

ECE 3510 Final Exam Study Guide

The Final will be **open book, open notes** exam with calculators.

The exam will cover

1. Signals and blocks in a feedback loop
2. Laplace transforms, be prepared to look up and adapt a table entries
Initial and final values
3. Inverse Laplace transforms (partial fractions)
4. Relationship of signals to pole locations Figs 2.1 & 2.2 on page 7
5. Boundedness and convergence of signals
Bounded if all poles in LHP, no double poles on $j\omega$ -axis
Converges to 0 if all poles LHP. Converges to a non-zero value if a single pole is at zero

6. H(s) of circuits

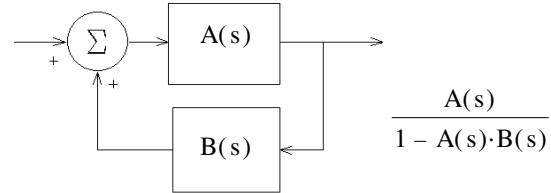
$Z(s)$ R Ls $\frac{1}{Cs}$ Be able to find $\frac{V_{out}(s)}{V_{in}(s)}$ or any other output over input.
Review voltage dividers and current dividers

7. Block Diagrams & their transfer functions

Standard feedback loop transfer function

8. BIBO Stability (Systems)

BIBO if all poles in LHP, no poles on $j\omega$ -axis



9. Impulse & step responses $h(t)$ $\frac{1}{s} \cdot H(s)$

10. Steady-state (DC gain = H(0)) & transient step responses

11. Effects of pole locations on step response, see Fig 3.12, p.36.

12. Sinusoidal responses, effects of poles & zeros, etc.

Review complex math relations

Steady-state AC analysis to get $Y(j\omega)$ & $y_{ss}(t)$
(Sinusoidal steady-state transfer function = H(j ω))

Conversions
Add & Subtract
Multiply and divide

13. Transient sinusoidal response

You should be ready to do partial fraction expansion to the first (transient) term from:

$$H(s) \times \left[A \cdot \frac{s}{s^2 + \omega^2} \right] \quad \text{or} \quad \left[B \cdot \frac{\omega}{s^2 + \omega^2} \right]$$

$$A \cdot \cos(\omega t) \qquad \qquad B \cdot \sin(\omega t)$$

14. Effect of initial conditions

$$Y(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \cdot X(s) + \frac{s \cdot y(0) + \frac{d}{dt} y(0) + a_1 \cdot y(0) - b_2 \cdot s \cdot x(0) - b_2 \cdot s \cdot \frac{d}{dt} x(0) - b_1 \cdot s \cdot x(0)}{s^2 + a_1 s + a_0}$$

May ask question like points on p. 43

May give H(s), a's & b's x(0)s and y(0)s. and ask for effect of initial conditions

15. The advantages of state space over classical frequency-domain techniques.

- Multiple input / multiple output systems
- Can model nonlinear systems
- Can model time varying systems
- Can be used to design optimal control systems
- Can determine controllability and observability

16. Electrical analogies of mechanical systems, particularly translational and rotational systems.

Review the handout and homeworks 8 & 9.

17. Control system characteristics and the objectives of a "good" control system. See pgs. 57 - 58

- Stable
- Tracking
 - fast
 - smooth
 - minimum error (often measured in steady state)
- Reject disturbances
- Insensitive to plant variations
- Tolerant of noise

Be able to relate these to poles and zeros on the real and Imaginary axis (where possible)

18. Elimination of steady-state error, p. 61.

DC

- 1 System stable
- 2 $C(s)$ or $P(s)$ has pole @ 0
- 3 $C(s)$ or $P(s)$ No zero @ 0

19. Rejection of constant disturbances, p. 63.

DC

- 1 System stable
- 2 $C(s)$ has pole @ 0
- 3 or $P(s)$ has zero @ 0 But bad for above

