AC Notes

ECE 2210 / 00

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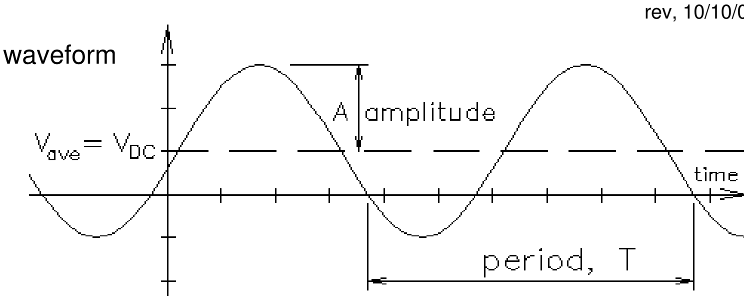
AC = **A**lternating **C**urrent

This term is used for any time-varying voltage or current waveform

Periodic waveforms: Waveshape repeats

- T = Period = repeat time
- f = frequency, cycles / second $\quad f = \frac{1}{T} = \frac{\omega}{2\cdot\pi}$
- ω = radian frequency, radians/sec $\quad \omega = 2\cdot\pi\cdot f$
- A = amplitude
- DC = average



Sinusoidal AC

$y(t) = A\cdot\cos(\omega\cdot t + \phi)$

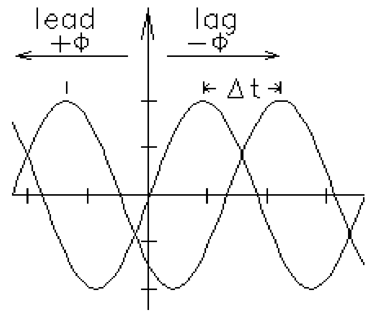
Phasor

voltage: $v(t) = V_p\cdot\cos(\omega\cdot t + \phi) \qquad \mathbf{V}(\omega) = V_p\cdot e^{j\cdot\phi}$

current: $i(t) = I_p\cdot\cos(\omega\cdot t + \phi) \qquad \mathbf{I}(\omega) = I_p\cdot e^{j\cdot\phi}$

Phase: $\phi = \frac{\Delta t}{T}\cdot 360\cdot\text{deg}$ or: $\phi = \frac{\Delta t}{T}\cdot 2\cdot\pi\cdot\text{rad}$

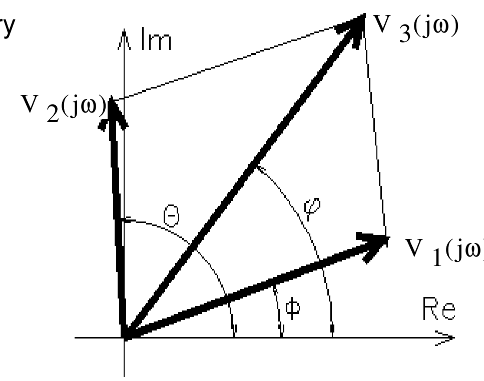
Phase:



Adding and subtracting Sinusoidal AC voltages or currents

Voltages and currents are represented by phasors draw on a real - imaginary "phasor" diagram. These phasors add and subtract just like vectors.

$v_1(t) = V_{p1}\cdot\cos(\omega\cdot t + \phi) \qquad v_2(t) = V_{p2}\cdot\cos(\omega\cdot t + \theta)$



The problem: $v_3(t) = v_1(t) + v_2(t) = ?$

Draw phasors as shown, $\mathbf{V_1}(j\omega) = V_{p1}\ \underline{/\phi}$ and $\mathbf{V_2}(j\omega) = V_{p2}\ \underline{/\theta}$

Add the phasors just as you would vectors to get: $\mathbf{V_3}(j\omega) = V_{p3}\ \underline{/\psi}$

$$V_{p3} = \sqrt{\left(V_{p1}\cdot\cos(\phi) + V_{p2}\cdot\cos(\theta)\right)^2 + \left(V_{p1}\cdot\sin(\phi) + V_{p2}\cdot\sin(\theta)\right)^2}$$

$$\psi = \arctan\left(\frac{V_{p1}\cdot\sin(\phi) + V_{p2}\cdot\sin(\theta)}{V_{p1}\cdot\cos(\phi) + V_{p2}\cdot\cos(\theta)}\right)$$

