## ECE 3510 Exam 2 given: Spring 06

(The space between problems has been removed.)

1. (13 pts) Find $\mathbf{Z e q}(\mathrm{j} \omega$ ) Reduce your answer to a simple complex number.
$\omega:=2000 \cdot \frac{\mathrm{rad}}{\mathrm{sec}}$
$\mathbf{Z}_{\mathbf{e q}}(\mathrm{j} \omega)=?$

2. (12 pts) Find the steady-state (sinusoidal) magnitude and phase of the following transfer function.
$|\mathrm{H}(\mathrm{j} \cdot \omega)|=$ ?
$\underline{H}(\mathrm{j} \omega)=$ ?
$H(s)=\frac{\frac{40}{\sec } \cdot s-\frac{300}{\sec ^{2}}}{s^{2}+\frac{90}{\sec ^{2}}}$
The phase angle may be reported as $\tan ^{-1}\left(\frac{b}{a}\right)$
$\omega:=10 \cdot \frac{\mathrm{rad}}{\mathrm{sec}}$
$Y(s)=3+0.5 \cdot j$
$\omega:=10 \cdot \frac{\mathrm{rad}}{\mathrm{sec}}$
3. (13 pts) Find the equivalent electric circuit for the mechanical system shown. $\mathrm{T}_{\mathrm{in}}$ is the input.

a) show the circuit with a transformer.
b) Show the circuit without a transformer, just like you did in the homework.
4. (6 pts) The transfer functions of $\mathrm{C}(\mathrm{s})$ and $\mathrm{P}(\mathrm{s})$ are given below. In each case determine if the steady-state tracking error will go to zero and whether disturbances will be completely rejected. You may assume
closed-loop stability. Give reasons for your answers.

0 steady-state err.?
a) $\mathrm{C}(\mathrm{s})=\frac{\mathrm{s}+1}{\mathrm{~s}^{3}+7 \cdot \mathrm{~s}^{2}+12 \cdot \mathrm{~s}} \quad \mathrm{P}(\mathrm{s})=\frac{\mathrm{s}+1}{\mathrm{~s}+3}$
yes no
Why? $\qquad$
b) $C(s)=\frac{s+4}{s^{2}+3 \cdot s+2}$

$$
\mathrm{P}(\mathrm{~s})=\frac{\mathrm{s}+1}{\mathrm{~s}^{2}+3 \cdot \mathrm{~s}}
$$

Why? $\qquad$
6. (12 pts) Sketch the root-locus plots for the following open-loop transfer functions:

Use only the main rules, that is, don't sweat the details like breakaway points and departure angles.
If you calculate anything (like a centroid) be sure to show your work.
a) $\mathrm{G}(\mathrm{s})=\frac{(\mathrm{s}-1) \cdot(\mathrm{s}+3)}{(\mathrm{s}+1)^{2} \cdot(\mathrm{~s}+5)}$
b) $\mathrm{G}(\mathrm{s})=\frac{1}{\mathrm{~s} \cdot(\mathrm{~s}+2) \cdot(\mathrm{s}+4)}$
7. (20 pts) Sketch the root-locus plots for the following open-loop transfer functions:
Use only the main rules, that is, don't sweat the details like breakaway points and departure angles.

If you calculate anything (like a centroid) be sure to show your work.

a) $\mathrm{G}(\mathrm{s})=\frac{\mathrm{s}+2}{\mathrm{~s} \cdot(\mathrm{~s}+5) \cdot(\mathrm{s}+7)}$
b) A compensator is added with a pole at -1 and a zero at -3 , how does this change the root locus? (draw again)
c) Can the system response be faster with the compensator? yes no
Why or why not?


$$
G(s)=\frac{(s+2) \cdot(s+3)}{s \cdot(s+1) \cdot(s+5) \cdot(s+7)}
$$

(18 pts) A root - locus is sketched at right.
The open - loop transfer function has one zero at $\mathrm{s}=-1$ and two poles at $\mathrm{s}=1$.

$$
G(s)=\frac{s+1}{(s-1)^{2}}
$$

a) Find the "break-away" point on the real axis.
b) Assume that the root-locus crosses the $\mathrm{j} \omega$ axis at $\sqrt{3}$

Determine if this is true. Show your work.
c) Regardless of what you found in part b, continue to assume that the root-locus crosses the $\mathrm{j} \omega$ axis at $\sqrt{3}$
Give the range of gain $k(k>0)$ for which the system is closed-loop stable.


## Answers

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1. $0.25+0.55 \cdot \mathrm{j} \Omega$
2. $|\mathrm{H}(\mathrm{j} \cdot \omega)|=50$

$$
\mu H(j \omega)=\tan ^{-1}\left(-\frac{4}{3}\right)
$$

3. $3 \cdot \cos \left(10 \cdot \frac{\mathrm{rad}}{\mathrm{sec}} \cdot \mathrm{t}\right)-0.5 \cdot \sin \left(10 \cdot \frac{\mathrm{rad}}{\mathrm{sec}} \cdot \mathrm{t}\right)$
4. a)

b)

$$
\frac{1}{\mathrm{~b}_{1}} \quad\left(\frac{\mathrm{r}_{2}}{\mathrm{r}_{1}}\right)^{2} \cdot \frac{1}{\mathrm{k}} \quad\left(\frac{\mathrm{r}_{2}}{\mathrm{r}_{1}}\right)^{2} \cdot \frac{1}{\mathrm{~b}_{2}}
$$




5. a) Yes, $C(s)$ has pole at zero

Yes, $\mathrm{C}(\mathrm{s})$ has pole at zero
b) Yes $\mathrm{P}(\mathrm{s})$ has pole at zero

No, $\mathrm{C}(\mathrm{s})$ has no pole at zero


7. a)

8. a) -3
b) Yes
c) $\mathrm{k}>2$

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c) Yes You can move the pole nearest the $j \omega$ axis farther from the $j \omega$ axis.

