ECE 3510 homework RL3

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2/25/10

1. Problem 4.5 (p.99) in the Bodson text.

   a) Sketch (by hand) the root-locus plot for the following open-loop transfer function:

   Apply only the main rules (Section 4.4.2 in text or the first page of class notes)

   $$G(s) = \frac{s\cdot(s+1)}{(s+2)^2\cdot(s+3)}$$

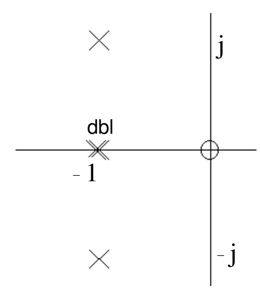
   b) Repeat part a) for: $G(s) = \frac{(s+3)}{s\cdot(s+9)^3}$

   c) Repeat part a) for: $G(s) = \frac{(s+a)}{(s+b)\cdot\left(s^2-2\cdot s+2\right)}$ $a > 0$ $b > 0$ $k > 0$

   a, b, & k are all positive, real numbers

   Also give condition(s) that a and b must satisfy for the closed-loop system to be stable for sufficiently high gain (k) (note that you do not need to apply the Routh-Hurwitz criterion, nor provide the range of k for which the system is closed-loop stable).

2. Problem 4.12 in the Bodson text.
   Sketch the root-locus for the open-loop poles shown at right, using only the main rules.
   There is a zero at $s = 0$, two poles at $s = -1$ and two poles at $s = -1 \pm j$.



The following review questions and problems come from the Nise 3rd Ed., starting on page 471,
Or 4th Ed., starting p 473. If you are using the 4th Ed., <u>clearly</u> state that on your homework.

3. Nise, Ch.8, review question 3, rephrased here: If $G(s_1) = 5 \angle 180°$, is the point s_1 on the root locus?
   If yes, what gain factor would place a closed-loop pole at s_1?

4. Nise, Ch.8, review questions 4, 6, 7, 8, 9, 10.

5. Nise, Ch.8, problem 3

6. a) Find the break-in point for Nise, problem 3a, above. Note, the math here may drive you nuts, but you may simply test and prove that a point you guess is correct.

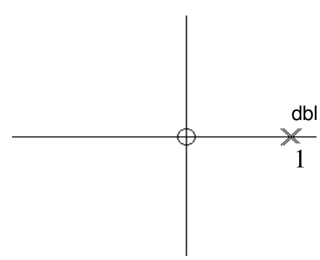
   b) Find the break-away point for Nise, problem 3d, above

7. Problem 4.11 in the Bodson text. (Hint: do part c before b)

   a) Sketch the root-locus for the open-loop poles shown at right.
   There is one zero at $s = 0$ and two poles at $s = 1$.

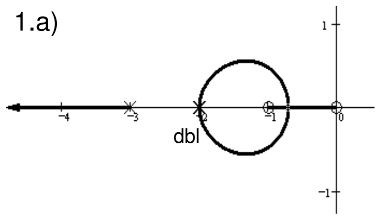
   c) Give the location(s) of the break-away point(s) (or arrival) on the real axis.

   b) Give the range of gain k ($k > 0$) for which the system is closed-loop stable, and give the locations of the $j\omega$ axis crossings.

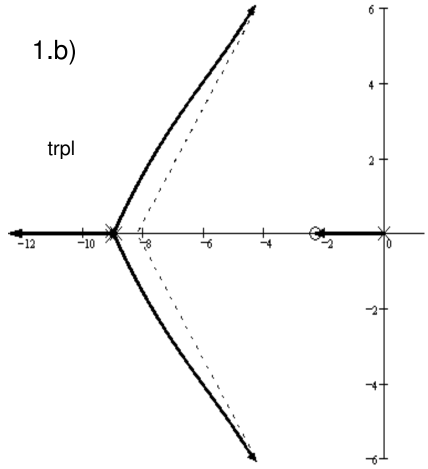


Answers

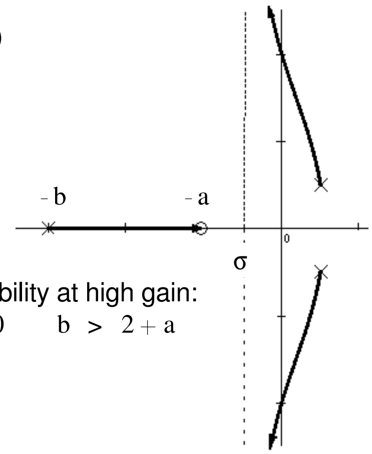
1.a)



