1. (6 pts) The output of a system is given by:

\[ Y(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \cdot X(s) + \frac{s y(0) + \frac{d}{dt} y(0) + a_1 y(0) - b_1 s x(0)}{s^2 + a_1 s + a_0} \\
\]

a) Identify the “zero-input response” and the “zero-state response” on the expression above.
b) Which of these two is more likely to be unbounded? Why?

2. (5 pts) List three advantages of state space over classical frequency-domain techniques.

3. (8 pts) a) Give the one characteristic of a feedback system that is more important than all others.

Without this nothing else matters, you haven’t even got a useable system.

b) To meet the requirement of part a), the system poles must lie in a certain region of the s-plane.

Show that area on drawing at right. Make it clear where the poles must lie. Both axes have the same scale.

c) "Tracking" is considered an objective of a feedback system.

List two characteristics of “good” tracking.
d) List one more characteristic or objective of a "good" feedback system.

Sketch the root-locus plots for the following open-loop transfer functions:

Use only the rules you were told to memorize, that is, you may estimate details like breakaway points and departure angles from complex poles. Show your work where needed (like calculation of the centroid).

Draw things like the asymptote angles carefully.

4. (13 pts)

a) Sketch

b) Find the range of gain \( k \) for which the system is closed-loop stable. Assume \( k > 0 \).

5. (11 pts) \( G(s) = \frac{1}{(s^2 + 2s + 10)(s + 4)(s + 6)} \)
Open-book part

1. (14 pts) A compensator circuit is shown with its transfer function.

\[ \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{R_2}{R_1 + R_2} \cdot \frac{s + \frac{1}{R_2 C}}{(s + \frac{1}{R_1 + R_2} C)} \]

a) Show the pole(s) and/or zero(s) on the s-plane. Pay attention to the relative locations of each and label them in terms of \( R_1, R_2, \) and \( C \).

b) For the following parts and input, find the steady-state output.

\[ v_{\text{in}}(t) = u(t) \cdot 8 \cdot V \cdot \cos(400t + 30\,\text{deg}) \]

Give your answer in the time-domain.

2. (18 pts) Find the equivalent electric circuit for the mechanical system shown. \( T_{\text{in}} \) is the input.

a) Show the circuit with a transformer. Show the parts in terms of \( J \)'s, \( k \)'s, \( b \)'s, etc., above.

b) Show the circuit without a transformer, just like you did in the homework. Show the parts in terms of \( J \)'s, \( k \)'s, \( b \)'s, etc., above.

3. (15 pts) A plant with the transfer shown below is part of a standard unity feedback system with gain \( k \).

a) Use the Routh-Hurwitz method to determine the stability of the whole feedback system.

b) Does \( k \) play a role in the stability?

If yes, determine the value range of \( k \) that will produce a stable system.

\[ G(s) = \frac{s^2 + 4}{s^3 + 8s^2 + 2s + 10} \]

4. (10 pts) A root-locus is sketched at right. \( G(s) = \frac{3(s + 2)}{s(s + 5)(s^2 + 6s + 25)} \)

Find the departure angle from the complex pole \(-3 + 4j\).
1. a) before the "+" sign
   zero-state response
   after the "+" sign
   zero-input response
   b) zero-state response
   c) $X(s)$ may have poles of its own, which may produce an unbounded output, either alone or by interaction with system poles on the imaginary axis. The initial conditions have no poles.

2. Multiple input / multiple output systems
   Can be used to design optimal control systems
   Can model nonlinear systems
   Can determine controllability and observability
   Can model time varying systems
   3 of these

3. a) Stability
   b) Anywhere in the left-half-plane, excluding the imaginary axis.
   c) fast smooth minimum error Often measured in steady state but also means minimum overshoot, etc.
   d) Any of these: Reject disturbances Or item left out of c) Like overshoot
   Insensitive to plant variations
   Tolerant of noise

4. a) [Diagram of a control system with transfer function $\frac{-1}{R_2^i C}$]
   b) $k < 16$

5. Open-book part
   1. a) [Graph showing the Nyquist plot]
      $\frac{-1}{R_2^i C}$
      $\frac{-1}{(R_1 + R_2)^i C}$
   b) $3.32 \cdot \cos(400\cdot t - 11.6\text{-deg})$

2. a) [Diagram of a control system with transfer function $\frac{1}{k_1} \cdot \frac{1}{b_1}$]
   $J_1 \frac{1}{r_1}$
   $J_2 \frac{1}{r_2}$
   $J_3 \frac{1}{k_2}$
   $N = \frac{r_2}{r_1}$
   $\frac{1}{k_1} \cdot \frac{1}{b_1}$

   b) [Diagram of a control system with transfer function $\frac{1}{k_1} \cdot \frac{1}{b_1}$]
   $J_1 \frac{1}{r_1}$
   $J_2 \frac{1}{r_2}$
   $J_3 \frac{1}{k_2}$
   $\frac{r_2}{r_1}^2 \cdot \frac{1}{b_2}$

3. a) b) $-2.5 < k < 3$
4. 3.73-deg