ECE 3510
Bode Plot Examples

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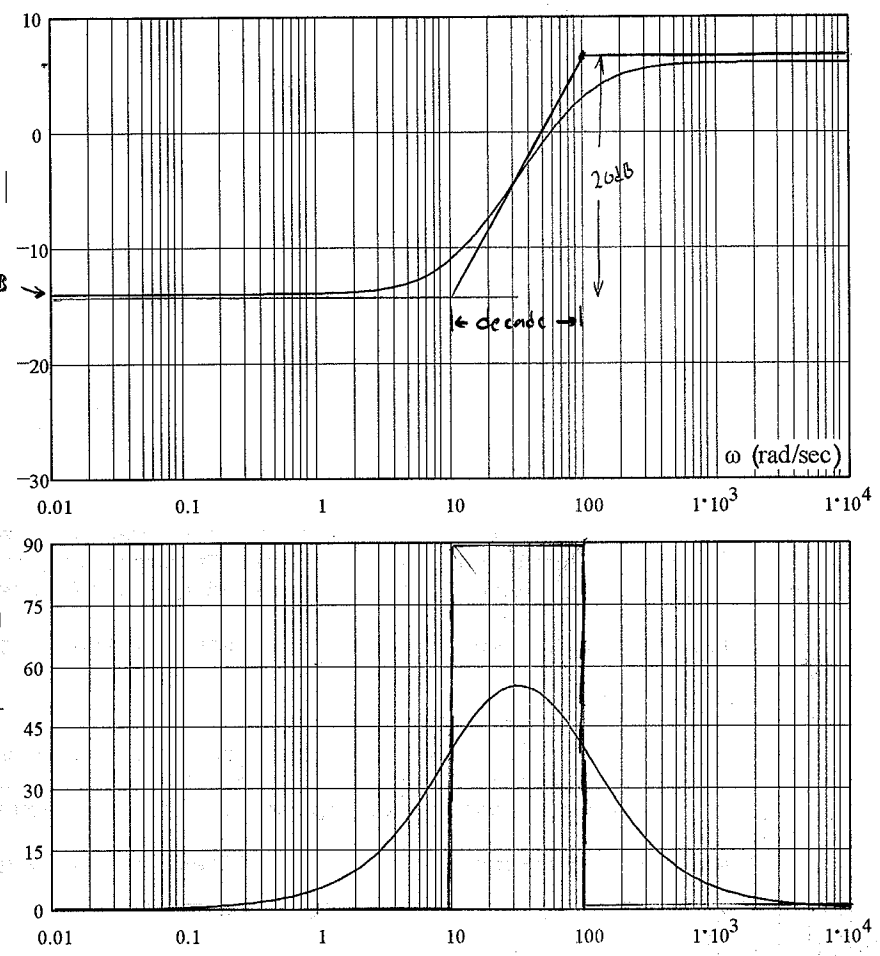
3/11/08

