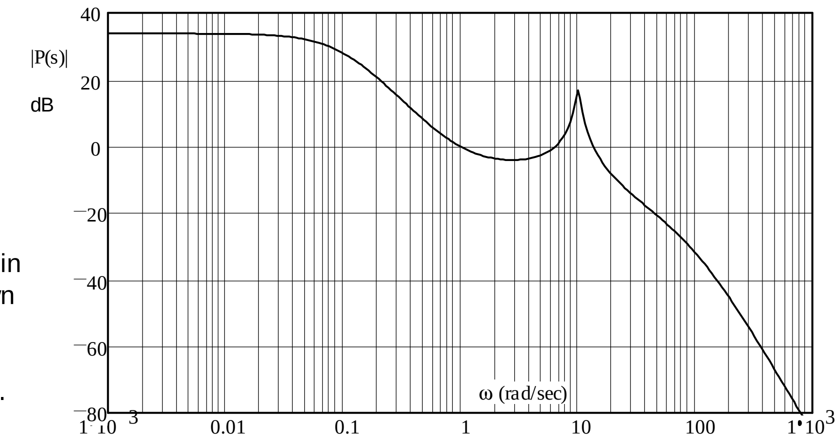
Name: ______________________________

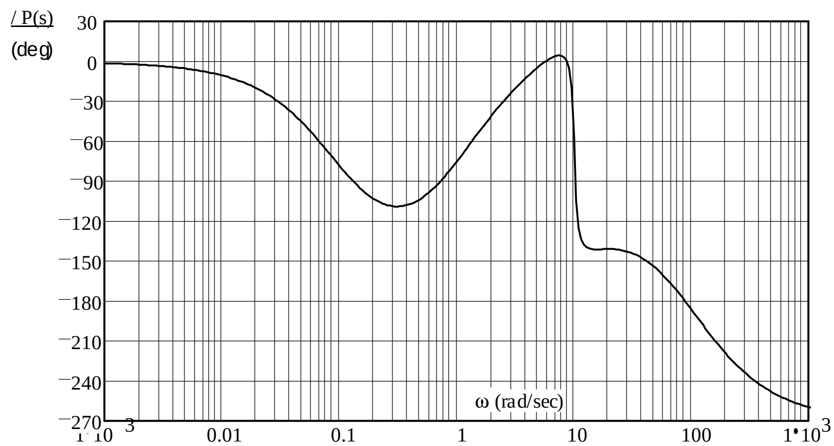
Hand in this page showing how you used the drawings to find answers to 1a and 3b.

You must show the work needed to get the answers below

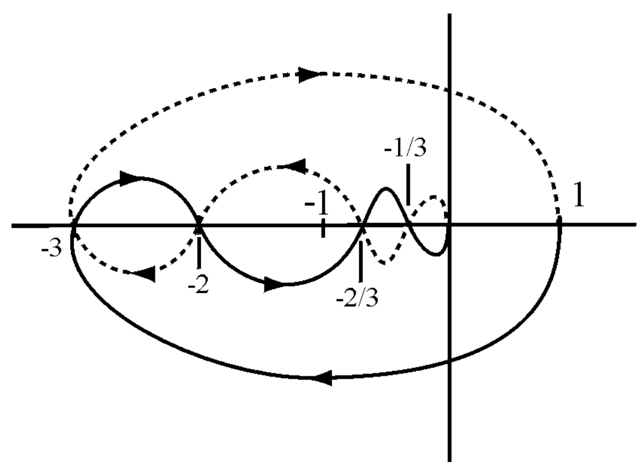
1. Problem 5.9 (p.147) in Bodson the text.

   a) Give the gain margin and the phase margin of the system whose Bode plots are shown at right (the plots are for the open-loop transfer function and the closed-loop transfer function is assumed to be stable).





