1. (6 pts) List Three advantages of state space over classical frequency-domain techniques.

2. (4 pts) The output of a system is given by:

\[
Y(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} X(s) + \frac{s y(0) + y(0) - a_1 y(0) - b_2 x(0) - b_1 x(0)}{s^2 + a_1 s + a_0}
\]

List the variables that together fully describe the state of the system at time \( t = 0 \) (the initial state).

3. (13 pts) Find the equivalent electric circuit for the mechanical system shown. \( F_{in} \) is an input force.

![Mechanical System Diagram]

4. (17 pts) Find the equivalent electric circuit for the mechanical system shown.

a) Show the circuit with a transformer.

b) Show the circuit without a transformer, just like you did in the homework.

![Mechanical System Diagram]

5. (12 pts) The transfer functions of \( C(s) \) and \( P(s) \) (Controller and Plant) are given below. In each case determine if the DC steady-state tracking error will go to zero and whether DC disturbances will be completely rejected. You may assume closed-loop stability. Give reasons for your answers.

a) \( C(s) = \frac{s + 2}{s^3 + 8s^2 + 20s} \) \( P(s) = \frac{s + 4}{s + 6} \)  

<table>
<thead>
<tr>
<th>0 steady-state err.?</th>
<th>Reject disturbance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

Why?

b) \( C(s) = \frac{s + 4}{s^2 + 4s + 3} \) \( P(s) = \frac{s + 2}{s^2 + 3s} \)

<table>
<thead>
<tr>
<th>0 steady-state err.?</th>
<th>Reject disturbance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Why?

c) \( C(s) = \frac{s + 2}{s(s + 3)(s + 5)} \) \( P(s) = \frac{s}{s + 3} \)

<table>
<thead>
<tr>
<th>0 steady-state err.?</th>
<th>Reject disturbance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Why?

d) \( C(s) = \frac{s + 2}{(s + 3)(s + 5)} \) \( P(s) = \frac{1}{s + 3} \)

<table>
<thead>
<tr>
<th>0 steady-state err.?</th>
<th>Reject disturbance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Why?
6. (12 pts) Characteristic equation of a feedback system is shown below. Use the Routh-Hurwitz method to determine the value range of $k$ that will produce a stable system.

$$0 = 9s^3 + 6s^2 + 10s + ks + 4 + k$$

7 - 9 Sketch the root-locus plots for the following open-loop transfer functions:
Use only the main rules, that is, the first page of my root locus notes. You may estimate details like breakaway points and departure angles from complex poles. Show your work where needed (like calculation of the centroid).

7. (12 pts)  
   a) \( G(s) = \frac{(s - 1)(s + 4)}{(s + 2)^2(s + 5)} \)

b) Find the range of gain ($k$) for which the system is closed-loop stable. Assume $k > 0$.

8. (11 pts) \( G(s) = \frac{1}{(s^2 + 6s + 25)(s + 6)(s + 8)} \)

9. (13 pts) \( G(s) = \frac{(s + 5)}{[(s + 2)^2 + 4^2][(s + 5)^2 + 3^2]} \)
1. Multiple input / multiple output systems
   - Can model nonlinear systems
   - Can model time varying systems
   - Can be used to design optimal control systems
   - Can determine controllability and observability

2. \( y(0), \quad \dot{y}(0) = \frac{d}{dt}y(0), \quad x(0), \quad \dot{x}(0) = \frac{d}{dt}x(0) \)

3. \[
\begin{align*}
0 & 0 & 1/k_1 \\
0 & 0 & 1/k_2 \\
1/k_2 & 1/f & 1/b \\
\end{align*}
\]

4. a) Velocity input
   \[
   \begin{align*}
   N &= \frac{r_1}{r_2} \\
   \end{align*}
   \]

b) Velocity input

5. a) YES \( C(s) \) has pole at zero
   b) YES \( P(s) \) has pole at zero
   c) NO \( P(s) \) has zero at zero
   d) NO No poles at zero

6. \[
\begin{array}{c|ccc}
   s^3 & 9 & (10+k) & 0 \\
   s^2 & 6 & (4+k) & \text{ } \\
   s^1 & 6(10+k) - 9(4+k) & 6 - 9(4+k) & = 0 \\
   s^0 & 6 & 6 & = \frac{24 - 3k}{6} \\
   \end{array}
\]

   Ans: \( -4 < k < 8 \)

7. By real-axis rule only

8. \[
\begin{array}{c|c|c}
   k & \text{Region} & \text{Phase} \\
   \hline
   5 & 1 & 0 \\
   6 & 2 & 90 \\
   7 & 3 & 180 \\
   8 & 4 & 270 \\
   \end{array}
\]

   b) \( k < 5 \)

9. \[
\begin{array}{c|c|c}
   k & \text{Region} & \text{Phase} \\
   \hline
   5 & 1 & 0 \\
   6 & 2 & 90 \\
   7 & 3 & 180 \\
   8 & 4 & 270 \\
   \end{array}
\]