Ex. 1 $P(s) = \dfrac{2\cdot(s+10)}{s+100}$

$\dfrac{2(+10)}{(+100)} = \dfrac{20}{100} = .2 \Rightarrow -14\text{dB}$, $0°$

$\dfrac{2(j\omega+10)}{+100}$ ↗ $\frac{20\text{dB}}{\text{dec}}$ $90°$

$\dfrac{2(j\omega+10)}{(j\omega+100)} \rightarrow 6\text{dB}, 0°$



Ex. 2 $P(s) = \dfrac{s+20}{4\cdot(s+1)^2}$

$\dfrac{+20}{4(+1)^2} = \dfrac{20}{4} = 5 \Rightarrow 14\text{dB}$, $0°$

$\dfrac{+20}{4(j\omega+1)^2}$ ↘ $\frac{-40\text{dB}}{\text{dec}}$

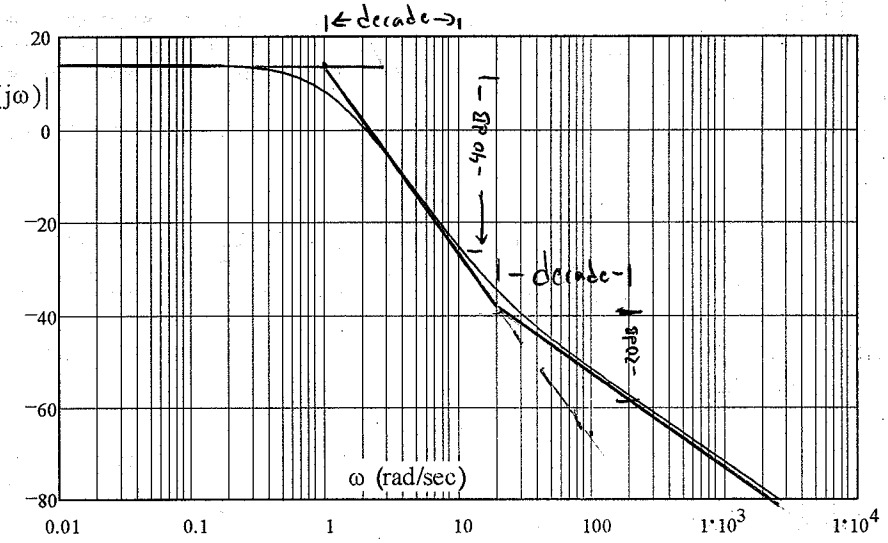
$-90 - 90° = -180°$

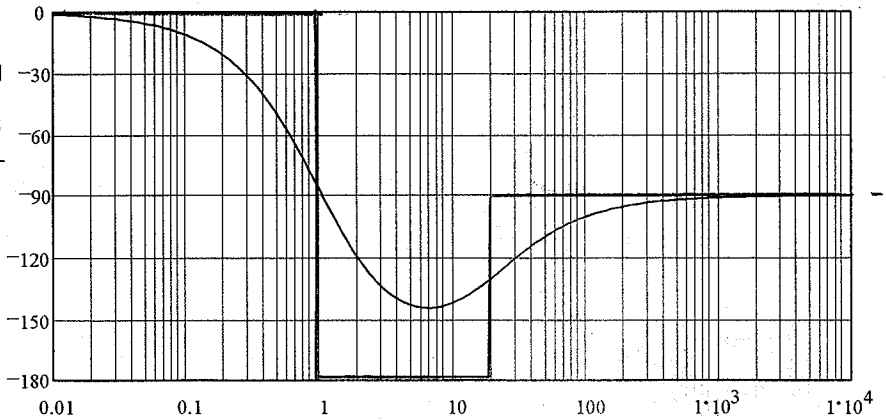
@ $\omega = 20$

$\dfrac{20}{4\cdot 20^2} = \dfrac{1}{80} = -38\text{dB}$

$\dfrac{j\omega+20}{4\cdot(j\omega+1)^2}$