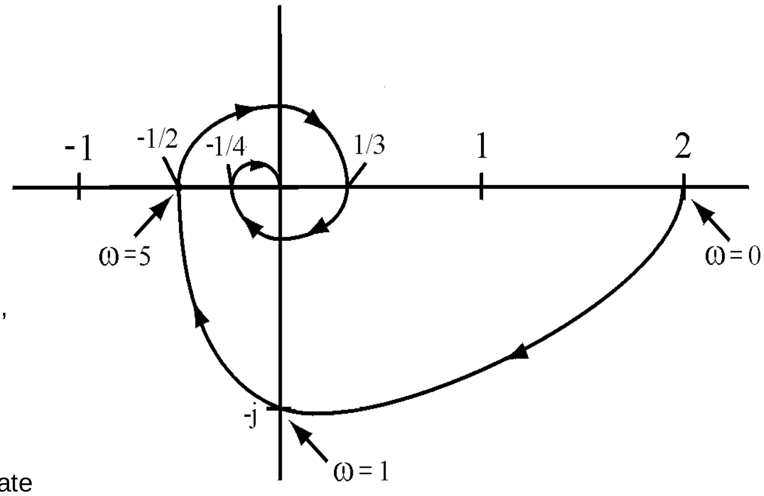
   b) Indicate whether the system whose Nyquist curve is shown is closed-loop stable, given that it is open-loop stable.



   c) What are the values of the gain $g > 0$ by which the open-loop transfer function of part (b) may be multiplied, with the closed-loop system being stable?

d) Sketch an example of a Nyquist curve for a system which has three unstable open-loop poles, and which is closed-loop stable.

2. Problem 5.11 (p.148) in the text.
All parts of this problem refer to the system whose Nyquist curve is shown at right (only the portion for $\omega > 0$ is plotted). Recall that the Nyquist curve represents the frequency response of the open-loop system, or $G(j\omega)$. If $G(s)$ is the open-loop transfer function. The closed-loop transfer function is $G(s)/(1 + G(s))$.



a) Knowing that the closed-loop system is stable, can one say for sure that the open-loop system is stable?

b) Given the closed-loop system is stable, estimate the gain margin and the phase margin of the closed-loop system.

c) How many unstable poles does the closed-loop system have if the open-loop gain is multiplied by 5?

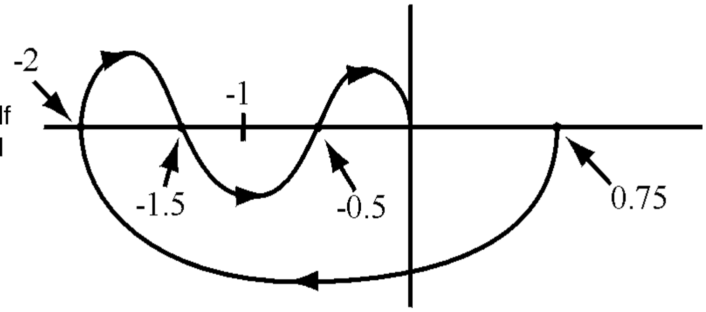
d) Give the steady-state response $y_{ss}(t)$ of the open-loop system to an input $x(t) = 2$.
Repeat for $x(t)=3\cos(t)$, and $x(t)=4\cos(5t)$.

e) Repeat part (d) for the closed-loop system.
Hint: remember that the output of the closed-loop system is $\text{input}\cdot\dfrac{G(s)}{1+G(s)}$

3. Problem 5.13 (p.149) in the text.

a) Consider the Nyquist diagram of a transfer function $G(s)$ shown at right. Only the portion for $\omega > 0$ is plotted.

