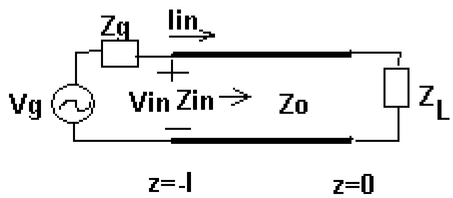
ECE 3300 INPUT IMPEDANCE

Input Impedance

Z_{in} (z) = V(z) / I(z)



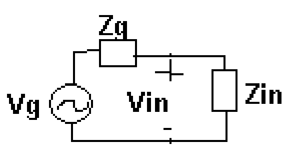
Zin(z) is ratio of TOTAL voltage V(z) and current I(z) on line.
Zo is ratio of forward or negative traveling wave Vo^+ / Io^+ or Vo^- / Io^-

Z_{in} (z) = V(z) / I(z)
$= Vo^+ [e^{-j\beta z} + \Gamma e^{j\beta z}] / (Vo^+ / Zo) [e^{-j\beta z} - \Gamma e^{j\beta z}]$
Divide by $e^{-j\beta z}$
$= Zo [1+ \Gamma e^{j2\beta z}] / [1- \Gamma e^{j2\beta z}]$ ohm

Zin(at the generator) = Zin(z= -l)
$= Zo [1+ \Gamma e^{-j2\beta l}] / [1- \Gamma e^{-j2\beta z}]$ ohm

Using: $e^{j\beta l} = \cos(\beta l) + j \sin(\beta l)$ and $e^{-j\beta l} = \cos(\beta l) - j \sin(\beta l)$

$Zin(-l) = Zo [Z_L \cos(\beta l) + j Zo \sin(\beta l)] / [Z_o \cos(\beta l) + j Z_L \sin(\beta l)]$
$= Zo [Z_L + j Zo \tan(\beta l)] / [Z_o + j Z_L \tan(\beta l)]$



Voltage Divider for Input Voltage

Vin = *Iin* Zin = *Vg* Zin / (Zg + Zin)
Vin = $V(-l) = Vo^+ [e^{j\beta l} + \Gamma e^{-j\beta l}]$

Solve for $Vo^+ = [Vg\ Zin / (Zg + Zin)] / [e^{j\beta l} + \Gamma e^{-j\beta l}]$

Now we know the positive traveling wave based on the generator voltage!
Remember we found relation for Vo^- from Vo^+ last time.
Given the generator voltage and impedance and the load impedance, we can now completely solve for the voltages (and currents) everywhere on the transmission line.

Example: Complete Solution
There is one example in your text. Here is a different one.

Given: a lossless transmission line
Zo = 50 ohms
Length = 90 meters
V_p = 3e8 m/s (air-filled transmission line)
Source: 2 GHz generator with R_g = 150 ohms connected to input terminal
Load: Output terminal left open $R_L = \infty$

λ = vp / f = 3e8 / 2e9 = 0.15 m
$\beta = 2\pi / \lambda$ = 41.89 rad/m
Voltage Reflection Coefficient (at the load)
$\Gamma = (\infty - 50) / (\infty + 50) = 1 \angle 0$

Input Impedance
Zin = Zo $[Z_L + j\,Zo\tan(\beta l)] / [Z_o + j\,Z_L \tan(\beta l)]$ = 0 - j 45 ohms

Generator voltage
v_g (t) = 10 sin(ωt + 30°) volts
convert to cosine
v_g (t) = 10 cos(ωt + (30-90) °) volts
convert to phasor
$V_g = 10 \angle -60°$ volts

Input voltage (total voltage at input terminal)
Vin = Vg Zin / (Zin + Rg) = (4)(- j 45) / (- j 45 + 150) = 2.87 $\angle$46.7° Volt

Magnitude of Forward-traveling wave
$Vo^+ = Vin / [e^{j\beta l} + \Gamma e^{-j\beta l}]$ = 2.14 $\angle$46.7° Volt
$|Vo^+|$ may be <> |Vin |

Phasor (total) Voltage
$V(z) = Vo^+ [e^{-j\beta z} + \Gamma e^{j\beta z}] = 3.20 \angle 180° [e^{-j\beta z} + 1 \angle 0\, e^{j\beta z}]$

Instantaneous (total) Voltage
v(z,t) = Real [V(z)] = 3.20 cos(ωt - βz + 180°) + 3.20 cos(ωt + βz + 180°) volts

Phasor (total) Current
$I(z) = (Vo^+ / Zo) [e^{-j\beta z} - \Gamma e^{j\beta z}]$
$= (3.20 \angle 180° / 50) [e^{-j\beta z} - 1 \angle 0\, e^{j\beta z}] = 0.64 \angle 180° [e^{-j\beta z} - e^{j\beta z}]$

Instantaneous (total) Current
i (z,t) = 0.64 cos(ωt - βz + 180°) + 0.64 cos(ωt + βz + 180°) amps

Electrical Length

Why did Zin = Zo? Because line is electrically so long that generator doesn't "see" the load.