$-90°$





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Ex. 3

$$P_3(s) = \frac{20000\cdot(s+0.1)}{(s-4)\cdot(s+1000)}$$

$\frac{20{,}000\,(\_ + .1)}{(\_ - 4)(\_ + 1000)} \rightarrow .5 = -6dB$ dB

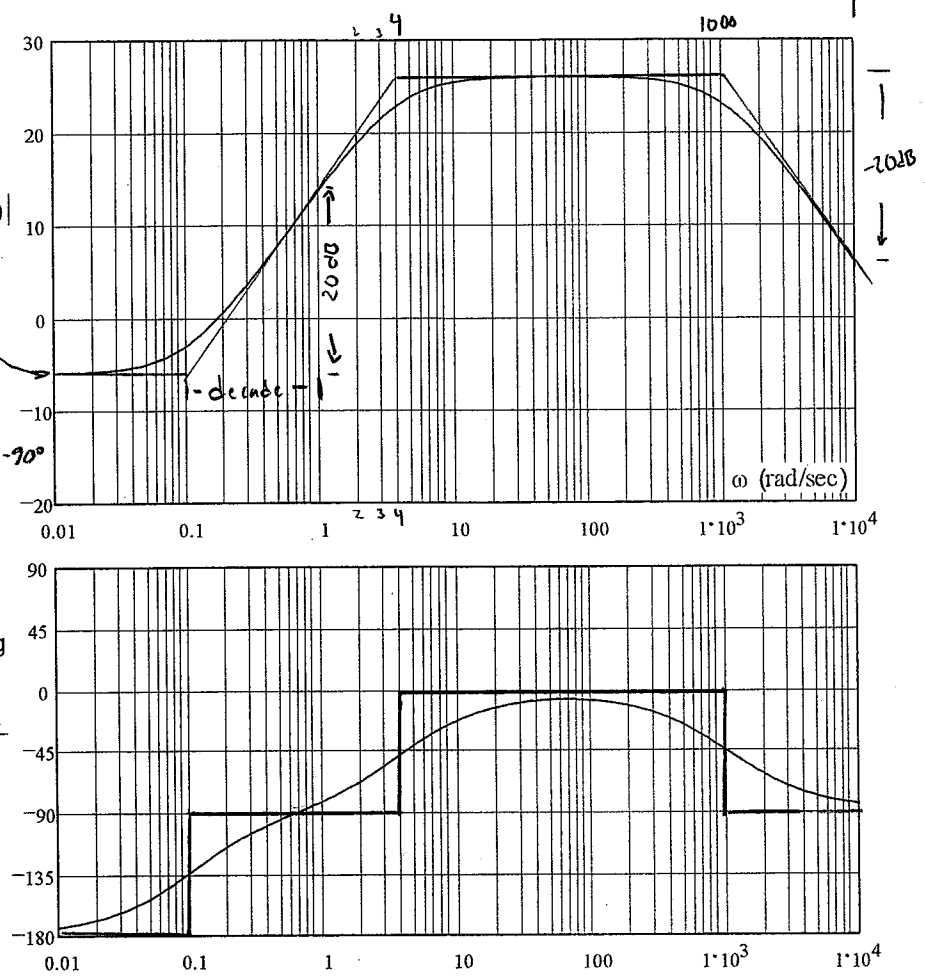
$\angle -180°$

$\frac{20{,}000\,(j\omega + \cancel{X})}{(\_ - 4)(\_ + 1000)}$ $\nearrow \frac{20dB}{dec}$ $\angle 90° - 180° = -90°$

$\frac{20{,}000\,(j\omega + \cancel{X})}{(j\omega - \cancel{X})(\_ + 1000)} \rightarrow 20 = 26dB$ phase deg

$\angle 0°$

$\frac{20{,}000\ \ j\omega + \cancel{X}}{(j\omega - \cancel{X})(j\omega + 1000)}$ $\searrow \frac{20dB}{dec}$ $\angle -90°$ $\angle P(j\omega)$