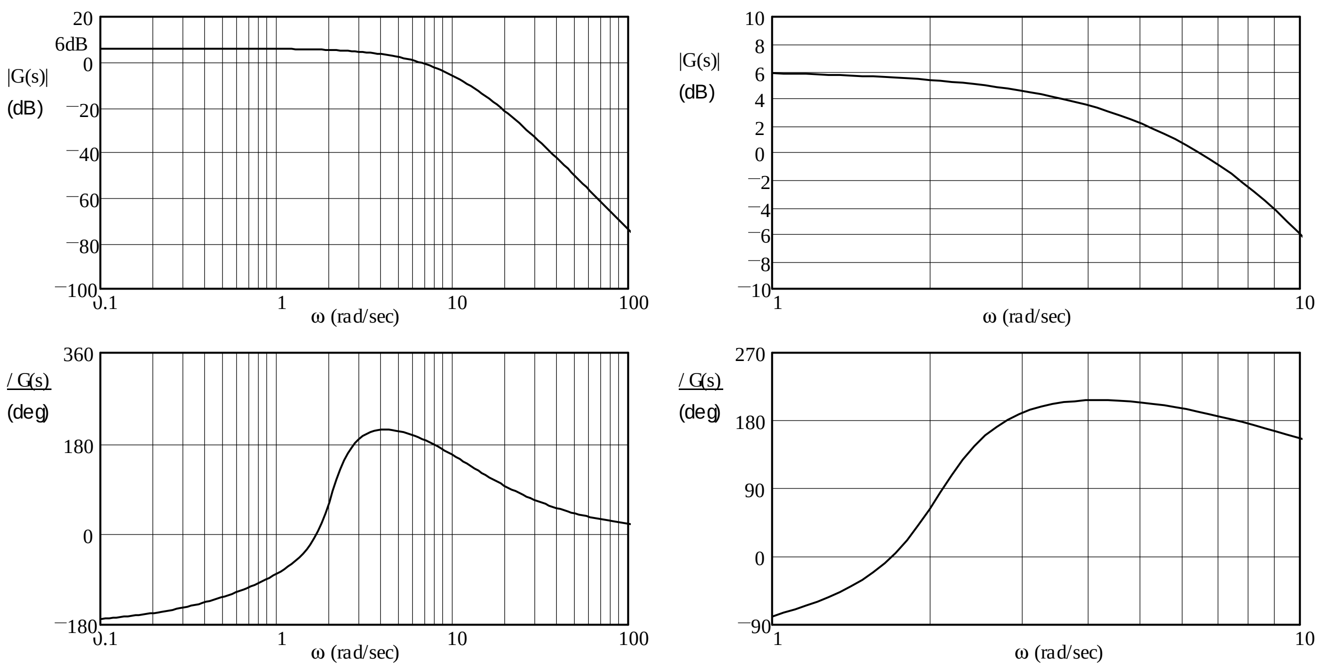
Assume that $G(s)$ has no poles in the open right-half plane, and that a gain K is cascaded with $G(s)$. Find the ranges of positive K for which the closed-loop system is stable.



b) Bode plots of the open-loop transfer function of a feedback system are shown below, with the detail from 1 to 10 rad/sec shown on the right. For this system:

• How much can the open-loop gain be changed (increased and/or decreased) before the closed-loop system becomes unstable ?

• What is a rough estimate of the phase margin of the feedback system? Show on the graph how the results are obtained. The numerical results do not have to be precise.



c) For the system of part (b), give the steady-state response of the open-loop system and of the closed-loop system to an input $x(t) = 2$.

4. Problem 5.14 (p.150) in the text. You'll need another sheet of paper for this one.

   a) Consider the lead controller for the double integrator. For the design that makes the crossover frequency equal to ω_C, obtain the polynomial that specifies the closed-loop poles (as a function of a/b and ω_C). Show that one closed-loop pole is at s = - ω_C no matter what a/b is.

   Hints: Find: $G(s) = P(s)\cdot C(s)$

   Find the denominator of the closed-loop transfer function: $D_G + N_G$

   Substitute in a, b, and k_c like eq. 5.62 in book, but use $\sqrt{\frac{a}{b}}$ instead of 3 (as mentioned in class).

   Use polynomial division to show that $D_G + N_G$ can be divided by $(s + \omega_c)$ with no remainder.

   b) Compute the other closed-loop poles, as functions of ω_C, when $a/b = 5.83$, 9, and 13.9.
   Hint: The "other" roots are the roots of the quotient.

Answers

1. a) $GM \simeq 30\cdot dB$ $PM \simeq 40\cdot deg$ b) yes c) $0 < g < \frac{1}{3}$, $\frac{1}{2} < g < \frac{3}{2}$ or $g > 3$
   d) Need 3 CCW encirclements of -1

2. a) yes b) $GM \simeq 2 \ (6\cdot dB)$ $PM \simeq 90\cdot deg$ c) 4

   d) 4 , $3\cdot\cos(t - 90\cdot deg)$, $-2\cdot\cos(5\cdot t)$ e) $\frac{4}{3}$, $\frac{3\cdot\sqrt{2}}{2}\cdot\cos(t - 45\cdot deg)$, $-4\cdot\cos(5\cdot t)$

3. a) $k < \frac{1}{2}$, $\frac{2}{3} < k < 2$ b) Gain may be increased by $\simeq 2dB$ and reduced by $\simeq 5dB$. $PM = 10^o$ to 15^o

   c) Open loop: -4 Closed loop: 4

4. b) $(-0.7071 - 0.7071\cdot j)\cdot\omega_c$ & $(-0.7071 + 0.7071\cdot j)\cdot\omega_c$, $-\omega_c$ & $-\omega_c$ (double) , $-0.436\cdot\omega_c$ & $-2.292\cdot\omega_c$