

## ECE 3300 SPECIAL CASES OF TRANSMISSION LINES

Motivation: Recall that a "capacitor" does not function as a capacitor, and an "inductor" does not function as an inductor at microwave frequencies. But if we want to build a filter, for instance, at microwave frequencies, we need C and L. How do we get them? We will see that lengths of shorted or open transmission lines can be used for C and L.

### SPECIAL CASES OF TRANSMISSION LINES

#### Short-Circuited Line



For a short-circuited transmission line:

$$Z_L = 0$$

$$\Gamma = [Z_L - Z_0] / [Z_L + Z_0] = -1 = 1 \angle 180^\circ$$

$$S \text{ (standing wave ratio)} = |V|_{\max} / |V|_{\min} = \infty$$

$$V_{sc}(z) = V_o^+ [e^{-j\beta z} - e^{j\beta z}] = -2j V_o^+ \sin(\beta z) \text{ volts}$$

$$I_{sc}(z) = (V_o^+ / Z_0) [e^{-j\beta z} + e^{j\beta z}] = 2 (V_o^+ / Z_0) \cos(\beta z) \text{ amps}$$

If you think of this from "ordinary" circuit perspective, what should  $V_{sc}(z)$  be on the load?  
ZERO!

$$\text{For } z=0 \quad V_{sc}(0) = -2j V_o^+ \sin(\beta 0) = 0!$$

What should  $I_{sc}(z)$  be?

MAXIMUM!

$$I_{sc}(0) = 2 (V_o^+ / Z_0) \cos(\beta 0) \text{ amps} = 2 (V_o^+ / Z_0)$$

Input Impedance (impedance at input terminals)

$$Z_{in}^{sc} = V_{sc}(z=-1) / I_{sc}(z=-1) = j Z_0 \tan(\beta l) \leftarrow \text{Strictly reactive}$$

Other reactive impedances:

$$\text{Capacitor} \quad Z_C = 1 / (j\omega C) = -j / \omega C$$

$$\text{Inductor} \quad Z_L = j\omega L$$

So, if  $Z_{in}$  has POSITIVE reactive value, it can act like an equivalent inductor  $Leq$

If it has NEGATIVE reactive value, it acts like an equivalent capacitor  $Ceq$

**Equivalent Inductor:**

$$j\omega Leq = jZ_0 \tan(\beta l) \quad \text{if } \tan(\beta l) \geq 0$$

$$L_{eq} = Z_0 \tan(\beta l) / \omega \quad \text{henries}$$

To create an equivalent inductor, we use the shortest possible line:

$$l_{min} = (1/\beta) \tan^{-1} (\omega L_{eq} / Z_0) \quad \text{meters}$$

### **Equivalent Capacitor**

$$1 / (j\omega C_{eq}) = jZ_0 \tan(\beta l) \quad \text{if } \tan(\beta l) \leq 0$$

$$C_{eq} = -1 / [ Z_0 \omega \tan(\beta l) ] \quad \text{farads}$$

To create an equivalent capacitor, we use the shortest possible line (must have positive physical length):

$$l_{min} = (1/\beta) [\pi - \tan^{-1} (\omega C_{eq} / Z_0) ] \quad \text{meters}$$

**Example** (book has example of capacitance)

Create a stub-line which will act as an equivalent inductance of 1 nHenry

Given:  $Z_0 = 50 \text{ ohms}$ ,  $F = 3 \text{ GHz}$ , air-filled line

$$\lambda = c / f = 3 \times 10^8 / 3 \times 10^9 = 10 \text{ cm}$$

$$\beta = 2\pi / \lambda = 62.83 \text{ rad/m}$$

$$\omega = 2\pi F = 1.88 \times 10^{10} \text{ rad/meter}$$

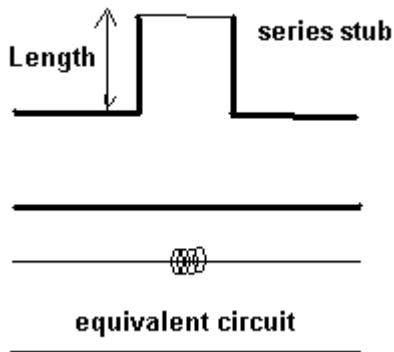
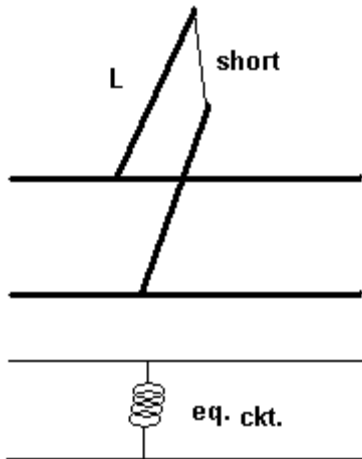
$$l_{min} = (1/\beta) \tan^{-1} (\omega L_{eq} / Z_0) \quad \text{meters}$$

$$= (1/ 62.83 \text{ rad/m}) \tan^{-1} (1.88 \times 10^{10} \text{ rad/meter} * 1 \text{ nHenry} / 50 \text{ ohms})$$

$$= 5.7 \text{ mm}$$

**Connection of parallel and series stubs** (not covered in text)

How do you connect these things?



What controls whether a stub is capacitive or

How do you tell if a stub is capacitive or inductive?

The LENGTH

### Open-circuited line

$$\Gamma=1$$

$$V_{oc}(z) = V_o^+ [ e^{-j\beta z} + e^{j\beta z} ] = 2 V_o^+ \cos(\beta z) \text{ volts}$$

$$I_{oc}(z) = (V_o^+ / Z_o) [ e^{-j\beta z} - e^{j\beta z} ] = -2j ( V_o^+ / Z_o) \sin(\beta z) \text{ amps}$$

$$Z_{in}^{oc} = V_{oc}(z=-1) / I_{oc}(z=-1) = -j Z_o \cot(\beta l) \leftarrow \text{Strictly reactive}$$

Open circuited line can ALSO be used to create inductance and capacitance.

In practice this is rarely done, because an "open" line is difficult to create. Fringing fields (fields fringing around the edges of an open end) create capacitance at the end of the open.

### Measurement Techniques using Open and Shorts

How do you measure the characteristic impedance  $Z_0$ ? (We BUY lines with this spec.)

$$Z_{in}^{oc} = V_{oc}(z=-1) / I_{oc}(z=-1) = -j Z_0 \cot(\beta l)$$

$$Z_{in}^{sc} = V_{sc}(z=-1) / I_{sc}(z=-1) = j Z_0 \tan(\beta l)$$

$$Z_{in}^{oc} Z_{in}^{sc} = Z_0^2 \text{ (multiplying equations)}$$

$$Z_0 = \sqrt{Z_{in}^{oc} Z_{in}^{sc}}$$

$$\tan(\beta l) = \sqrt{-Z_{in}^{sc} / Z_{in}^{oc}}$$