ECE 3510 Final given: Spring 07

1. (14 pts) A system has this transfer function: $H(s) = \frac{36}{s+100}$

   a) What is the steady-state response of this system to the step input: $x(t) = 10 \cdot u(t)$

   b) What is the steady-state response of this system to the input: $x(t) = 5 \cdot \cos(400 \cdot t + 20 \cdot \deg) \cdot u(t)$

2. (14 pts) This system: $H(s) = \frac{3}{s+12}$

   Has this input: Cosine input: $x(t) = 5 \cdot \cos(20 \cdot t) \cdot u(t)$ $X(s) = \frac{5 \cdot s}{\left(s^2 + 20^2\right)}$

   Resulting in this output: $Y(s) = \frac{3}{(s+12)} \cdot \frac{5 \cdot s}{\left(s^2 + 400\right)}$

   a) This separates into 3 partial fractions that you can find in the laplace transform table.
   Show what they are, but don't find the coefficients.

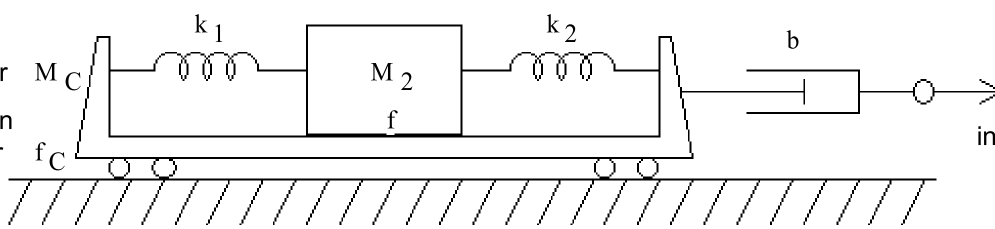
   $$Y(s) = \frac{3}{s+12} \cdot \frac{5 \cdot s}{s^2 + 400} =$$

   b) Continue with the partial fraction expansion just far enough to find the **transient** coefficient as a number.

   c) Express the transient part as a function of time. $y_{tr}(t) = ?$

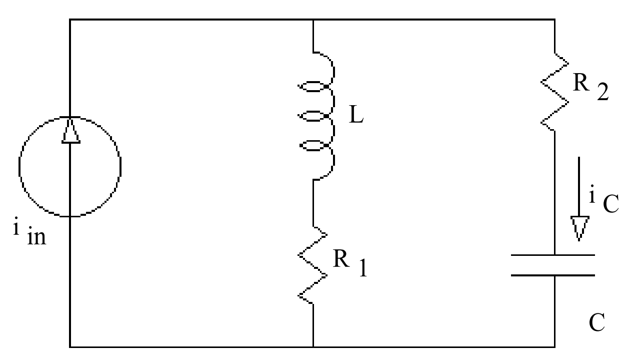
   d) What is the time constant of this expression? $\tau = ?$

3. (16 pts) A rail car carrying a mass is shown below. The mass of the car is M_C and the friction of the car (both air and rolling) is f_C. f is the friction between the mass and the car. V_{in} is an input velocity.



   Find the electrical analogy of this mechanical system as done in my notes.

4. (20 pts) Find the transfer function $H(s) = \frac{I_C(s)}{I_{in}(s)}$ for this circuit.



   Write H(s) in the normal form, as a ratio of polynomials.
   Don't divide the polynomials. Initial conditions are 0.

   You MUST show work to get credit.

   Hint: Since the input and the "output" are both currents, think in terms of a current divider instead the voltage divider you may be more used to.

   H(s) = ?

   b) Find the characteristic equation of the circuit shown.

   c) If the transfer function has one or more zeros express it (them) in terms of R_1, R_2, C, & L.

