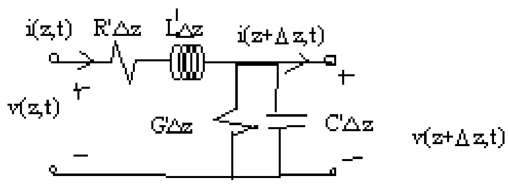
ECE3300 -- LECTURE 6

<u>Transmission Line Equations (Telegraphers Equations):</u>



Lumped element model represents voltage and current on transmission line. Voltage and current?? What about fields??? Fortunately, they are dual (proportional).

Kirchoff Loop Equation:

$V(z,t) – R\ I(z,t) – L\ di(z,t)/dt – v(z+\Delta z,t) = 0$

$R = R'\Delta z,\ \ L = L'\ \Delta z$

Divide by Δz, combine V terms, and rearrange:

$V(z,t) / \Delta z – R'\ I(z,t)\ \ – L'\ dI(z,t) /dt – V(z+\Delta z,t) / \Delta z = 0$

$-[\ V(z+\Delta z,t) - V(z,t)\ ] / \Delta z = R'\ I(z,t) + L'\ dI(z,t)/dt$

Limit as $\Delta z \rightarrow 0$ (difference goes to differential):
$-dV(z,t)/dz = R'\ I(z,t) + L'\ dI(z,t)/dt$

Kirchoff Node Equation:

$I(z,t) – G\ v(z+\Delta z,t) – C\ dv(z+\Delta z,t)/dt – I(z+\Delta z,t) = 0$

$I(z,t) – G'\Delta z\ v(z+\Delta z,t) – C'\Delta z\ dv(z+\Delta z,t)/dt – I(z+\Delta z,t) = 0$

Divide by Δz and rearrange :

$-[\ I(z+\Delta z,t) - I(z,t)] / \Delta z = G'\ v(z+\Delta z,t) + C'\ dv(z+\Delta z,t)/dt$

Limit as $\Delta z \rightarrow 0$:
$-dI(z,t) / dz = G'\ v(z+\Delta z,t) + C'\ dv(z+\Delta z,t)/dt$

Telegraphers equations:
$-dV(z,t)/dz = R'\ I(z,t) – L'\ dI(z,t)/dt$
$-dI(z,t) / dt = G'\ v(z+\Delta z,t) + C'\ dv(z+\Delta z,t)/dt$

Sinusoidal Steady-State Form:
Convert to Phasor form.

$-dV(z)/dz = (R' + j\omega L')\ I(z)$
$-dI(z)/dz = (G' + j\omega C')\ V(z)$
What is difference between this and "regular" circuit equations?? They are a function of z…. distance down the transmission line.

Wave Equation form:
Take d/dz of one equation, and substitute it into the other equation. (Doesn't matter which one)

$-d^2 I(z)/dz^2 = (G' + j\omega C')\ dV(z)/dz$
$dV(z)/dz = -d^2 I(z)/dz^2 / (G' + j\omega C')$

$-dV(z)/dz = (R' + j\omega L')\ I(z)$
$-d^2 I(z)/dz^2 / (G' + j\omega C') = (R' + j\omega L')\ I(z)$
$-d^2 I(z)/dz^2 - ((R' + j\omega L')(G' + j\omega C'))\ I(z) = 0$

$\mathbf{-d^2 I(z)/dz^2 - \gamma^2 I(z) = 0}$

alternate form (derived in text)
$\mathbf{-d^2 V(z)/dz^2 - \gamma^2 V(z) = 0}$

Complex Propagation Constant:
$\gamma = \mathrm{sqrt}((R' + j\omega L')(G' + j\omega C')) = \alpha + j\beta$

attenuation constant:
$\alpha = \mathrm{Re}[(R' + j\omega L')(G' + j\omega C')]$ (Neper/meter)
Passive Lines (all shown here) = zero or positive … exp ($-\alpha z$) attenuation occurs
Active Lines (active region of laser) = negative … gain occurs