Ex. 4 $P_4(s) = \frac{0.5\cdot(s+1)\cdot(s-20)}{s\cdot(s+100)}$

$\frac{0.5\,(\_ + 1)(\_ - 20)}{j\omega\ (\_ + 100)}$ $|P(j\omega)|$ dB

$j\omega$: .01, try .1 $\rightarrow$ 1 = 0dB

$= 10 = 20dB$

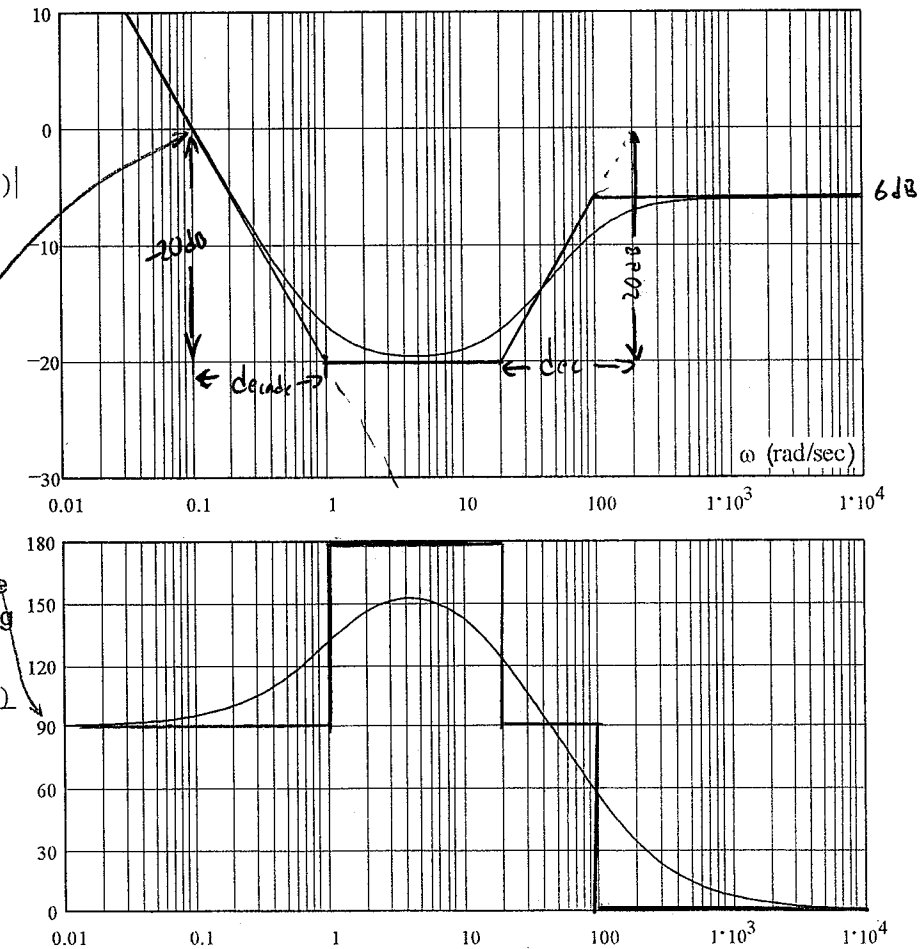
$\searrow \frac{-20dB}{dec}$ $\angle 180° - 90° = 90°$

$\frac{0.5\,(j\omega + \cancel{X})(\_ - 20)}{j\omega\ (\_ + 100)} = .1 = -20dB$ phase deg

$\angle +180°$

$\frac{0.5 \times (j\omega \cancel{-20})}{(\_ + 100)}$ $\nearrow \frac{20dB}{dec}$ $\angle 90°$ $\angle P(j\omega)$

$\frac{0.5 \times j\omega}{j\omega + \cancel{100}} = -6dB$ $\angle 0°$



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Ex. 5

$$P_5(s) := \frac{5000 \cdot s \cdot (s-4)}{(s+0.2)\cdot(s+20)\cdot(-s+1000)}$$

$\frac{5000\, j\omega\, (-4)}{(+.2)(+20)(+1000)}$ $90°$ $+180°$ @ $\omega = .01$ $= \frac{5000 \cdot .01 \cdot 4}{.2 \cdot 20 \cdot 1000}$ $|P(j\omega)|$

