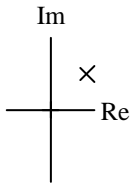
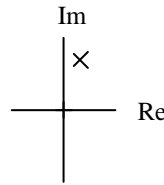


ECE 3510 How Nyquist Plots Work see pages 615 and 616 in Nise

For any point on the s-plane

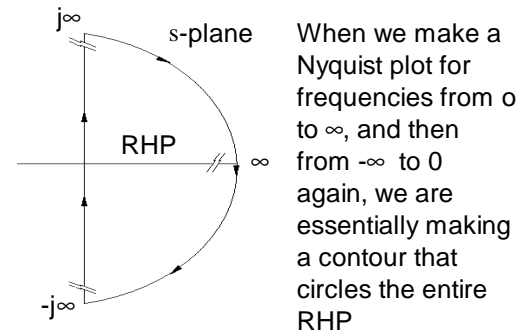
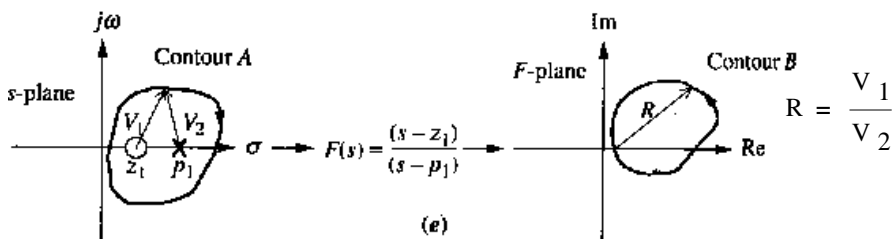
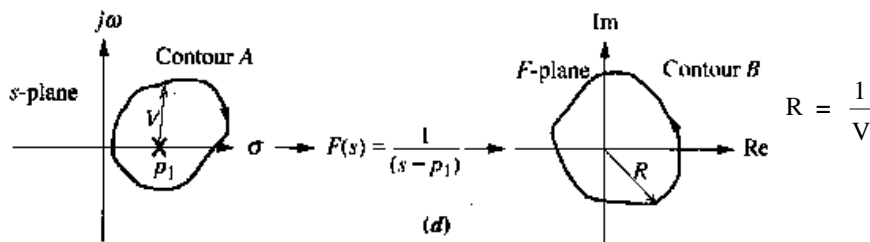
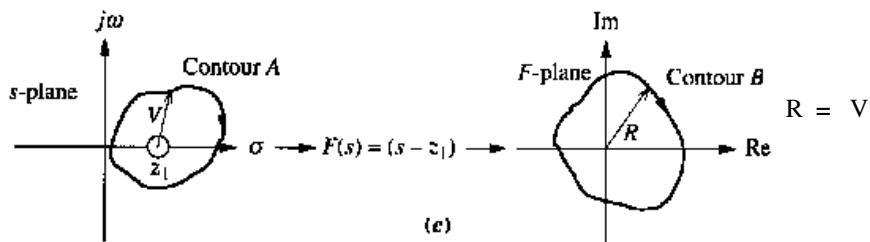
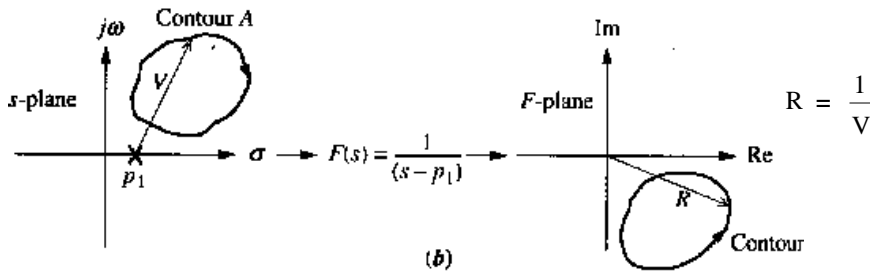
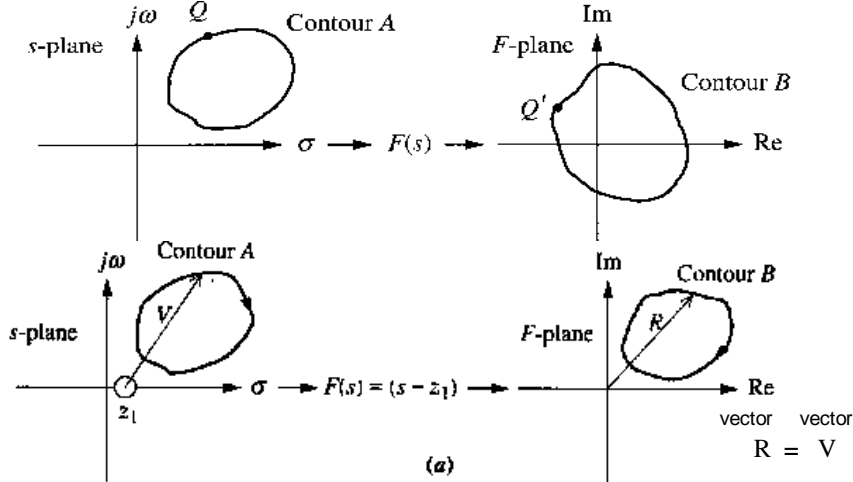


