ECE 3510 Exam 1 given: Spring 07

(The space between problems has been removed.)

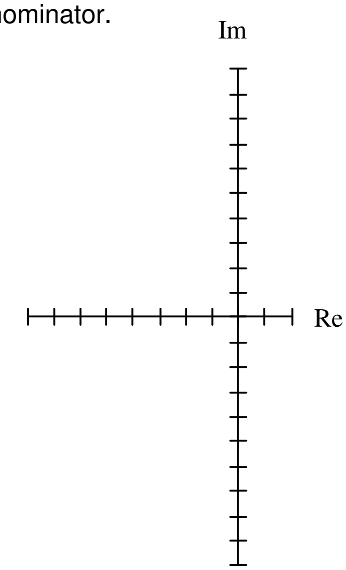
1. (15 pts) a) Find the Laplace transform of the time function, below. Use the table attached to this exam.

   Combine any pieces you may find so that they have one common denominator.

   $$f(t) = e^{-3\cdot t}\cdot(2\cdot\cos(6\cdot t) - 4\cdot\sin(6\cdot t))\cdot u(t)$$

   b) List the poles of the Laplace transform.
   Indicate multiple poles if there are any.

   c) Draw the poles on the s-plane at right.
   Make sure I can tell the values of the real & imaginary parts.



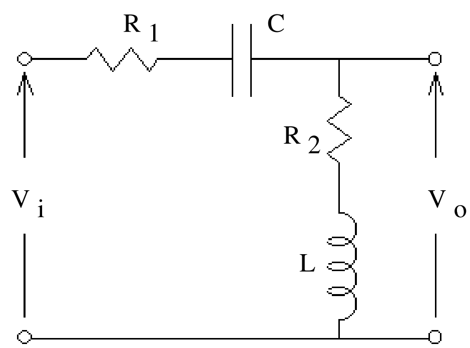
2. (18 pts) Find the inverse Laplace transform of the following function:

   Use partial fraction expansion and the tables.

   Show all your work to get credit.

   $$F(s) := \frac{s^2}{(s+1)\cdot(s+2)^2}$$

3. (15 pts) a) Find the transfer function, $H(s) = \dfrac{V_o(s)}{V_i(s)}$ for this circuit.



   Write $H(s)$ in the normal form (as a ratio of polynomials with no coefficient before the highest-order term in the denominator). Don't divide the polynomials.
   Initial conditions are 0.

   You <u>MUST</u> show work to get credit.

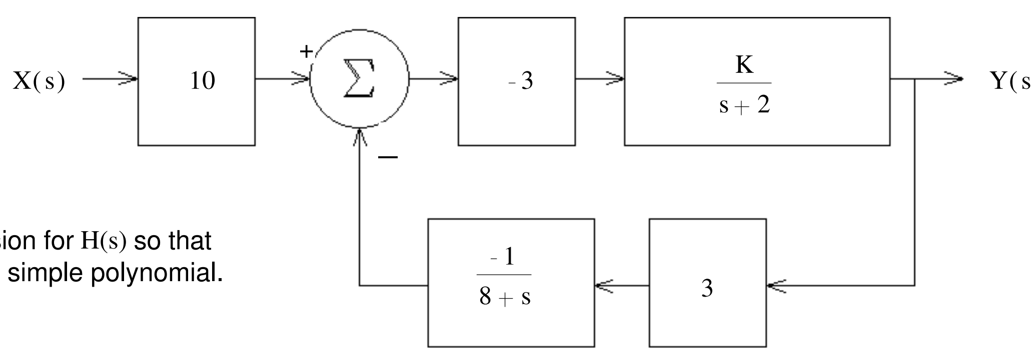
   $H(s) = ?$

   b) Find the characteristic equation of the circuit shown.

   c) The solutions to the characteristic equation are called the ____________ of the transfer function.

   d) Does the transfer function have one or more zeros? If yes, express it (them) in terms of R_1, R_2, C, & L.