$= .05 \Rightarrow -26\text{dB}$

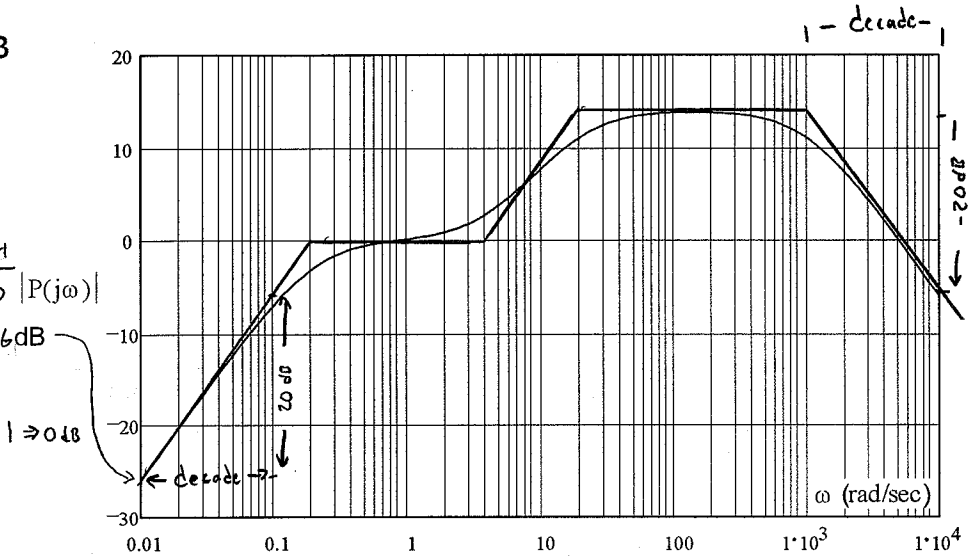
$\frac{5000\, j\omega\, (-4)}{(j\omega+.2)(+20)(+1000)} = \frac{5000 \cdot 4}{20 \cdot 1000} = 1 \Rightarrow 0\text{dB}$ $+180°$

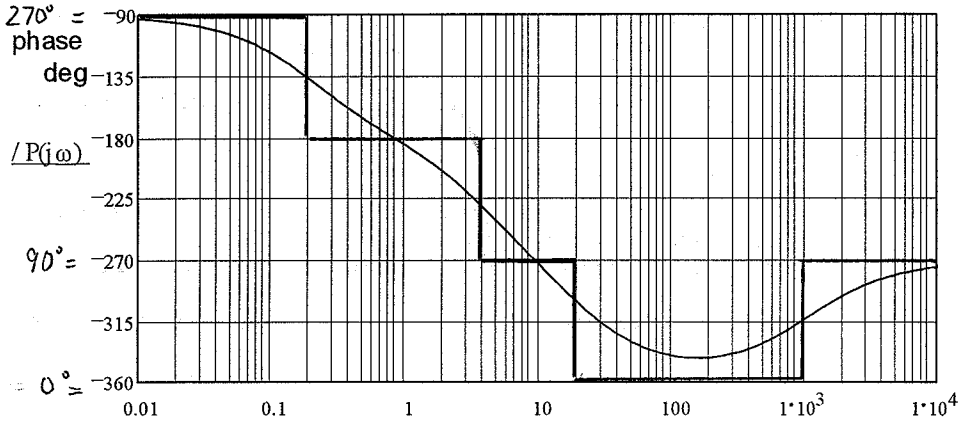
$\frac{5000\, (j\omega)}{(+20)(+1000)}$ $90°$ $\nearrow \frac{20\text{dB}}{\text{dec}}$

$\frac{5000\, j\omega}{(j\omega+20)(+1000)} = 5 \Rightarrow 14\text{dB}$

$\frac{5000}{(j\omega+1000)}$ $\searrow \frac{-20\text{dB}}{\text{dec}}$

$-(90°)$





Ex. 6

$$P_6(s) := \frac{20000 \cdot (s+0.1)}{(s+2)\cdot\left(s^2+10\cdot s+10000\right)}$$