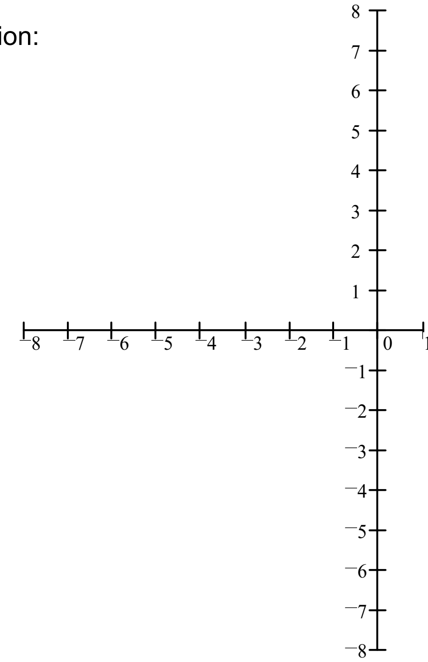
d) $I_{in} = 100 \cdot mA \cdot u(t)$ The following are the only initial conditions: $i_c(0) = 20 \cdot mA$ $\frac{d}{dt} i_c(0) = 5 \cdot \frac{A}{sec}$

   Find $I_C(s)$ **Hint:** See p. 42

5. (14 pts) Sketch the root-locus plots for the following open-loop transfer function:
   Use only the main rules, that is, the first page of my root locus notes.

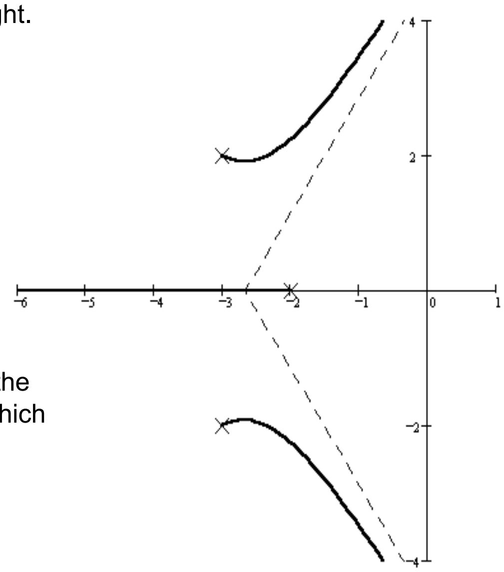
   Show your work.

$$G(s) = \frac{(s+5)}{\left[(s+2)^2+4^2\right]\cdot\left[(s+5)^2+3^2\right]}$$



6. (12 pts) The root locus for the transfer function below is shown at right.

$$G(s) = \frac{1}{\left[(s+3)^2+4\right]\cdot(s+2)}$$



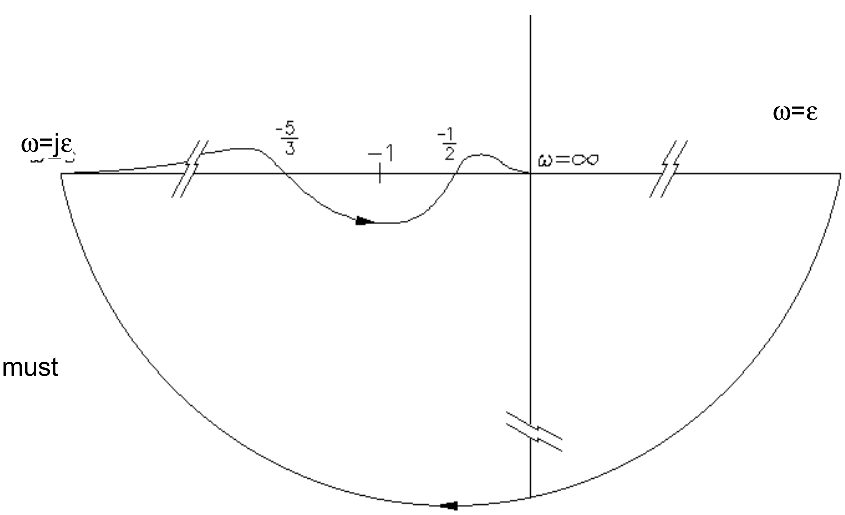
a) Does the root locus cross the $j\omega$ axis at 5?
   Be sure to show the work and method you used to decide.

b) Regardless of what you found in part a, continue to assume that the root-locus crosses the $j\omega$ axis at 5. Give the range of gain k for which the system is closed-loop stable.

   Remember, I asked for a **range** for stability

7. (10 pts) Refer to the Nyquist curve at right (only the portion for $\omega > 0$ is plotted).

