The Final will be **open book, open notes** exam with calculators.

**The exam will cover**

1. Laplace transforms (simple forms only)
2. Inverse Laplace transforms (partial fractions)
3. Relationship of signals to pole locations
   - Figs 2.1 & 2.2 on page 7
4. Boundedness and convergence of signals
   - Bounded if all poles in LHP, no double poles on jω-axis
   - Converges to 0 if all poles LHP. Converges to a non-zero value if a single pole is at zero
5. **H(s) of circuits**
   \[ Z(s) = \frac{1}{Cs} \]  
   \[ R \]  
   \[ \frac{1}{Ls} \]  
   Be able to find \[ \frac{V_{\text{out}(s)}}{V_{\text{in}(s)}} \] or any other output over input.
   Review voltage dividers and current dividers
6. Block Diagrams & their transfer functions. Standard feedback loop transfer function
7. **BIBO Stability (Systems)**
   - BIBO if all poles in LHP, no poles on jω-axis
8. Impulse & step responses
   \[ h(t) = \frac{1}{s} H(s) \]
9. Steady-state (DC gain) & transient step responses
   \[ H(0) \]
10. Effects of pole locations on step response, see Fig 3.12, p.36.
11. Steady-state sinusoidal response. You should be ready to do some complex arithmetic.
   \[ H(j\omega) \]
12. Transient sinusoidal response. You should be ready to do partial fraction expansion to the first (transient) term.
   \[ H(s) \times \frac{A}{s^2 + \omega^2} \]  
   \[ \text{or } \frac{B}{s^2 + \omega^2} \]  
   \[ A \cdot \cos(\omega t) \]  
   \[ B \cdot \sin(\omega t) \]
13. Effect of initial conditions
   \[ Y(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \cdot X(s) \]  
   \[ + \frac{s \cdot y(0) + \frac{d}{dt} y(0) + a_1 y(0) - b_2 s \cdot x(0) - b_2 s \cdot \frac{d}{dt} x(0) - b_1 s \cdot x(0)}{s^2 + a_1 s + a_0} \]
   May ask question like points on p. 42
   May give H(s), a’s & b’s and y(0).. and ask for effect of initial conditions
14. The advantages of state space over classical frequency-domain techniques.
   - Multiple input / multiple output systems
   - Can model nonlinear systems
   - Can model time varying systems
   - Can be used to design optimal control systems
   - Can determine controllability and observability
15. Electrical analogies of mechanical systems, particularly translational and rotational systems.
   Review the handout and homeworks 8 & 9.

16. Control system characteristics and the objectives of a "good" control system. See pgs. 57 - 58
   Stable
   Tracking
   fast
   smooth
   minimum error (often measured in steady state)
   Reject disturbances
   Insensitive to plant variations
   Tolerant of noise

17. Elimination of steady-state error, p. 61.
   1 System stable
   DC
   2 \( C(s) \) or \( P(s) \) has pole @ 0
   3 No zero @ 0

18. Rejection of constant disturbances, p. 63.
   1 System stable
   DC
   2 \( C(s) \) has pole @ 0
   3 or \( P(s) \) has zero @ 0 But bad for above

   \[
   D(s) = s^3 + 20s^2 + 59s + 32
   \]
   \[
   s^3 | 1 \quad 59 \quad 0
   s^2 | 20 \quad 32
   s^1 | \frac{20 \cdot 59 - 1 \cdot 32}{20} = 57.4 \quad \frac{20 \cdot 0 - 1 \cdot 0}{20} = 0
   s^0 | \frac{57.4 \cdot 32 - 20 \cdot 0}{57.4} \quad \frac{57.4 \cdot 0 - 20 \cdot 0}{57.4} = 0
   \]
   Be able to do this with variable such as "k"

20. Root - Locus method
   a) Main rules
   1. Root-locus plots are symmetric about the real axis.
   2. On the real axis, spaces left of an odd number of O-L poles and zeros are always part of the locus.
      (Essentially, every other space on the real axis (counting leftward) is part of the plot.)
   3. Each O-L pole originates \((k = 0)\) one branch. \((n)\)
      Each O-L zero terminates \((k = \infty)\) one branch. \((m)\)
      All remaining branches go to \(\infty\). \((n - m)\)
      These remaining branches approach asymptotes as they go to \(\infty\).
   4. The origin of the asymptotes is the centroid.
      \[
      \text{centroid} = \sigma = \frac{\sum \text{OLpoles} - \sum \text{OLzeros}}{n - m}
      \]
   5. The angles of the asymptotes
      \[
      \begin{array}{c|ccc}
      n - m & \text{angles (degrees)} \\
      2 & 90 & 270 \\
      3 & 60 & 180 & 300 \\
      4 & 45 & 135 & 225 & 315 \\
      \end{array}
      \]
   6. The angles of departure (and arrival) of the locus are almost always:
      \[
      \text{OR:}
      \]
      \[
      \text{OR:}
      \]
      \[
      \text{OR:}
      \]
b) Additional Root locus rules. Review the handout.