Electrical Length = length / wavelength = 900 meters / .15 meters = 6000 wavelengths long.

What if this generator had been a battery? (DC)

Then wavelength = ∞

$\beta = 0$

$\tan(\beta l) = 0$

$Zin = Zo\ [Z_L + j\ Zo \tan(\beta l)] / [Z_o + j\ Z_L \tan(\beta l)]$

$= 50\ [\infty + 0] / [50 + 0] = \infty = Z_L$!

The line is electrically so SHORT that it is invisible, and the input impedance of the line is the load. This is exactly what happens when you are considering "ordinary" (low frequency) circuits.

Transient Effects

Does this mean there will be NO reflection?? No. There will be a reflection, when the wave finally reaches the end.

BUT? Our cos functions show the incident and reflected waves simultaneously. There is no 6000 cycle delay shown here!

That's because the phasor solution is a STEADY STATE solution. Transients and transient effects are not shown. Are transient effects important? Yes. We'll learn about them next week.

Summary of Lossless Transmission Lines

Lossless Transmission Line

$R' << \omega L'$, $G' << \omega C'$

$\gamma = \alpha + j\ \beta = j\omega \sqrt{(L'C')}$

$\alpha = 0$

$\beta = \omega \sqrt{(L'C')}$

$Zo = \sqrt{(L'/C')}$ → strictly real

$\lambda = 2\pi / \beta = 2\pi / \omega \sqrt{(L'C')}$

$v_p = \omega / \beta = 1/ \sqrt{(L'C')}$

For TEM line:

$L'C' = \mu\varepsilon$ (properties of insulation material)

Lossless TEM line:

$\beta = \omega / \sqrt{(\mu\varepsilon)}$
$v_p = 1/ \sqrt{(\mu\varepsilon)} = c_o / \sqrt{\varepsilon_r}$
$\lambda = v_p / f = \lambda_o / \sqrt{\varepsilon_r}$

Phasor (total steady-state) voltage and current
$V(z) = Vo^+ e^{-j\beta z} + Vo^- e^{j\beta z}$
$= Vo^+ e^{-j\beta z} + \Gamma Vo^+ e^{j\beta z}$
$I(z) = (Vo^+ /Zo) e^{-j\beta z} - (Vo^- /Zo) e^{j\beta z}$
$= (Vo^+ /Zo) [e^{-j\beta z} - \Gamma e^{j\beta z}]$

Voltage and Current at Load
$Z_L = V_L / I_L = [(Vo^+ + Vo^-) / (Vo^+ - Vo^-)] * Zo$
$V_L = V(z=0) = Vo^+ + Vo^-$
$I_L = I(z=0) = Vo^+ / Zo - Vo^- / Zo$

Voltage Reflection Coefficient (at load)
$\Gamma = Vo^- / Vo^+$
$= [(Z_L - Zo) / (Z_L + Zo)]$
$= (Z_L/Zo - 1) / (Z_L/Zo + 1)$
$= |\Gamma| \angle \theta_r = |\Gamma| e^{j\theta_r}$
$= (S-1) / (S+1)$

Current Reflection Coefficient (at load)
$Io^- / Io^+ = - Vo^- / Vo^+ = - \Gamma$

Magnitude of voltage maxima:
$| V(z)|_{max} = | Vo^+| [1 + |\Gamma|]$

Location of voltage maxima:
$-z = l_{max} = (\theta_r \lambda / 4\pi + n\lambda / 2)$ for n=1,2,… if $\theta_r < 0$; n=0,1,2,… if $\theta_r \geq 0$
First voltage Maximum: $L_{max} = \theta_r \lambda / 4\pi$

Location of voltage minima:
$L_{min} = (-(2n+1)\pi + \theta_r) / 2\beta$
First minima at n=0.

Voltage Standing Wave Ratio (VSWR):
$S = | V |_{max} / | V |_{min} = (1 + |\Gamma|) / (1 - |\Gamma|)$ dimensionless

Input Impedance
$Z_{in} (z) = V(z) / I(z)$
$= Zo [1+ \Gamma e^{j2\beta z}] / [1- \Gamma e^{j2\beta z}]$ ohm

Zin(at the generator) = Zin(z= -l)

$= Zo\ [\ 1+ \Gamma\ e^{-j2\beta\ l}] / [\ 1- \Gamma\ e^{-j2\beta\ z}]$ ohm
$= Zo\ [Z_L + j\ Zo\ \tan(\beta l)] / [Z_o + j\ Z_L \tan(\beta l)]$

Input Voltage
$Vin = Iin\ Zin = Vg\ Zin / (Zg + Zin)$
$Vin = V(-l) = Vo^{+}\ [e^{j\beta\ l} + \Gamma\ e^{-j\beta\ l}]$

Positive-traveling wave
$Vo^{+} = [Vg\ Zin / (Zg + Zin)] / [e^{j\beta\ l} + \Gamma\ e^{-j\beta\ l}]$

Negative-traveling wave
$Vo^{-} = [(Z_L - Zo) / (Z_L + Zo)] * Vo^{+}$