$$s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^2$$

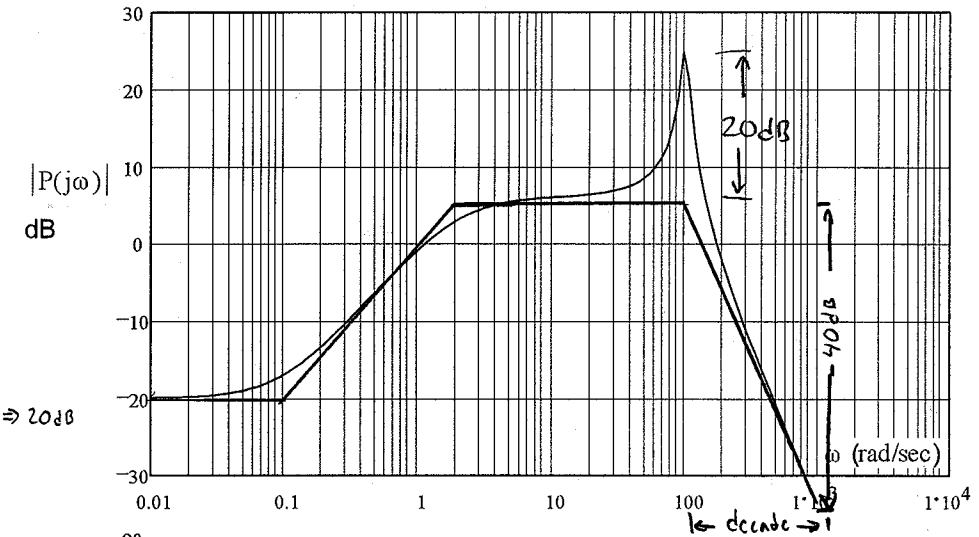
natural frequency

$$\omega_n = \sqrt{\omega_n^2} = \sqrt{10000} = 100$$

damping factor:

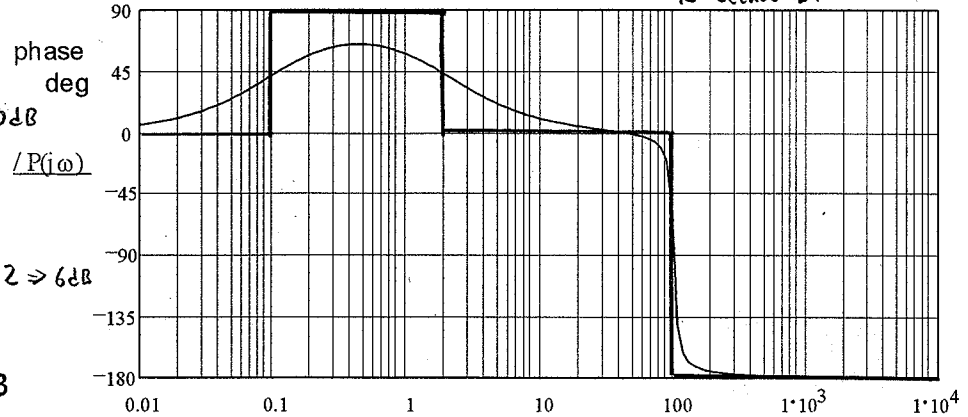
$$\zeta = \frac{2\cdot\zeta\cdot\omega_n}{2\cdot\omega_n} = \frac{10}{2\cdot 100} = 0.05$$

$\frac{1}{2(.05)} = .1 \Rightarrow 20\text{dB}$



$\frac{20{,}000\, (+.1)}{(+2)(+100)^2} = \frac{2000}{20000} \Rightarrow -20\text{dB}$ $0°$

$\frac{20{,}000 \cdot j\omega}{(+2)(+100^2)}$ $90°$ $\nearrow 20$ $\frac{20000\, j\omega}{(j\omega+2)(+100^2)} = 2 \Rightarrow 6\text{dB}$



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Ex. 7

$$P_7(s) := \frac{400\cdot(s+0.1)\cdot(s+100)}{\left[(s+0.4)^2+15.84\right]\cdot(s+1000)}$$

natural freq. $(s+a)^2+b^2$

$$\omega_n = \sqrt{a^2+b^2} = \sqrt{0.4^2+15.84} = 4$$

damping factor: $\zeta = \frac{a}{\zeta} = \frac{0.4}{4} = 0.1$

$\frac{1}{2\times.1} \Rightarrow 14\,dB$

$\frac{400(\_+.1)(\_+100)}{(\_+4)^2(\_+1000)} = \frac{4000}{16000} \Rightarrow -12dB$