Convert phasor back to time-domain voltage: $v_3(t) = v_1(t) + v_2(t) = V_{p3}\cdot\cos(\omega\cdot t + \psi)$ Done

Phasor analysis, For steady-state sinusoidal response ONLY

Phasors are used for much more than just adding and subtracting sinusoidal waveforms and are drawn on a complex plane for a very good reason. The math is all based on the Euler's equation:

Euler's equation $\quad e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$ $\quad$ OR: $\quad \cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2} \qquad \sin(\theta) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$

$$e^{j\cdot(\omega\cdot t+\theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$$

$$\mathrm{Re}\left[e^{j\cdot(\omega\cdot t+\theta)}\right] = \cos(\omega\cdot t + \theta)$$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$ That's the phasor

Capacitors and Inductors in AC circuits cause 90° phase shifts between voltages and currents because they integrate and differentiate. But... integration and differentiation is a piece-of-cake in phasors.

Drop the ωt (t=0) to get:

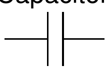
Calculus $\quad \frac{d}{dt}\left[A\cdot e^{j\cdot(\omega\cdot t+\theta)}\right] = j\cdot\omega\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta)} = \omega\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta+90\cdot\text{deg})} = \omega\cdot A\cdot e^{j\cdot(\theta+90\cdot\text{deg})}$

$$\int A\cdot e^{j\cdot(\omega\cdot t+\theta)}\,dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta)} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta-90\cdot\text{deg})} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta-90\cdot\text{deg})}$$

Phasor analysis with impedances, For steady-state sinusoidal response ONLY

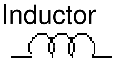
AC impedance

Capacitor



$i_C = C\cdot\frac{d}{dt}v_C \qquad v_C = \frac{1}{C}\cdot\int i_C(t)\,dt \qquad \mathbf{Z_C} = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C} \qquad \mathbf{V_C}(j\omega) = \frac{1}{j\cdot\omega\cdot C}\cdot\mathbf{I}(j\omega)$

Inductor



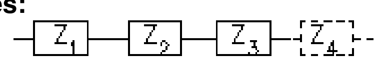
$v_L = L\cdot\frac{d}{dt}i_L \qquad i_L = \frac{1}{L}\cdot\int v_L(t)\,dt \qquad \mathbf{Z_L} = j\cdot\omega\cdot L \qquad \mathbf{V_L}(j\omega) = j\cdot\omega\cdot L\cdot\mathbf{I}(j\omega)$

Resistor

$v_R = i_R\cdot R \qquad i_R = \frac{v_R}{R} \qquad \mathbf{Z_R} = R \qquad \mathbf{V_R}(j\omega) = R\cdot\mathbf{I}(j\omega)$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:

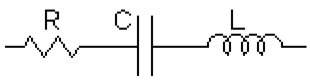


$$\mathbf{Z_{eq}} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots$$

Voltage divider:

$$\mathbf{V_{Zn}} = \mathbf{V_{total}}\cdot\frac{\mathbf{Z_n}}{\mathbf{Z_1}+\mathbf{Z_2}+\mathbf{Z_3}+\ldots}$$

Example:

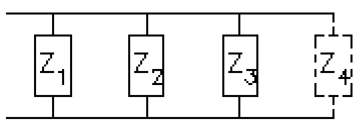


If: $R := 200\cdot\Omega \qquad C := 2\cdot\mu F \qquad L := 16\cdot mH$ and $\omega := 4000\cdot\frac{rad}{sec}$

$$\mathbf{Z_{eq}} = R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L$$

then $\mathbf{Z_{eq}} = R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L = 200\cdot\Omega - 125\cdot j\cdot\Omega + 64\cdot j\cdot\Omega = 200 - 61j\ \bullet\Omega$

parallel:

