

ECE3300 Lossless Transmission Lines

Lossless Transmission Line:

Transmission lines are characterized by: γ and Z_0 which are a function of R', L', G', C'

To minimize loss:

Use high conductivity materials for conductors and non-conductive materials for insulator.

Result: R', G' are very small.

$R' \ll \omega L', G' \ll \omega C' \rightarrow$ Set $R' = G' = 0$ in equations.

$$\gamma = \alpha + j\beta = j\omega \sqrt{L'C'} \text{ (lossless line)}$$

$$\alpha = 0 \text{ (lossless line)}$$

$$\beta = \omega \sqrt{L'C'} \text{ (lossless line)}$$

$$Z_0 = \sqrt{L'/C'} \text{ (lossless line)} \rightarrow \text{strictly real}$$

$$\text{Wavelength} = \lambda = 2\pi / \beta = 2\pi / \omega \sqrt{L'C'} \text{ (lossless line)}$$

$$\text{Velocity of propagation} = v_p = \omega / \beta = 1 / \sqrt{L'C'} \text{ (lossless line)}$$

For TEM line:

$$L'C' = \mu\epsilon$$

$$\beta = \omega / \sqrt{\mu\epsilon}$$

$$v_p = 1 / \sqrt{\mu\epsilon} = 2.996 \times 10^8 \text{ m/s for air!}$$

$$\epsilon = \epsilon_r \epsilon_0, \mu = \mu_r \mu_0$$

$$v_p = 1 / \sqrt{\mu\epsilon} = c / \sqrt{\epsilon_r}$$

$$\lambda = v_p / f = \lambda_0 / \sqrt{\epsilon_r}$$

THIS IS NOT IN YOUR BOOK:

Dispersion:

Most signals that are sent down transmission lines are not actually sine waves. Usually they are pulses, groups of pulses, etc. These pulses are made up of many different frequencies of sine waves added together. If all of the frequencies travel at the same velocity, then the pulse 'stays together' and keeps its shape. If the frequencies do not travel at the same velocity, the pulse starts to broaden and separate, and is said to "disperse". This is generally not a good thing, because it corrupts the shape of the pulse. Dispersion happens when the phase velocity v_p is a function of frequency. For a general line $v_p = \omega / \beta$, where β is a function of frequency, RLCG (and hence ϵ , σ and μ). If the line is lossless, then it is only a function of ϵ and μ . If ϵ and μ are constant (do not depend on frequency), then so is v_p . BUT (very big BUT), as you will see in your Lab 1, for water-based materials like human tissues, ϵ does depend on frequency (strongly). This means that v_p depends on frequency, and the lines are NOT dispersionless. The pulses will definitely experience dispersion.

Wave Propagation in Lossless Transmission line:

$\alpha, \beta, \gamma, v_p$ do not depend on shape of transmission line! (for lossless TEM lines)

But Z_0 does. Z_0 is strictly real (no phase change).

So, once started V,I propagate the same way. Z_0 controls relative magnitudes of V,I.

Reflected Wave :

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z} \text{ (Observe that the loss term } e^{-\alpha z} \text{ is left out of the equation.)}$$

$$I(z) = (V_o^+ / Z_o) e^{-j\beta z} - (V_o^- / Z_o) e^{j\beta z} \text{ (check signs in previous lecture!)}$$

To find the complete solution, we need to find the voltages of the +z and -z traveling waves.

+z traveling wave comes from the generator

-z traveling wave is reflected from the load.

At the load, $z = 0$ (see figures in book or previous lectures) .. plug into equations above

$$V(z=0) = V_L = V_o^+ e^{-j\beta 0} + V_o^- e^{j\beta 0} = V_o^+ + V_o^-$$

$$I(z=0) = I_L = (V_o^+ / Z_o) e^{-j\beta 0} - (V_o^- / Z_o) e^{j\beta 0} = (V_o^+ / Z_o) - (V_o^- / Z_o)$$