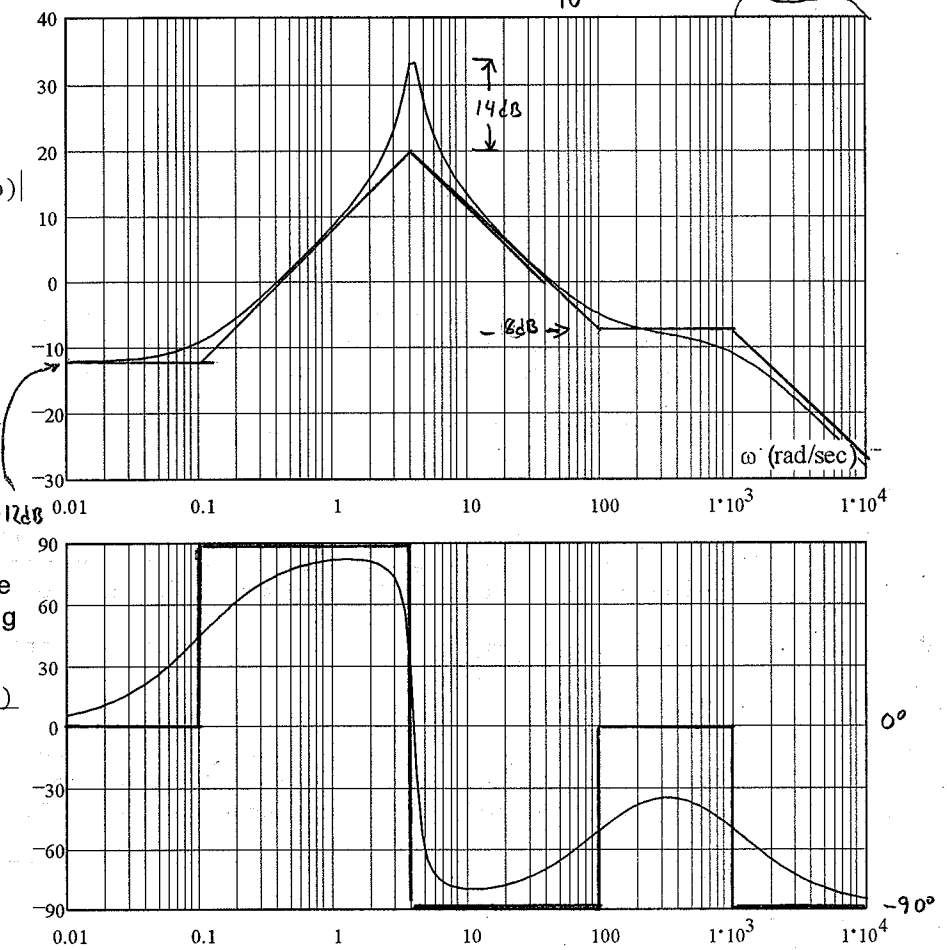
$\frac{400(jw+.1)(\_+100)}{(\_+4)^2(\_+1000)}$ $\nearrow 20dB/dec$ @ $90°$

$\frac{400(jw)(\_+100)}{(jw+4)^2(\_+1000)}$ $\searrow 20dB/dec$ @ $-90°$

$\frac{400(jw+.1)(\_+100)}{(jw+4)^2(\_+1000)}$ @ $0°$

$\frac{400}{(jw+10)}$ @ $-90°$ $\searrow -20dB/dec$

$\frac{400(jw+10)}{jw(\_+1000)} = .4 \Rightarrow -8dB$



Ex. 8

$$P_8(s) := \frac{25\cdot\left[(s+10)^2+9900\right]}{\left(s^2+s+4\right)\cdot(s+2000)}$$