20. Routh-Hurwitz method.

$$D(s) = s^3 + 20s^2 + 59s + 32$$

| | | | |
|-------|--|--|---|
| s^3 | 1 | 59 | 0 |
| s^2 | 20 | 32 | |
| s^1 | $\frac{20 \cdot 59 - 1 \cdot 32}{20} = 57.4$ | $\frac{20 \cdot 0 - 1 \cdot 0}{20} = 0$ | |
| s^0 | $\frac{57.4 \cdot 32 - 20 \cdot 0}{57.4} = 32$ | $\frac{57.4 \cdot 0 - 20 \cdot 0}{57.4} = 0$ | |

Be able to do this with variable such as "k"

21. Root - Locus method

a) Main rules

1. Root-locus plots are symmetric about the real axis.
2. On the real axis, spaces left of an odd number of O-L poles and zeros are always part of the locus. (Essentially, every other space on the real axis (counting leftward) is part of the plot.)
3. Each O-L pole originates ($k = 0$) one branch. (n)
 Each O-L zero terminates ($k = \infty$) one branch. (m)
 All remaining branches go to ∞ . (n - m)

These remaining branches approach asymptotes as they go to ∞ .

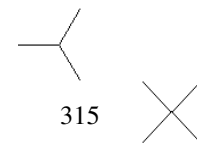
4. The origin of the asymptotes is the centroid.

$$\text{centroid} = \sigma = \frac{\sum_{\text{all}} \text{OLpoles} - \sum_{\text{all}} \text{OLzeros}}{n - m}$$

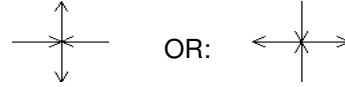
(# poles - # zeros)

5. The angles of the asymptotes

| n - m | angles (degrees) | | |
|-------|------------------|-----|-----|
| 2 | 90 | 270 | |
| 3 | 60 | 180 | 300 |
| 4 | 45 | 135 | 225 |



6. The angles of departure (and arrival) of the locus are almost always:



b) Additional Root locus rules. Review the handout.

1. The breakaway points are also solutions to: $\sum_{\text{all}} \frac{1}{(s + -p_i)} = \sum_{\text{all}} \frac{1}{(s + -z_i)}$

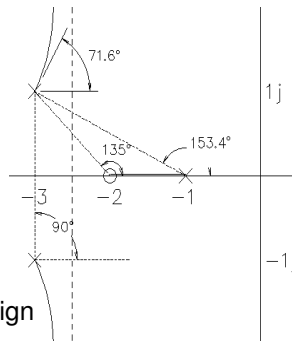
2. Gain at any point on the root locus: $k = \frac{1}{|G(s)|}$

3. Phase angle of G(s) at any point on the root locus: $\arg(G(s)) = \arg(N(s)) - \arg(D(s)) = \pm 180^\circ \pm 360^\circ \dots$

Or: $\arg\left(\frac{1}{G(s)}\right) = \arg(D(s)) - \arg(N(s)) = \pm 180^\circ \pm 360^\circ \dots$

4. Departure angles from complex poles:

$180 - 90 - 153.4 + 135 = 71.6 \text{ deg}$



c) Root Locus general, Interpretation and design

1. Concepts of what a root locus plot is and what it tells you. Movement of poles
2. Good vs bad, fast response vs slow, OK damping vs bad.
3. Effects of adding a compensator

Know pole & zero locations of P, PD, PI, & PID Compensators as well as Lag and Lead

4. Important conclusions from root locus, section 4.4.5, p. 84.
5. Simple root-locus design, the placement of additional poles and zeros in order to affect the root locus.

22. Be able to find a transfer function for a system with multiple multiple feedback paths, like you did in the PID lab and again in lab 8.

