## ECE 3510 Exam 1 given: Spring 06

1. (24 pts) Find the inverse Laplace transform of the following function:

Use partial fraction expansion and the tables.
Show all your work to get credit.

$$
F(s):=\frac{1}{(s+2) \cdot(s+1)^{2}}
$$

2. (15 pts) Find the transfer function $H(s)=\frac{\mathrm{V}_{\mathrm{o}}(\mathrm{s})}{\mathrm{V}_{\mathrm{i}}(\mathrm{s})}$ for this circuit.

Write $\mathrm{H}(\mathrm{s})$ in the normal form, as a ratio of polynomials.
Don't divide the polynomials. Initial conditions are 0.
You MUST show work to get credit.

3. (5 pts) In the schematic at right,

What is the purpose of diodes $D_{5}$ and $D_{6}$ ?

4. (21 pts) For the feedback system shown below, find the transfer function of the whole system, with feedback.

Find $\quad H(s)=\frac{\mathrm{Y}_{\text {out }^{(s)}}}{\mathrm{X}_{\mathrm{in}}(\mathrm{s})} \quad \begin{aligned} & \text { Simplify your expression for } \mathrm{H}(\mathrm{s}) \text { so that he numerator } \\ & \text { and denominator are both simple polynomials }\end{aligned}$


## ECE 3510 Exam 1 Spring 06 p2

5. ( 25 pts ) Consider the step response of a system has the following transfer function:
$H(s)=\frac{s+15 \cdot k}{s^{2}+6 \cdot s+5 \cdot k}$
Step input: $\quad x(t)=2 \cdot u(t)$
a) What is the DC gain of this system?
b) What is the final value of the output (steady-state value)?
c) What value of k will make the output critically damped? (you may leave k in the form of a fraction)
d) $k=1$, the output is: (Circle one)
A) underdamped
B) critically damped
C) overdamped
e) $k=5$, the output is: (Circle one)
A) underdamped
B) critically damped
C) overdamped
f) For the underdamped case above (d or e), Express H(s) in the following form. Insert the value of k and find the values of a and b .

$$
H(s)=\frac{s+15 \cdot k}{(s+a)^{2}+b^{2}}
$$

g) Draw the poles from the system of part f).

Make sure I can tell the values of the real \& imaginary parts.
6. (10 pts) This system:

$$
H(s)=\frac{k}{s+a}
$$

Has this input:

$$
\text { Cosine input: } \quad x(t)=\cos \left(\omega_{0} \cdot t\right) \cdot u(t)
$$

Resulting in this output: $\quad \mathrm{Y}(\mathrm{s})=\frac{\mathrm{k}}{(\mathrm{s}+\mathrm{a})} \cdot \frac{\mathrm{s}}{\left(\mathrm{s}^{2}+\omega_{\mathrm{o}}{ }^{2}\right)}$
This separates into 3 partial fractions that you can find in the laplace transform table. Show what they are, but don' t find the coefficients.


Continue with the partial fraction expansion just far enough to find the transient coefficient.

## Answers

1. $\left(e^{-2 \cdot t}-e^{-t}+t \cdot e^{-t}\right) \cdot u(t)$
2. $\frac{s^{2}}{s^{2}+\frac{1}{C \cdot R} \cdot s+\frac{1}{L \cdot C}}$
3. The motor is an inductive load, so its current cannot change instantaneously. This can be a significant problem when the transistors suddenly shut off. The "flyback" diodes provide a harmless path for the current to decay to zero.
4. $\frac{8 \cdot \mathrm{~s}+40}{\mathrm{~s}^{2}+15 \cdot \mathrm{~s}+38}$
5. a) 3
b) 6
c) 1.8
d) C) overdamped
e) A) underdamped f) $\frac{s+75}{(s+3)^{2}+4^{2}}$
6. $\frac{-\mathrm{k} \cdot \mathrm{a}}{\mathrm{a}^{2}+\omega_{\mathrm{o}}{ }^{2}}$