The impedance at the load (which you KNOW because you built or measured the load) is:

$$Z_L = V_L / I_L$$

$$V_L = V(z=0) = V_o^+ + V_o^-$$

$$I_L = I(z=0) = V_o^+ / Z_o - V_o^- / Z_o$$

$$Z_L = V_L / I_L = [(V_o^+ + V_o^-) / (V_o^+ / Z_o - V_o^- / Z_o)] * Z_o$$

$$V_o^- = [(Z_L - Z_o) / (Z_L + Z_o)] * V_o^+$$

Voltage Reflection Coefficient:

(proportion of wave reflected)

$$\Gamma = V_o^- / V_o^+ = [(Z_L - Z_o) / (Z_L + Z_o)] \text{ (ratio – unitless)}$$

$$= (Z_L / Z_o - 1) / (Z_L / Z_o + 1)$$

$$= |\Gamma| \angle \theta_r = |\Gamma| e^{j\theta_r}$$

$$V(z) = V_o^+ e^{-j\beta z} + \Gamma V_o^+ e^{j\beta z}$$

$$I(z) = (V_o^+ / Z_o) [e^{-j\beta z} - \Gamma e^{j\beta z}]$$

Current Reflection Coefficient:

$$I_o^- / I_o^+ = - V_o^- / V_o^+ = - \Gamma$$

Example:

100-ohm line (Z_o) connected to a 50-ohm termination (R_L)

$$\Gamma = (50/100 - 1) / (50/100 + 1) = -1/3 = V_o^- / V_o^+$$

For a 1V sine wave, 1/3 V sine wave is reflected back, out of phase.

Draw. ($V_o^+ = 1$ at open end)

Example:

100-ohm line (Z_o) left open ($R_L = \infty$)

$$\Gamma = (\infty / 100 - 1) / (\infty / 100 + 1) = 1 = V_o^- / V_o^+$$

For a 1V sine wave, 1 V sine wave is $\frac{1}{2}$ reflected back, in phase.

Draw. ($V_o^+ = 1$ at open end, sum = 2V at open end)

Example:

100-ohm line (Z_o) with a short ($R_L = 0$)

$$\Gamma = (0/100 - 1) / (0/100 + 1) = -1 = V_o^- / V_o^+$$

For a 1V sine wave, 1 V sine wave is reflected back, out of phase.

Draw as function of time. ($V_o^+ = 1$ at open end, sum = 0V at open end)

What about elsewhere?

Example: (so how do you get rid of these reflections!)

100-ohm line (Z_o) with $R_L = 100$ ohms

$$\Gamma = (100/100 - 1) / (100/100 + 1) = 0! = V_o^- / V_o^+$$

For a 1V sine wave, 0 V is reflected back

Draw as function of time. ($V_o^+ = 1$ at open end, sum = 1V at open end)

Standing Waves:

We have calculated what happens at one location (end of line), as a function of time.

What about the rest of the line?

Sum of V_o^- and V_o^+ creates a "standing wave".

The total wave on the transmission line is the "interference" of the two waves.

At any point, it is a sine wave (same frequency as incident).

At this point, the magnitude and phase is controlled by the sum of the two waves.

The largest magnitude is when they add in phase (max 2V for 1V incident)

The smallest magnitude is when they add out of phase (min 0V for 1V incident)

Understanding Standing Wave Diagrams:

The diagrams in Figure 2-11,2-12 show the wave at several different times. The ENVELOPE of this wave is often all that is plotted, and inside this envelope, the summed wave is still going up and down.

Velocity of propagation of a pure standing wave? ZERO.

Wavelength of a standing wave? λ

Repetition period? $\lambda / 2$

THIS INFORMATION IS NOT IN YOUR TEXT (see extra notes on standing waves):

Nulls/Peaks

$$V(z) = V_o^+ e^{-j\beta z} + \Gamma V_o^+ e^{j\beta z}$$

$$|V(z)| = |V(z) V^*(z)|^{1/2} = \{ (V_o^+) (e^{-j\beta z} + |\Gamma| e^{j\theta_r} e^{j\beta z}) \} \{ (V_o^+)^* (e^{j\beta z} + |\Gamma| e^{-j\theta_r} V_o^+ e^{-j\beta z}) \}^{1/2}$$

$$= |V_o^+| [1 + |\Gamma|^2 + |\Gamma| (e^{j(2\beta + \theta_r)} + e^{-j(2\beta + \theta_r)})]^{1/2} = |V_o^+| [1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta z + \theta_r)]^{1/2}$$