23. Bode Plots

Be able to draw both magnitude and phase plots

I may ask you to start with a circuit

Basic rules

Complex poles and zeros $s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2 = (s + a)^2 + b^2 = s^2 + 2 \cdot a \cdot s + a^2 + b^2$

natural frequency $\omega_n = \sqrt{a^2 + b^2}$ damping factor $\zeta = \frac{a}{\omega_n}$ max at approx ω_n , $\frac{1}{2 \cdot \zeta}$ $20 \cdot \log\left(\frac{1}{2 \cdot \zeta}\right)$ dB

Bode to transfer function (like problem 5.2b)

GM, PM & DM

24. Nyquist plots

You may be asked to draw a simple one. At minimum you should;
 Be able to find the start point (DC gain ($s = 0 = \omega$)) from the transfer function)
 Find the final value ($\omega = \infty$) and the approach angle to the final value.

Concepts of what a Nyquist plot is and what it tells you.

Be able to count encirclements, with or without the $\omega < 0$ part of the plot.

GM & PM

25. Phase-lead compensator, section 5.3.9 & lab 10

Material new to the Final: Discrete-time Signals & Systems

- | 1. Discrete signals | $x(k)$ | $\mathbf{f(k)}$ | $\mathbf{F(z)}$ |
|---|--|---|---|
| 2. z-transform | $X(z) = \sum_{k=0}^{\infty} x(k) \cdot z^{-k}$ | $\delta(k)$ | 1 |
| | Finite-length signals have all poles at zero | $u(k)$ | $\frac{z}{z-1}$ |
| 3. Relationship of signals to pole locations, Fig 6.9, p159. | | p^k | $\frac{z}{z-p}$ |
| lines of constant damping | | $\cos(\Omega_o \cdot k)$ | $\frac{z \cdot (z - \cos(\Omega_o))}{z^2 - 2 \cdot \cos(\Omega_o) \cdot z + 1}$ |
| Speed of decay | | $\sin(\Omega_o \cdot k)$ | $\frac{z \cdot \sin(\Omega_o)}{z^2 - 2 \cdot \cos(\Omega_o) \cdot z + 1}$ |
| 4. Properties of the z-transform | | | |
| linear | | | |
| Right-shift = delay = multiply by z^{-1} | $= \frac{1}{z}$ | | |
| Left-shift = advance = multiply by z | | | |
| Initial value = $x(0) = X(\infty)$ | | | |
| Final value (DC) = $x(\infty) = (z-1) \cdot X(z) \Big _{z:=1}$ | | $\mathbf{F(z)}$ | $\mathbf{f(k)}$ |
| 5. Inverse z-transforms (partial fractions & long division) | | A | $A \cdot \delta(k)$ |
| Divide by z first: $\frac{X(z)}{z}$ | Poles on real axis (not at zero): | $\frac{B \cdot z}{(z-p)}$ | $B \cdot p^k$ |
| | Complex poles: | $\frac{B \cdot z}{(z-p)} + \frac{\bar{B} \cdot z}{(\bar{z}-\bar{p})}$ | $2 \cdot B \cdot (p)^k \cdot \cos(\theta_p \cdot k + \theta_B)$ |
| 6. Boundedness and convergence of signals, relate to continuous-time signals | | | |
| Bounded if all poles in inside unit circle, no double poles on unit circle | | | |
| Converges to 0 if all poles inside unit circle. Converges to a non-zero value if a single pole is at 1 | | | |
| 7. Difference equations, be able to get $H(z)$ | | | |
| 8. Discrete-time systems, FIR (all poles at zero), IIR (some poles not at zero) | | | |
| 9. BIBO Stability, all poles inside unit circle. | | | |
| 10. Step & Sinusoidal responses, effects of poles & zeros, etc. | | | |
| DC gain = $H(1)$ | sinusoidal: $H(e^{j\Omega_o}) = H \angle \theta_H$ | | multiply magnitudes and add angles |
| 11. Initial Conditions, p. 179 | | | |
| 12. Implementations, p181 - 183, be able to go back and forth to $H(z)$ | | | |
| 13. Same Feedback system as in continuous-time and Root locus works the same but is interpreted very differently. | | | |

All Homeworks

All Labs