1. (27pts) This problem refers to the crude servo that you worked with in the first lab. It is essentially the same as the servo that I demonstrated in class the very first day.

a) Draw a system block diagram, identifying each of the parts in generic terms (controller, plant, feedback sensor, and possibly others) and in specific terms (Motor, amplifier, etc.).

b) Identify the input and output on the drawing. Show what the input and output are by giving me the units of each. (Don't worry if more than one answer is possible, just give one.)

c) Identify all the other lines between all the blocks by the signal at that part of the block diagram. Include units.

In the lab you made the system responded in 4 different ways, depending on the gain. Give your answers below in order from lowest to highest gain. (One of these you probably didn't actually see, so pretend you did.)

d) What were the responses that you saw. Use the word "damped" in 3 of your answers

e) Show a typical time-domain step response in the for each.

f) Show transfer-function pole locations for each. The real and Imaginary axis have the same scale. Make your pole locations fit your time-response.

1. (lowest gain)

2.

3.

4. (Highest gain)
2. (10 pts) Find the inverse Laplace transform of the following function: 
\[ F(s) = \frac{s + 3}{s(s + 2)(s + 4)} \]

Use partial fraction expansion and the tables. 
Show all your work to get credit.

3. (3 pts) What is the time constant of this expression? 
\[ \frac{1}{5} e^{-10t} \] 
\[ \tau = ? \]

4. (18 pts) a) Find the transfer function, 
\[ H(s) = \frac{V_o(s)}{V_i(s)} \]
for this circuit.

Write \( H(s) \) in the normal form (as a ratio of polynomials with no coefficient before the highest-order term in the denominator). Don't divide the polynomials.
Initial conditions are 0.
You MUST show work to get credit.

\[ H(s) = ? \]

b) Find the characteristic equation of the circuit shown.
c) The solutions to the characteristic equation are called the _________ of the transfer function.
d) Does the transfer function have one or more zeros? If yes, express it (them) in terms of \( R_1, R_2, C, \) & \( L \).

5. (14 pts) a) A feedback system is shown in the figure. What is the transfer function of the whole system, with feedback.

\[ H(s) = \frac{X_{out}(s)}{X_{in}(s)} = ? \]

Simplify your expression for \( H(s) \) so that the denominator is a simple polynomial.
b) Find the value of \( F \) to make the transfer function critically damped.

c) Does the transfer function have one or more zeros? If yes, express it (them) in terms of \( R_1, R_2, C, \) & \( L \).

6. (8 pts) A system has this transfer function:
\[ H(s) = \frac{8s + 2}{s^2 + 6s + 4} \]

What is the steady-state response \( y_{ss}(t) \) of this system to the input 
\[ x(t) = (3 + 5e^{-4t})u(t) \]

7. (12 pts) This system:
\[ H(s) = \frac{2}{s + 10} \]
Has this input:
Cosine input: \[ x(t) = 5\cos(20t)u(t) \]
\[ X(s) = \frac{5s}{s^2 + 200} \]

Resulting in this output:
\[ Y(s) = \frac{2}{(s + 10)} \cdot \frac{5s}{s^2 + 400} \]

a) This separates into 3 partial fractions that you can find in the laplace transform table.

\[ \text{Show what they are, but don't find the coefficients.} \]

\[ Y(s) = \frac{2}{s + 10}, \frac{5s}{s^2 + 400} = \]

b) Continue with the partial fraction expansion just far enough to find the transient coefficient as a number.
c) Express the transient part as a function of time. 
\[ y_{tr}(t) = ? \]
8. (10 pts) This system: \( H(s) = \frac{4}{s + 6} \) Has this Cosine input: 
\[ x(t) = 5 \cdot \cos(8 \cdot t + 40 \text{deg}) \cdot u(t) \]

Use steady-state AC analysis to find the steady-state response \( y_{ss}(t) \) of the system.

**Answers**

1. \[ \theta_{in} \] to \[ \theta_{out} \]

- **Controller**
- **Plant**

- **Input Position**
- **Feedback Signal**
- **Output Position**

2. \( \left( 3 \cdot \frac{1 - e^{-2t}}{8} - \frac{e^{-4t}}{8} \right) \cdot u(t) \)

3. **Under damped**

4. **Critically Damped**

5. **(Highest gain)**

6. **100 ms**

7. \[ \frac{A}{s + 10} + \frac{B}{(s^2 + 400)} + \frac{C}{s} \]

8. \( 0.2 \cdot \cos(8 \cdot t - 13.3 \text{deg}) \)