Magnitude of Peak (maxima):

When cos term is 1

$$|V(z)|_{\max} = |V_o^+| [1 + |\Gamma|^2 + 2|\Gamma| \cos(\theta_r)]^{1/2} = |V_o^+| [1 + |\Gamma|]$$

Location of Peak (minima):

cos term is 1 when: $(2\beta z + \theta_r) = (2\beta (-l_{\max}) + \theta_r) = -2n\pi$ (could be \pm but only - is useful)

Solve for l_{\max} : $-z = l_{\max} = (\theta_r + 2n\pi) / 2\beta$

$$\beta = 2\pi / \lambda$$

$-z = l_{\max} = (\theta_r \lambda / 4\pi + n\lambda / 2)$ for $n=1,2,\dots$ if $\theta_r < 0$; $n=0,1,2,\dots$ if $\theta_r \geq 0$

First voltage Maximum:

$$l_{\max} = \theta_r \lambda / 4\pi$$

Example:

Open circuit

$$\Gamma = (\infty / 100 - 1) / (\infty / 100 + 1) = 1 \angle 0$$

$$l_{\max} = 0$$

See Figure 2-12

Example:

Short circuit

$$\Gamma = (0/100 - 1) / (0/100 + 1) = -1 = 1 \angle \pi = 1 \angle 180^\circ$$

$$l_{\max} = \pi \lambda / 4\pi = \lambda / 4$$

See Figure 2-12

Current Standing Wave

Recall that $I_o^- / I_o^+ = -V_o^- / V_o^+ = -\Gamma$

Current wave is 180 degrees out of phase from voltage wave.

Voltage maximum is current minimum.

Voltage and Current Minima:

Read this section (p. 51-52) in your text.

Theory is same as for maxima.

$$l_{\min} = (-(2n+1)\pi + \theta_r) / 2\beta$$

First minima at $n=0$.

Voltage Standing Wave Ratio (VSWR):

$$S = |V|_{\max} / |V|_{\min} = (1 + |\Gamma|) / (1 - |\Gamma|) \text{ dimensionless}$$

$$\text{Also: } \Gamma = (S-1) / (S+1)$$

S is large when there is a large reflected wave, small when there is a small reflected wave.

It measures the mismatch.

Large S is "bad" for most things.

There is a VSWR meter!

Example:

Open circuit

$$\Gamma = 1 \angle 0$$

$$S = 2 / 0 = \infty$$

Example:

Short circuit

$$\Gamma = (0/100 - 1) / (0/100 + 1) = -1 = 1 \angle \pi = 1 \angle 180^\circ$$

$$S = 2 / 0 = \infty$$

Example:

Perfectly matched line (no reflection)

$$\Gamma = (100/100 - 1) / (100/100 + 1) = 0! = V_o^- / V_o^+$$

$$S = 1 / 1 = 1 \leftarrow \text{This is the best we can do.}$$

Measurements on Transmission Lines:

Measuring Z_L -- Slotted Line Measurements have a probe which will give you $|V(z)|$

Example:

Short circuit shown in Figure 2-12b. For $F=1\text{GHz}$, $\lambda = 0.3\text{m}$

Measure $|V|_{\max}$ and $|V|_{\min}$ and their locations

$$|V|_{\max} = 2$$

$$|V|_{\min} = 0$$

First voltage minima occurs at $z=0$

Second occurs at $z=-0.15\text{m}$

Know Z_o from type of line used ($Z_o = 50$ ohms)

$$S = |V|_{\max} / |V|_{\min} = \infty$$

$$|\Gamma| = (S-1) / (S+1) = 1$$

$$\beta = 2\pi / \lambda = 20.94$$

$$\theta_r - 2\beta l_{\min} = -\pi \text{ (for } n=0\text{)}$$

$$l_{\min} = 0$$

$$\theta_r = -\pi$$

$$\Gamma = 1 e^{j\pi}$$

$$7) Z_L = Z_o [(1 + \Gamma) / (1 - \Gamma)] = 0! \text{ short}$$