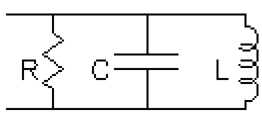


$$\mathbf{Z_{eq}} = \frac{1}{\frac{1}{\mathbf{Z_1}}+\frac{1}{\mathbf{Z_2}}+\frac{1}{\mathbf{Z_3}}+\ldots}$$

Current divider:

$$\mathbf{I_{Zn}} = \mathbf{I_{total}}\cdot\frac{\frac{1}{\mathbf{Z_n}}}{\frac{1}{\mathbf{Z_1}}+\frac{1}{\mathbf{Z_2}}+\frac{1}{\mathbf{Z_3}}+\ldots}$$

Example:



If: $R := 200\cdot\Omega \qquad C := 2\cdot\mu F \qquad L := 16\cdot mH$ and $\omega := 4000\cdot\frac{rad}{sec}$

$$\mathbf{Z_{eq}} = \frac{1}{\frac{1}{R}+\frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)}+\frac{1}{j\cdot\omega\cdot L}}$$

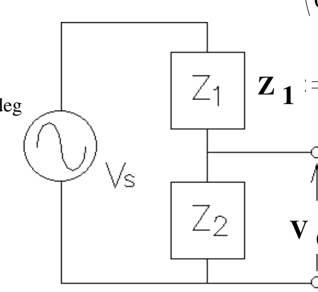
then $\mathbf{Z_{eq}} = \frac{1}{\frac{1}{R}+j\cdot\omega\cdot C-\frac{j}{\omega\cdot L}} = \frac{1}{\frac{1}{200\cdot\Omega}+8\cdot10^{-3}\cdot j\cdot\frac{1}{\Omega}-0.01563\cdot j\cdot\frac{1}{\Omega}}$

$= 60.1\cdot\Omega + 91.7\cdot\Omega = 109.7\Omega\ \underline{/56.7^o}$

$\sqrt{(60.1\cdot\Omega)^2+(91.7\cdot\Omega)^2} = 109.6\bullet\Omega \qquad \operatorname{atan}\left(\frac{91.7\cdot\Omega}{60.1\cdot\Omega}\right) = 56.76\bullet deg$

Example

Find $\mathbf{V_O}$ in the circuit shown. Express it as a magnitude and phase angle (the way $\mathbf{V_S}$ is expressed).



$\mathbf{V_S} := 6\cdot V\cdot e^{j\cdot18\cdot deg} \qquad \mathbf{Z_1} := 25\cdot\Omega + 35j\cdot\Omega \qquad \mathbf{Z_2} := 80\cdot\Omega\cdot e^{-j\cdot60\,deg}$

$\mathbf{V_O} = ?$

$\mathbf{V_O} := \frac{\mathbf{Z_2}}{\mathbf{Z_1}+\mathbf{Z_2}}\cdot\mathbf{V_S}$ Simple voltage divider

$\left|\mathbf{Z_2}\right|\cdot\cos(-60\cdot deg) = 40\bullet\Omega \qquad \left|\mathbf{Z_2}\right|\cdot\sin(-60\cdot deg) = -69.282\bullet\Omega$

$\mathbf{Z_2} = 40 - 69.282j\ \bullet\Omega \qquad \mathbf{Z_1}+\mathbf{Z_2} = 25\cdot\Omega + 35j\cdot\Omega + 40\cdot\Omega - 69.282\cdot j\cdot\Omega = 65 - 34.282j\ \bullet\Omega = 73.486\cdot\Omega\cdot e^{-j\cdot27.81\cdot deg}$

$$\mathbf{V_O} := \frac{\mathbf{Z_2}}{\mathbf{Z_1}+\mathbf{Z_2}}\cdot\mathbf{V_S} = \frac{80\cdot\Omega\cdot e^{-j\cdot60\,deg}}{73.486\cdot\Omega\cdot e^{-j\cdot27.81\cdot deg}}\cdot\left(6\cdot V\cdot e^{j\cdot18\cdot deg}\right) = \frac{80\cdot\Omega}{73.486\cdot\Omega}\cdot6\cdot V\cdot e^{j\cdot(-60-(-27.81)+18)\cdot deg} = 6.53\cdot V\cdot e^{-j\cdot14.2\cdot deg}$$