   This system has a pole at zero, so the large arc is actually at "∞".



a) Can the closed-loop system be stable? Show why

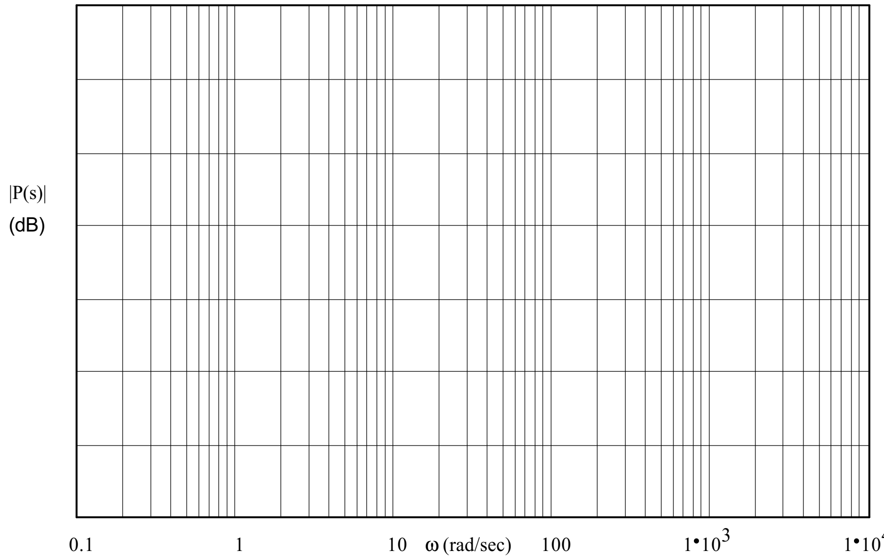
b) If the answer to (a) is yes, what condition must the open-loop system satisfy?

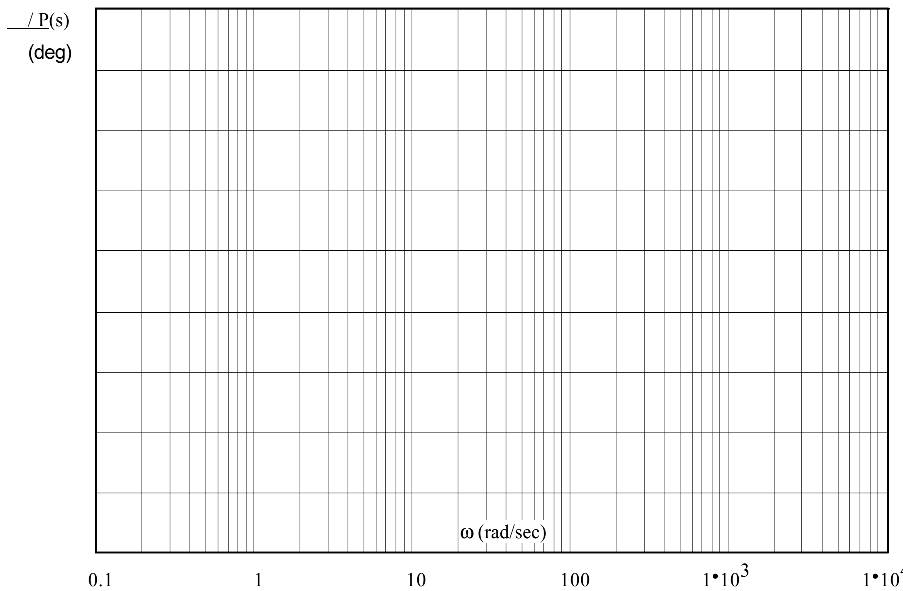
c) What is (are) the gain margin(s)?

d) Does the open-loop system have more poles than zeros? If yes, how many?