Phase constant
$\beta = \mathrm{Im}[(R' + j\omega L')(G' + j\omega C')]$ (radians/meter)
phase shift term

Forward and Backward Waves
Solutions of the basic wave equations have solutions of the form:

$V(z) = Vo^{+} e^{-\gamma z} + Vo^{-} e^{\gamma z}$ volts

$I(z) = Io^{+} e^{-\gamma z} + Io^{-} e^{\gamma z}$ amps

Later, we will see:
$e^{-\gamma z}$ represents forward(+) traveling wave
$e^{\gamma z}$ represents backward(-) traveling wave

Characteristic Impedance

$d/dz\, [V(z)] = d/dz[\, Vo^+ e^{-\gamma z} + Vo^- e^{\gamma z}\ \text{volts}\,] = \gamma\, [Vo^+ e^{-\gamma z} + Vo^- e^{\gamma z}\ \text{volts}]$

Plug into wave equation:

$\gamma\, [-Vo^+ e^{-\gamma z} + Vo^- e^{\gamma z}\ \text{volts}] = (R' + j\omega L')I(z)$

$I(z) = \gamma\, [Vo^+ e^{-\gamma z} - Vo^- e^{\gamma z}\ \text{volts}] / (R' + j\omega L') = [\gamma / (R' + j\omega L')]\, [Vo^+ e^{-\gamma z} - Vo^- e^{\gamma z}\ \text{volts}]$

$Zo = Vo^+ / Io^+ = Vo^- / Io^- = (R' + j\omega L') / \gamma = \text{sqrt}(\, (R' + j\omega L') / (G' + j\omega C')\,)$

Characteristic impedance is the ratio of V/I for forward and backward waves, SEPARATELY. It is not the ration of V/I for the total wave.

$Vo^+ = |\, Vo^+ |\, e^{j\phi+}$

$Vo^- = |\, Vo^- |\, e^{j\phi-}$

$v(z,t) = \text{Real}(\, V(z)\, e^{j\omega t}\,) = \text{Real}[(Vo^+ e^{-\gamma z} + Vo^- e^{\gamma z}\,)\, e^{j\omega t}\,]$
$= \text{Real}\, [|\, Vo^+ |\, e^{j\phi+} e^{j\omega t} e^{-(\alpha + j\beta)z} + |\, Vo^- |\, e^{j\phi-} e^{j\omega t} e^{(\alpha + j\beta)z}\,]$
$= |\, Vo^+ | e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |\, Vo^- | e^{\alpha z} \cos(\omega t + \beta z + \phi^-)$

First term: Traveling wave in +z direction (coefficients of t and z have opposite signs)
Second term: Traveling wave in -z direction (coefficients have same signs)

Propagation Velocity: $v_p = \omega / \beta = f\, \lambda$

Standing Wave (next section):

Combination of +z and -z traveling waves

Example: (exercise 2.4)

A 2-wire air line has R'=.404 mohm/m, L'=2. μH/m , G'=0, C'=5.56 pF
For operation at 5 kHz, determine

(a) Attenuation constant

$\alpha = \text{Re}[\, \text{sqrt}(\, (R' + j\omega L')\, (G' + j\omega C'))] = 3.37 \times 10^{-7}$ Np/m
$\omega = 2\pi f = 2\pi\, 5 \times 10^3$

(b) Phase constant

β = Im[sqrt((R' + jωL') (G' + jωC'))] = 1.05×10^{-4} rad/m

(c) Phase velocity

$v_p = \omega / \beta = 2.99 \times 10^8$ m/s

(d) Characteristic impedance

Zo = sqrt((R' + jωL') / (G'+jωC')) = 600 - j2 ohms = 600 $\angle$-.19° ohms

How would you increase Zo using the same materials?

R' is function of resistance of conductors. Using same materials, you would need MORE metal to increase this. Make wires thicker.

L' is function of inductance between conductors. Bring them closer together to increase L'.

C' similar, but want to decrease it to increase Zo…. TRADEOFFS!