4. (19 pts) a) A feedback system is shown in the figure. What is the transfer function of the whole system, with feedback.



   $$H(s) = \frac{Y(s)}{X(s)} = ?$$

   Simplify your expression for $H(s)$ so that the denominator is a simple polynomial.

   b) Find the value of K to make the transfer function critically damped. Answer may be left as a fraction.

   c) If K is **Greater** than this value the system will be: underdamped or overdamped

   Circle one

   d) Does the transfer function have any zeros? Answer no or find the s value(s) of the zero(s).

5. (5 pts) A system has this transfer function: $H(s) = \dfrac{8}{s^2 + 5 \cdot s + 4}$

   What is the steady-state response ($y_{ss}(t)$) of this system to the input: $x(t) = 3 \cdot u(t)$

6. (9 pts) A system has this transfer function: $H(s) = \dfrac{8}{s + 4}$

   What is the steady-state response ($y_{ss}(t)$) of this system to the input: $x(t) = \sqrt{2} \cdot \cos(4 \cdot t) \cdot u(t)$

7. (7 pts) Express the following steady-state signal in the time domain, as a sum of cosine and sine with no phase angles:

   $Y(j\omega) = 4 + 2 \cdot j$ $\qquad \omega := 12 \cdot \dfrac{\text{rad}}{\text{sec}}$

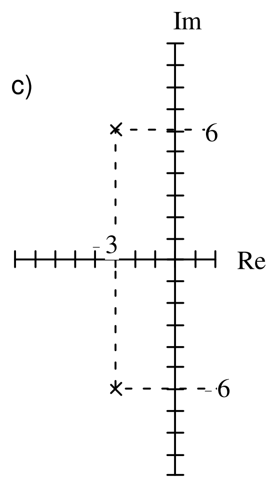
8. (12 pts) Find the steady-state (sinusoidal) magnitude and phase of the following transfer function.

   $\left|H(j \cdot \omega)\right| = ?$ $\qquad \underline{/H(j\omega)} = ?$ The phase angle may be reported as $\tan^{-1}\left(\dfrac{b}{a}\right)$ where b and a are shown as numbers

   $\omega := 8 \cdot \dfrac{\text{rad}}{\text{sec}}$ $\qquad H(s) = \dfrac{\dfrac{5}{\text{sec}} \cdot s - \dfrac{30}{\text{sec}^2}}{s^2 + \dfrac{54}{\text{sec}^2}}$

Answers

1.a) $\dfrac{2 \cdot s - 18}{(s+3)^2 + 6^2}$ OR: $\dfrac{2 \cdot s - 18}{s^2 + 6 \cdot s + 45}$ $\qquad$ b) $-3 + 6 \cdot j \qquad -3 - 6 \cdot j$ $\qquad$ c)



2. $\left(e^{-1 \cdot t} - 4 \cdot t \cdot e^{-2 \cdot t}\right) \cdot u(t)$

3. a) $\dfrac{s^2 + \dfrac{R_2}{L} \cdot s}{s^2 + \left(\dfrac{R_1 + R_2}{L}\right) \cdot s + \dfrac{1}{L \cdot C}}$ $\qquad$ b) $0 = s^2 + \left(\dfrac{R_1 + R_2}{L}\right) \cdot s + \dfrac{1}{L \cdot C}$

   c) poles $\qquad$ d) 0 and $-\dfrac{R_2}{L}$

4. a) $\dfrac{-30 \cdot K \cdot (s+8)}{s^2 + 10 \cdot s + 16 + 9 \cdot K}$ $\qquad$ b) 1

   c) underdamped $\qquad$ d) -8

5. 6

6. $2 \cdot \cos(4 \cdot t - 45 \cdot \text{deg})$

7. $4 \cdot \cos(12 \cdot t) - 2 \cdot \sin(12 \cdot t)$

8. $5 \qquad \tan^{-1}\left(-\dfrac{4}{3}\right)$

ECE 3510 Midterm #1 Arn Stolp

Name________________________________

Scores:

Pages 1&2 _______ of a possible 33 pts

Pages 3&4 _______ of a possible 34 pts

Pages 5&6 _______ of a possible 33 pts

Total _______ of a possible 100 pts

Information

Complex numbers $\mathtt{A} = a + b\cdot j$ $\quad \mathtt{A} = A\cdot e^{j\cdot\theta}$ $\quad \mathtt{D} = c + d\cdot j$ $\quad \mathtt{D} = D\cdot e^{j\cdot\phi}$

Conversions

$$A = |\mathtt{A}| = \sqrt{a^2 + b^2} \qquad \theta = \arg(\mathtt{A}) = \operatorname{atan}\left(\frac{b}{a}\right)$$

$$a = A\cdot\cos(\theta) \qquad b = A\cdot\sin(\theta)$$

Addition and Subtraction

$$\mathtt{A} + \mathtt{D} = (a + b\cdot j) + (c + d\cdot j) = (a + c) + (b + d)\cdot j$$

$$\mathtt{A} - \mathtt{D} = (a + b\cdot j) - (c + d\cdot j) = (a - c) + (b - d)\cdot j$$

Multiplication

$$\mathtt{A}\cdot\mathtt{D} = (a + b\cdot j)\cdot(c + d\cdot j) = (a\cdot c - b\cdot d) + (b\cdot c + a\cdot d)\cdot j$$

$$\mathtt{A}\cdot\mathtt{D} = A\cdot e^{j\cdot\theta}\cdot D\cdot e^{j\cdot\phi} = A\cdot D\cdot e^{j\cdot(\theta+\phi)}$$

Division

Rectangular:

$$\frac{\mathtt{A}}{\mathtt{D}} = \frac{a + b\cdot j}{c + d\cdot j} = \frac{a + b\cdot j}{c + d\cdot j}\cdot\frac{c - d\cdot j}{c - d\cdot j} = \frac{a\cdot c + b\cdot d}{c^2 + d^2} + \frac{b\cdot c - a\cdot d}{c^2 + d^2}\cdot j$$

$$\frac{1}{c + d\cdot j}\cdot\frac{c - d\cdot j}{c - d\cdot j} = \frac{c}{c^2 + d^2} - \frac{d}{c^2 + d^2}\cdot j$$

Polar:

$$\frac{\mathtt{A}}{\mathtt{D}} = \frac{A\cdot e^{j\cdot\theta}}{D\cdot e^{j\cdot\phi}} = \frac{A}{D}\cdot e^{j\cdot(\theta-\phi)}$$

Misc Information

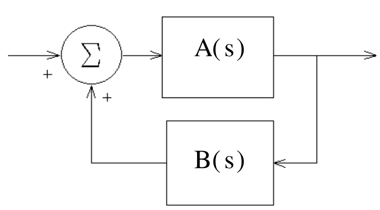
$$0 = a\cdot x^2 + b\cdot x + c \qquad x = \frac{-b + \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a}$$

$b^2 - 4\cdot a\cdot c > 0$ overdamped

$b^2 - 4\cdot a\cdot c = 0$ critically damped

$b^2 - 4\cdot a\cdot c < 0$ under damped

Standard feedback loop transfer function



$$\frac{A(s)}{1 - A(s)\cdot B(s)}$$

$$Y(s) = \frac{b_2\cdot s^2 + b_1\cdot s + b_0}{s^2 + a_1\cdot s + a_0}\cdot X(s) + \frac{s\cdot y(0) + \frac{d}{dt}y(0) + a_1\cdot y(0) - b_2\cdot s\cdot x(0) - b_2\cdot s\cdot\frac{d}{dt}x(0) - b_1\cdot s\cdot y(0)}{s^2 + a_1\cdot s + a_0}$$

A <u>Laplace transform table</u> and a <u>Laplace property table</u> were also included with the exam.