8. (16pts) Sketch the Bode plot for the following transfer function. Make sure to label the graphs, and plot the slopes and magnitudes accurately. Also draw the "smooth" lines. Don't forget that complex poles affect the shape of the "smooth" phase angle line

$$P(s) = \frac{10\cdot\left(s^2 + 40\cdot s + 40000\right)}{(s+20)^2}$$





9. (20 pts) Find the $x(k)$ whose z-transform is given. Use partial fraction expansion.

a) $X(z) = \dfrac{1}{(z-1)\cdot(z+2)}$

b) $X(z) = \dfrac{16\cdot z}{(z-(1+2\cdot j))\cdot(z-(1-2\cdot j))}$

10. (14 pts) $y(k) = 4\cdot x(k) - 5\cdot x(k-2) + 2\cdot y(k-1) + 3\cdot y(k-2)$

a) Find the $H(z)$ corresponding to the difference equation. Express as a ratio of polynomials.

b) List the poles of $H(z)$. Indicate multiple poles if there are any.

c) Draw the block diagram of a simple direct implementation of the difference equation.

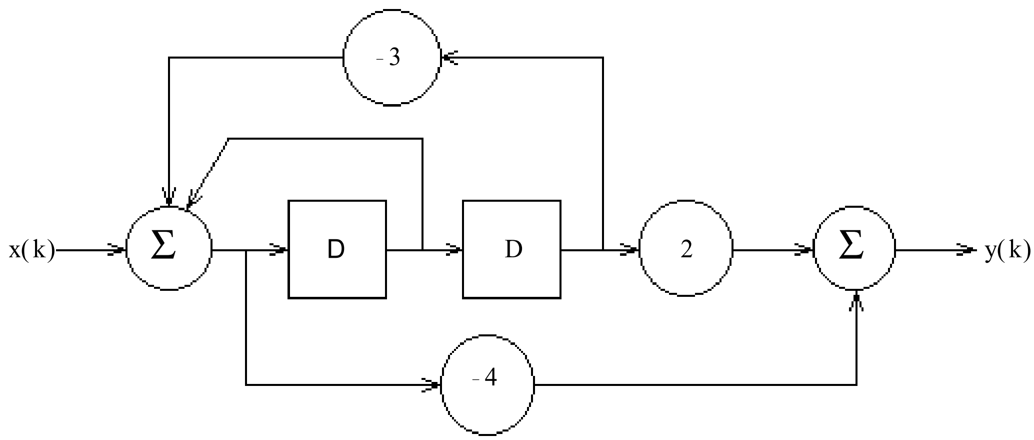
11. (4 pts) Find $x(\infty)$ if the z-transform of $x(k)$ is $X(z) = \dfrac{5\cdot\left(z^2+1\right)}{(z-1)\cdot(3\cdot z+2)}$

12. (12 pts) A signal $x(t)$ with transform $X(s) = \dfrac{24}{s\cdot(s+2)\cdot(s+4)} = \dfrac{3}{s} - \dfrac{6}{(s+2)} + \dfrac{3}{(s+4)}$

(partial fraction expansion)

is sampled at time instants $t = kT$ to obtain $x_d(k)$. Find the transform $X_d(z)$ of the resultant signal.
Express $X_d(z)$ as one numerator over one denominator. You do not have to simplify beyond that.

13. (14 pts) Find the transfer function $H(z) = Y(z)/X(z)$.



14. Do you want your grade and scores posted on my door and on the Internet? Yes No (Circle one)

If your answer is yes, then provide some sort of alias or password: ________________________________

The grades will be posted on my door in alphabetical order under the alias that you provide here. I will not post grades under your real name. The Internet version will be a pdf file which you can download. Both will show the homework, lab, and exam scores of everyone who answers yes here.

ECE 3510 Final

Name ________________________________

Scores:

pages 1-2 __________ / 44 pts

pages 3-4 __________ / 34 pts

pages 5-6 __________ / 38 pts

pages 7-8 __________ / 34 pts

pgs 9-10 __________ / 30 pts

Total __________ / 180 pts

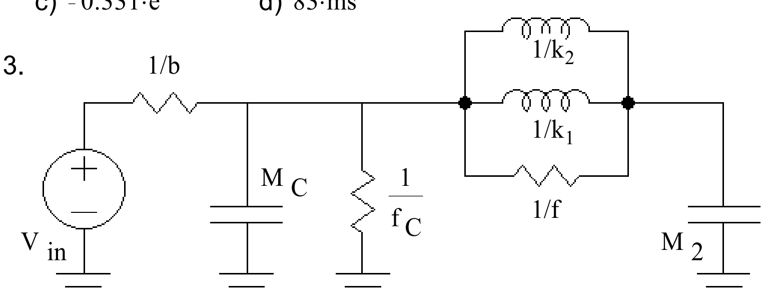
Answers

1. a) 3.6 b) $0.436\cdot\cos(400\cdot t - 56\cdot deg)$

2. a) $\dfrac{A}{s+12} + \dfrac{B\cdot s}{(s^2+400)} + \dfrac{C\cdot 20}{(s^2+400)}$ b) -0.331