1. The breakaway points are also solutions to:
\[ \frac{1}{\sum_{i=1}^{n} \frac{1}{s + p_i}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{s + z_i}} \]

2. Gain at any point on the root locus:
\[ k = \left| \frac{1}{G(s)} \right| \]

3. Phase angle of \( G(s) \) at any point on the root locus:
\[ \arg(G(s)) = \arg(N(s)) - \arg(D(s)) = \pm 180^\circ \pm 360^\circ \ldots \]
Or:
\[ \arg\left(\frac{1}{G(s)}\right) = \arg(D(s)) - \arg(N(s)) = \pm 180^\circ \pm 360^\circ \ldots \]

4. Departure angles from complex poles:
\[ 180 - 90 - 153.4 + 135 = 71.6 \text{ deg} \]

Root Locus general

a) Concepts of what a root locus plot is and what it tells you. Movement of poles
b) Good vs bad, fast response vs slow, OK damping vs bad.
c) Effects of adding a compensator
d) Important conclusions from root locus, section 4.4.5, p. 82.
e) Simple root-locus design, the placement of additional poles and zeros in order to affect the root locus.

21. Phase-locked loops
   How does it work   The loop block diagram   Material from labs

22. Bode Plots
   Be able to draw both magnitude and phase plots
   I may ask you to start with a circuit
   Basic rules
   Complex poles an zeros
   \[ s^2 + 2 \zeta \omega_n s + \omega_n^2 \]
   \[ (s + a)^2 + b^2 = s^2 + 2a \omega_n s + a^2 + b^2 \]
   \[ \text{max at } \omega_n \frac{1}{2 \zeta} \]
   Bode to transfer function (like problem 5.2b)
   GM & PM

23. Nyquist plots
   You may be asked to draw one, be able to find start, end, and end approach angle.
   Use a quicky Bode plot to estimate curve
   Concepts of what a Nyquist plot is and what it tells you. \[ Z = N + P \]
   Make sure you understand problem 5.11
   GM & PM

24. Phase-lead compensator
1. Discrete signals

\[ x(k) \]

2. z-transform

\[ X(z) = \sum_{k=0}^{\infty} x(k)z^{-k} \]

Finite-length signals have all poles at zero

3. Relationship of signals to pole locations, Fig 6.9, p155.

- Lines of constant damping
- Speed of decay

4. Properties of the z-transform

- Linear
- Right-shift = delay = multiply by \( z^{-1} = \frac{1}{z} \)
- Left-shift = advance = multiply by \( z \)
- Initial value = \( x(0) = X(\infty) \)
- Final value (DC) = \( x(\infty) = (z-1)X(z) |_{z=1} \)

5. Inverse z-transforms (partial fractions & long division)

Divide by \( z \) first:

- Poles on real axis (not at zero): \( \frac{A}{z-p} \) \( B \cdot z \)
- Complex poles: \( \frac{2 \cdot |B| \cdot (|p|) \cdot \cos(\theta_p + \theta_B)}{z-p} \)

6. Boundedness and convergence of signals, relate to continuous-time signals

- Bounded if all poles in inside unit circle, no double poles on unit circle
- Converges to 0 if all poles inside unit circle. Converges to a non-zero value if a single pole is at 1

7. Difference equations, be able to get \( H(z) \)

8. Discrete-time systems, FIR (all poles at zero), IIR (some poles not at zero)

9. BIBO Stability, all poles inside unit circle.

10. Step & Sinusoidal responses, effects of poles & zeros, etc.

DC gain = \( H(1) \)

Sinusoidal: \( H(e^{j\Omega_o}) = |H| / \theta_H \) multiply magnitudes and add angles

11. Initial Conditions, p. 174

12. Implementations, p176 & 177, be able to go back and forth to \( H(z) \)

13. Same Feedback system as in continuous-time and Root locus works the same but is interpreted very differently.

14. Sampled-data systems

\[ t = kT \quad x_d(k) = x(kT) \quad A/D \text{ converter} \]

\[ z = e^{kT} \quad s = \frac{\ln(z)}{T} \]

Compare s-plane to z-plane, Fig 7.3, p190

15. Conversions of continuous-time transfer functions to discrete-time transfer functions

- Step response

\[ y(s) = \frac{1}{s}H(s) \quad \text{step response} \]

\[ y(t) \quad \text{by partial frac. exp.} \]

- Discrete step response

\[ y_d(k) = y(kT) \quad \text{find:} \]

\[ Y(z) = \frac{z}{z-1}H(z) = \frac{Y(z)}{z-1} \]

16. Nyquist sampling criterion, at least twice the highest signal frequency

All Homeworks I’ll scan through for problems

All Labs

- Position & speed control
- DC motor characteristics
- PID
- Matlab
- Ball & Beam
- Flexible Beam
- PLL
- Inverted Pendulum

Discrete system should have the same step response