1.b)

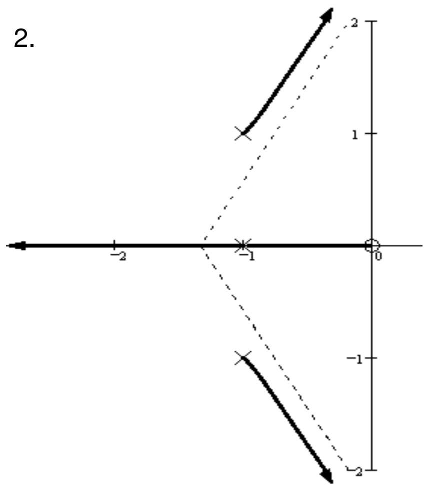


1.c)

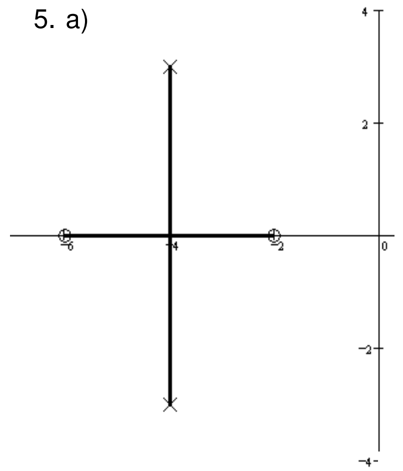


Stability at high gain:
$a>0 \quad b > 2+a$

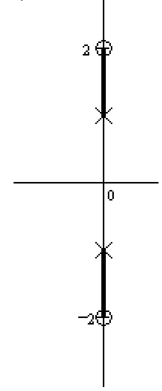
2.



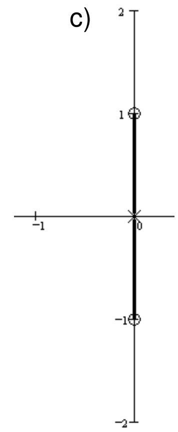
5. a)



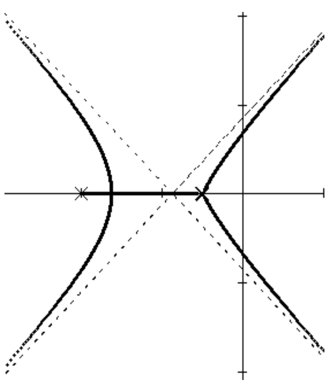
b)



c)



5. d)

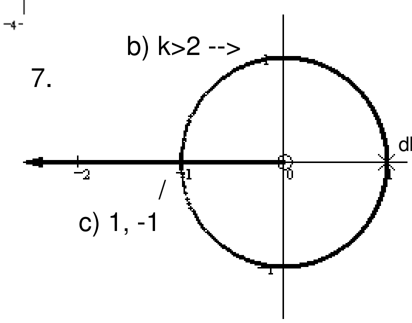


6. a) -4

b) -3.25

7.

b) k>2 -->



c) 1, -1

ANSWERS TO REVIEW QUESTIONS

1. The plot of a system's closed-loop poles as a function of gain
2. (1) Finding the closed-loop transfer function, substituting a range of gains into the denominator, and factoring the denominator for each value of gain.
   (2) Search on the s-plane for points that yield 180 degrees when using the open-loop poles and zeros.

3. 3. Yes, K = 1/5
4. 4. No
| 5. At the zeros of G(s) and the poles of H(s)
| 6. (1) Apply Routh-Hurwitz to the closed-loop transfer function's denominator.
\|/ (2) Search along the imaginary axis for $\angle G(s) = \pm 180°$.

7. If any branch of the root locus is in the rhp, the system may be unstable.
   If the gain places one of the closed-loop poles on that part of the branch, it will be unstable.
8. If the branch of the root locus is vertical, the settling time remains constant for that range of gain on the vertical section.
9. The natural frequency is the distance of a pole from the origin. If a region of the root locus is circular and the center of the circle is at the origin, then the natural frequency would not change over that region of gain.
10. Determine if there are any break-in or breakaway points
11. (1) Poles must be at least five times further from the imaginary axis than the dominant second order pair,
    (2) Zeros must be nearly canceled by higher order poles.
12. Number of branches, symmetry, starting and ending points
13. The zeros of the open loop system help determine the root locus. The root locus ends at the zeros.
    Thus, the zeros are the closed-loop poles for high gain.