c) $-0.331\cdot e^{-12\cdot t}$ d) $83\cdot ms$

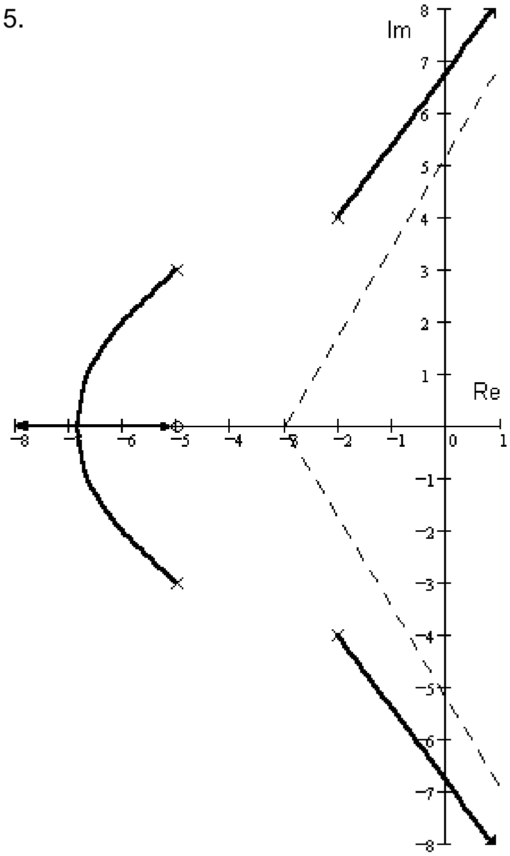
3.



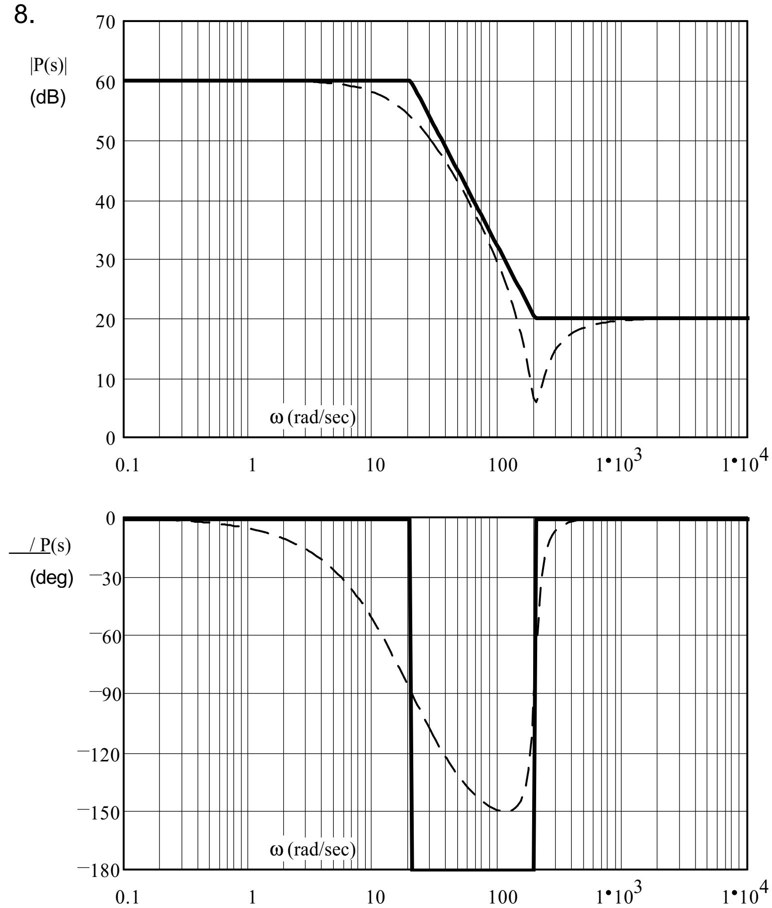
4. a) $\dfrac{s\cdot\left(s+\dfrac{R_1}{L}\right)}{s^2 + \dfrac{R_2+R_1}{L}\cdot s + \dfrac{1}{L\cdot C}}$ b) $0 = s^2 + \dfrac{R_2+R_1}{L}\cdot s + \dfrac{1}{L\cdot C}$ c) $s = 0$ & $s = -\dfrac{R_1}{L}$

