1. (13 pts) Find $Z_{\text{eq}}(j\omega)$. Reduce your answer to a simple complex number.

$$\omega := \frac{2000}{\text{rad/sec}}$$

$$Z_{\text{eq}}(j\omega) = ?$$

2. (12 pts) Find the steady-state (sinusoidal) magnitude and phase of the following transfer function.

$$|H(j\omega)| = ?, \quad \angle H(j\omega) = ?$$

$$\omega := \frac{10}{\text{rad/sec}}$$

$$H(s) = \frac{40}{s^2 + \frac{300}{\text{sec}^2}} - \frac{90}{s^2 + \frac{90}{\text{sec}^2}}$$

3. (6 pts) Express the following signal in the time domain, as a sum of cosine and sine with no phase angles:

$$Y(s) = 3 + 0.5j$$

$$\omega := \frac{10}{\text{rad/sec}}$$

4. (13 pts) Find the equivalent electric circuit for the mechanical system shown. $T_{\text{in}}$ is the input.

a) show the circuit with a transformer.

b) Show the circuit without a transformer, just like you did in the homework.

5. (6 pts) The transfer functions of $C(s)$ and $P(s)$ are given below. In each case determine if the steady-state tracking error will go to zero and whether disturbances will be completely rejected. You may assume closed-loop stability. Give reasons for your answers.

<table>
<thead>
<tr>
<th>$C(s)$</th>
<th>$P(s)$</th>
<th>0 steady-state err?</th>
<th>Reject disturbance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{s+1}{s^3 + 7s^2 + 12s}$</td>
<td>$\frac{s+1}{s+3}$</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Why?</td>
<td></td>
</tr>
<tr>
<td>$\frac{s+4}{s^2 + 3s + 2}$</td>
<td>$\frac{s+1}{s^2 + 3s}$</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Why?</td>
<td></td>
</tr>
</tbody>
</table>

6. (12 pts) Sketch the root-locus plots for the following open-loop transfer functions:

Use only the main rules, that is, don’t sweat the details like breakaway points and departure angles.

If you calculate anything (like a centroid) be sure to show your work.

<table>
<thead>
<tr>
<th>$C(s)$</th>
<th>$P(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{(s-1)(s+3)}{(s+1)^2(s+5)}$</td>
<td></td>
</tr>
</tbody>
</table>

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ECE 3510 Exam 2 given: Spring 06
b) \( G(s) = \frac{1}{s(s + 2)(s + 4)} \)

7. (20 pts) Sketch the root-locus plots for the following open-loop transfer functions:

Use only the main rules, that is, don't sweat the details like breakaway points and departure angles.

If you calculate anything (like a centroid) be sure to show your work.

a) \( G(s) = \frac{s + 2}{s(s + 5)(s + 7)} \)

b) A compensator is added with a pole at -1 and a zero at -3, how does this change the root locus? (draw again)

\( G(s) = \frac{(s + 2)(s + 3)}{s(s + 1)(s + 5)(s + 7)} \)

c) Can the system response be faster with the compensator? yes no

Why or why not?

8. (18 pts) A root-locus is sketched at right.

The open-loop transfer function has one zero at \( s = -1 \) and two poles at \( s = 1 \).

\( G(s) = \frac{s + 1}{(s - 1)^2} \)

a) Find the "break-away" point on the real axis.

b) Assume that the root-locus crosses the \( j\omega \) axis at \( \sqrt{3} \)

Determine if this is true. Show your work.

c) Regardless of what you found in part b, continue to assume that the root-locus crosses the \( j\omega \) axis at \( \sqrt{3} \)

Give the range of gain \( k (k > 0) \) for which the system is closed-loop stable.

Some angle relations you may find useful
1. $0.25 + 0.55j \ \Omega$

2. $|H(j\omega)| = 50 \quad \angle H(j\omega) = \tan^{-1}\left(\frac{4}{3}\right)$

3. $3 \cos\left(10\frac{\text{rad}}{\text{sec}}t\right) - 0.5 \sin\left(10\frac{\text{rad}}{\text{sec}}t\right)$

4. a)

$$\frac{1}{b_1} \quad N = \frac{r_2}{r_1} \quad \frac{1}{k} \quad \frac{1}{b_2}$$

b)

$$\frac{1}{b_1} \quad \frac{(r_2)^2}{r_1} \cdot \frac{1}{k} \quad \frac{(r_2)^2}{r_1} \cdot \frac{1}{b_2}$$

5. a) Yes, $C(s)$ has pole at zero

b) Yes, $P(s)$ has pole at zero

No, $C(s)$ has no pole at zero

6. a)

b)

7. a)

b)

8. a) -3  
   b) Yes 
   c) $k > 2$  
   c) Yes  
   You can move the pole nearest the $j\omega$ axis farther from the $j\omega$ axis.