$s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^2$

natural frequency $\omega_{n1} = \sqrt{\omega_{n1}^2} = \sqrt{4} = 2$

damping factor: $\zeta = \frac{2\cdot\zeta\cdot\omega_{n1}}{2\cdot\omega_{n1}} = \frac{1}{2\cdot2} = 0.25$

$\frac{1}{2\cdot.25} \Rightarrow 6dB$

natural freq. $\omega_{n2} = \sqrt{a^2+b^2} = \sqrt{10^2+9900} = 100$

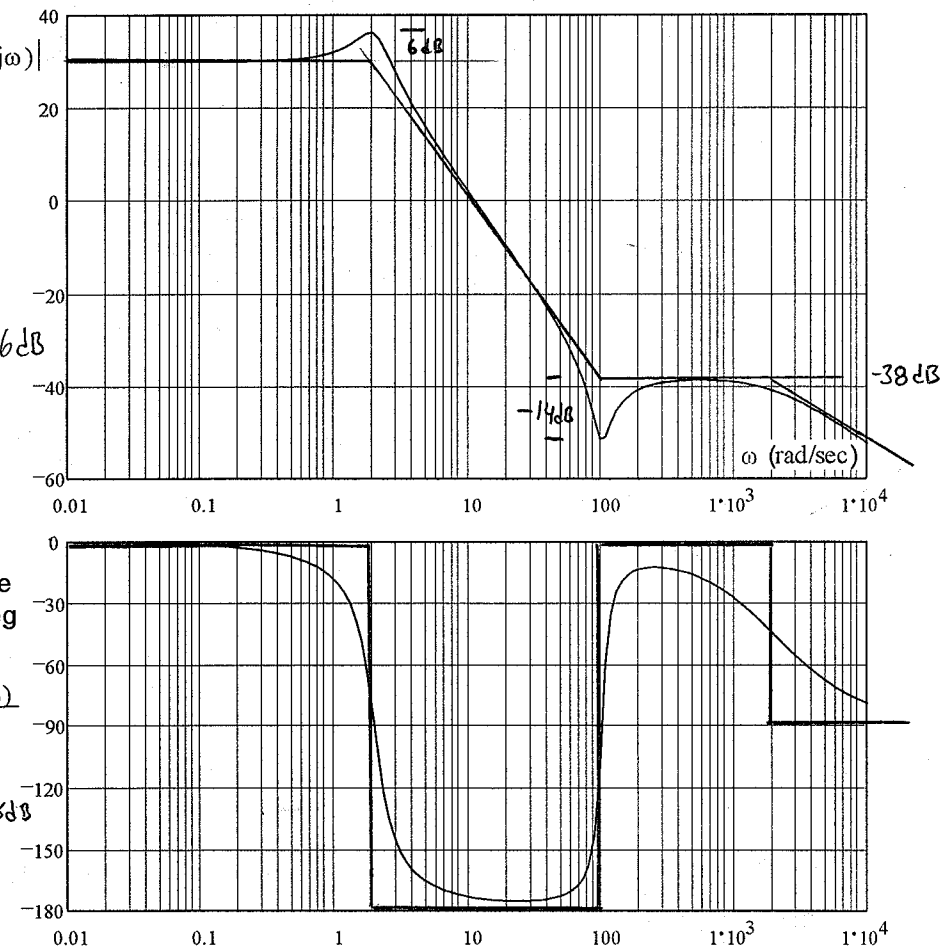
damping factor: $\zeta = \frac{10}{100} = 0.1$ $\frac{1}{2\cdot.1} \Rightarrow 14dB$

$\frac{25(\_+100)^2}{(\_+2)^2(\_+2000)} = \frac{250000}{4\cdot2000} \Rightarrow 30dB$

31.25

$\frac{25(\_+100)^2}{(jw+2)^2(\_+2000)}$ $\searrow \frac{40dB}{dec}$ @ $-90-90° = -180°$

@ $w=100$ $\frac{25\cdot100^2}{100^2\cdot2000} = .125 = -38dB$



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