d) $I_C(s) = \dfrac{s\cdot\left(s+\dfrac{R_1}{L}\right)}{s^2 + \dfrac{R_2+R_1}{L}\cdot s + \dfrac{1}{L\cdot C}}\cdot\dfrac{100\cdot mA}{s} + \dfrac{s\cdot 20\cdot mA + 5\cdot\dfrac{A}{sec} + \dfrac{R_2+R_1}{L}\cdot 20\cdot mA}{s^2 + \dfrac{R_2+R_1}{L}\cdot s + \dfrac{1}{L\cdot C}}$

5.



8.

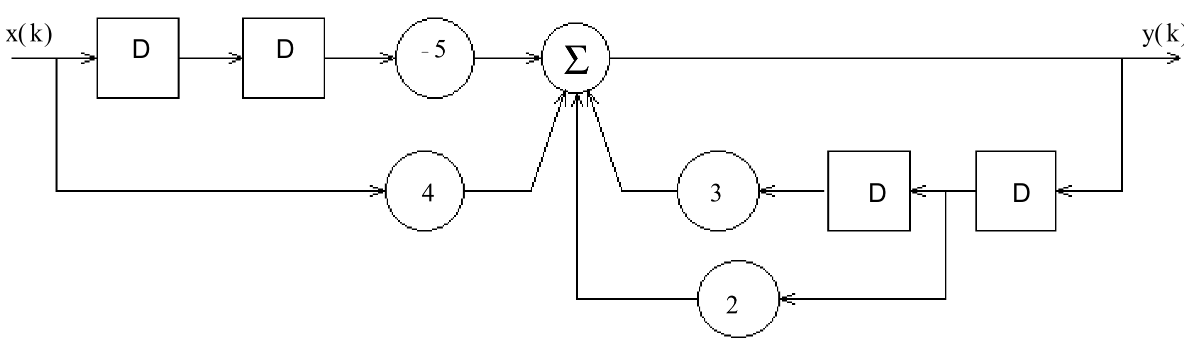


6. a) $G(5\cdot j) = \dfrac{1}{\left[(5\cdot j+3)^2+4\right]\cdot(5\cdot j+2)} = \dfrac{1}{-174}$ YES, The phase angle **is** equal to $\pm 180°$ b) $k < 174$

7. a) $N = 0$ b) The open-loop system must be stable, $P = 0$ c) $\dfrac{3}{5} <$ gain < 2 d) 2

9. a) $-\dfrac{1}{2}\cdot\delta(k)+\dfrac{1}{3}+\dfrac{1}{6}\cdot(-2)^k$ b) $8\cdot\left(\sqrt{5}\right)^k\cdot\sin(1.107\cdot k)$ 10. a) $\dfrac{4\cdot z^2-5}{z^2-2\cdot z-3}$ b) -1 and $+3$

10. c)



11. 2 12. $\dfrac{3\cdot z\cdot\left[\left(z-e^{-2\cdot T}\right)\cdot\left(z-e^{-4\cdot T}\right)-2\cdot(z-1)\cdot\left(z-e^{-4\cdot T}\right)+(z-1)\cdot\left(z-e^{-2\cdot T}\right)\right]}{(z-1)\cdot\left(z-e^{-2\cdot T}\right)\cdot\left(z-e^{-4\cdot T}\right)}$ 13. $\dfrac{-4\cdot z^2+2}{z^2-z+3}$