addition:
$Z_1 + Z_2 = (Ar_1 + Ar_2) + j\,(Ai_1 + Ai_2)$

Multiplication:
$Z_1 Z_2 = A_1 \exp(j\varphi_1)\, A_2 \exp(j\varphi_2) = A_1 A_2 \exp(j (\varphi_1+\varphi_2))$

Division: $(A_2 \neq 0)$
$Z_1 / Z_2 = A_1 \exp(j\varphi_1) / A_2 \exp(j\varphi_2) = A_1 / A_2 \exp(j (\varphi_1-\varphi_2))$

Exponentiation
$Z_1^n = (A_1 \exp(j\varphi_1))^n = A_1 \exp(j\, n\, \varphi_1)$

Useful Facts:
-1 = "180 degree phase shift" = $1\angle 180 = j^2 = e^{j\pi} = e^{-j\pi}$
j = "90 degree phase shft" = $1\angle 90 = e^{j\pi/2}$
$-j$ = "-90 degree phase shift" = $1\angle{-90} = e^{-j\pi/2}$
(show what this means on phasor plots)
**COMPLEX NUMBERS HAVE PHYSICAL MEANING

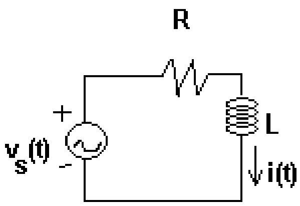
HP Calculators: FAIR GAME! BUT be SURE you understand the physics before you do the numbers.

Phasors:

Phasor technique is used to simplify solution of ANY linear system with sinusoidal forcing function.

Example: Series R-L. Find I(t)

vs(t) = R I(t) + L di(t)/dt



1. Define a reference (phase reference) point

V(t) = $A\cos(\omega t+\phi_o)$ ← Define ϕ_o as reference

Example: 1GHz source
V(t) = 1.0 cos(2π(1GHz)t +0) V/m
V(t) is the INSTANTANEOUS value

2. Express time-dependent variables as phasors

V(t) = real($V\, e^{j\omega t}$) where V(t) = 1.0 $e^{(j\,0))}$ V/m
Notes: *italic used in place of tilde phasor sign in these notes;* EEs use $e^{j\omega t}$ and physics uses $e^{-j\omega t}$

3. Integration and differentiation can also be replaced by phasor notation:

$\partial\, e^{j\omega t} / \partial\, t = j\omega\, e^{j\omega t}$
$\int e^{j\omega t} = e^{j\omega t} / j\omega$

Example: Series R-L
Vs(t) = R I(t) + L di(t)/dt
$V = RI + j\omega L I$

4. Solve (for I)

$I = V / (R + j\omega L) = V (R-j\omega L) / (R + j\omega L)(R-j\omega L) = V (R-j\omega L) / (R^2 + (\omega L)^2)$
$= [VR / (R^2 + (\omega L)^2)] - j\, [\omega L / (R^2 + (\omega L)^2)]$ = Real + j Imag !
$|I|$ = sqrt(Real**2 + Imag**2)
$\angle I$ = arctan(Imag/Real)

5. Convert to instantaneous value

I(t) = $|I| \cos(\omega t + \angle I)$ amps

Useful transformations given on p. 27 of text.

Example: Two 1GHz waves are propagating down a transmission line.
One has a magnitude of 1 V, and the other has a magnitude of 3 V. This second wave leads the first by 24 degrees. What is the total wave on the transmission line?

$V_1(t) = 1.\ \mathrm{Cos}(2\pi(1e9)t + 0°)$ V
$V_2(t) = 3.\ \mathrm{Cos}(2\pi(1e9)t + 24°)$ V

$V_1 = 1 \exp(j0)$ V$= 1\angle 0$ V$= 1\cos(0) + j\ 1\sin(0)$ V$= 1 + j0$ V
$V_2 = 3 \exp(j24°)$ V $= 3\ \angle 24°$ V$= 3\cos(24°) + j\ 3\sin(24°)$ V$= 2.74 + j\ 1.22$ V
$V = Z_1 + Z_2 = (\ Ar_1 + Ar_2\ ) + j\ (Ai_1 + Ai_2) = 3.74 + j\ 1.22$ V $= 3.93 \angle\ 18.1°$ V

Time domain? $V(t) = 3.93 \cos(2\pi(1e9)t + 18.1°)$ V