ECE3300 CURL of a Vector Field

Circulation of a vector field

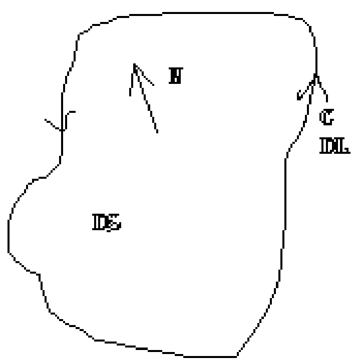
$$Circulation = \oint_C \overline{B} \bullet \partial \bar{l} = \text{scalar}$$

EXAMPLES

Uniform Field → Circulation = 0

B Field around a Current-carrying wire ≠ 0

Conversion from this to CURL:



$$Circulation = \oint_C \overline{B} \bullet \partial \bar{l}$$

$$Vector\ perpendicular\ to\ surface = \left[\bar{n}\oint_C \overline{B} \bullet \partial \bar{l}\right]$$

$$Take\ \max: \left[\bar{n}\oint_C \overline{B} \bullet \partial \bar{l}\right]_{\max}$$

$$Take\ \text{lim}it = \lim_{\Delta s \longrightarrow 0} \frac{1}{\Delta s}\left[\bar{n}\oint_C \overline{B} \bullet \partial \bar{l}\right]_{\max}$$

$$curl\overline{B} = \nabla x \overline{B} =$$

$$\lim_{\Delta s \longrightarrow 0} \frac{1}{\Delta s}\left[\bar{n}\oint_C \overline{B} \bullet \partial \bar{l}\right]_{\max}$$

curl **B** = circulation of **B** per unit area.

Defined at a POINT

∇x**B = NOT** a cross product (physical interpretation different because ∇ not a physical "vector")

$$\nabla x \overline{B} = \bar{x}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \bar{y}\left(\frac{\partial B_{zx}}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \bar{z}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = \begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix}$$

Other coordinate systems: See back cover.

Vector Identities:

$$\nabla x(\overline{A}+\overline{B}) = \nabla x\overline{A} + \nabla x\overline{B}$$

$$\nabla \bullet (\nabla x\overline{A}) = 0$$

$$\nabla x(\nabla V) = 0 \quad for\ scalar\ V$$

Stoke's Theorem:

$$\int_S \left(\nabla x\overline{B}\right) \bullet d\overline{S} = \oint_C \overline{B} \bullet d\bar{l}$$

Irrotational Field = Conservative Field

if ∇x**B**=0

EXAMPLE of STOKE's THEOREM

Laplacian Operator:

$$Gradient = \nabla V = \bar{x}\frac{\partial V}{\partial x} + \bar{y}\frac{\partial V}{\partial y} + \bar{z}\frac{\partial V}{\partial z}$$

$$Laplacian\ of\ \ scalar = \nabla^2 V = \nabla \bullet (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$Laplacian\ of\ \ vector = \nabla^2 \overline{E} = \bar{x}\nabla^2 E_x + \bar{y}\nabla^2 E_y + \bar{z}\nabla^2 E_z$$

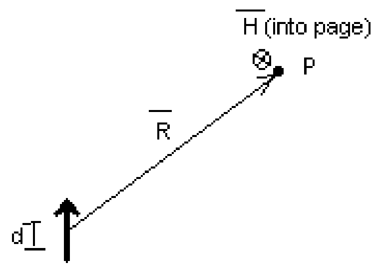
Vector Identity:

$$\nabla^2 \overline{E} = \nabla(\nabla \bullet \overline{E}) - \nabla x(\nabla x\overline{E})$$

Biot-Savart Law

Observation: Current-carrying wires deflected compass needle

Recall from electrostatics d**E** = **R'** dq / (4πε R'^{2})



The magnetostatic equivalent is d**H** = (I/4π) (d**I** x **R**) / R^2
When integrated:

$$\overline{H} = \frac{I}{4\pi}\int_{\ell}\frac{d\overline{I}\ x\ \overline{R}}{R^2}$$

Units (A/m)

EXAMPLE – square loop of current