You can find a corresponding point on the F(s) plane



For any contour on the s-plane

There is a corresponding contour on the F(s) plane



When we make a Nyquist plot for frequencies from 0 to ∞, and then from -∞ to 0 again, we are essentially making a contour that circles the entire RHP

This gives information about poles and zeros in the RHP. This information reveals itself in the encirclements of 0, OR if we want the same information about 1 + G(s), in the encirclements of -1.

Closed-loop transfer function
(G(s) includes gain, k)

$$\frac{G(s)}{1 + G(s)} = \frac{G(s)}{\left(\frac{D_G + N_G}{D_G} \right)}$$

poles of CL
= zeros of
1 + G(s)
= Z (RHP)

poles of
1 + G(s)
= poles of G(s)
= P (RHP)

We plot G(s), but count CW encirclements of -1, rather than 0 to get the CW encirclements that 1 + G(s) would have of 0. Thus.

$$N = \text{CW encirclements of } -1 \\ = Z - P$$

rearrange

$$Z = N + P$$

P = OL poles in RHP
(0 if open-loop stable)
(P cannot be negative)

N = CW encirclements of -1
CL System CANNOT be stable if N > 0

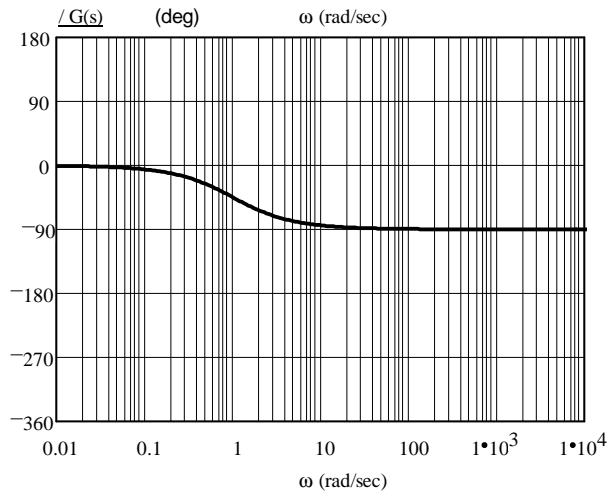
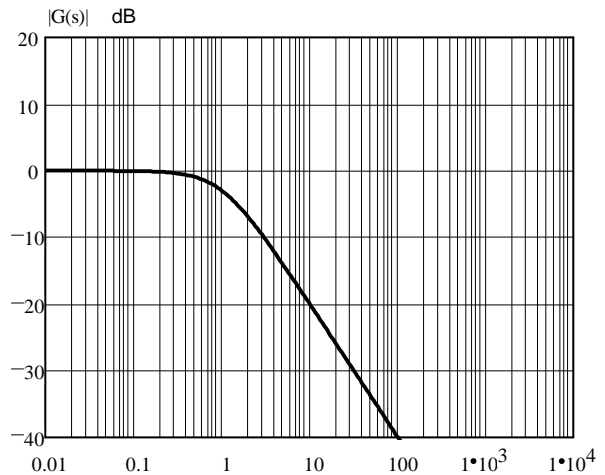
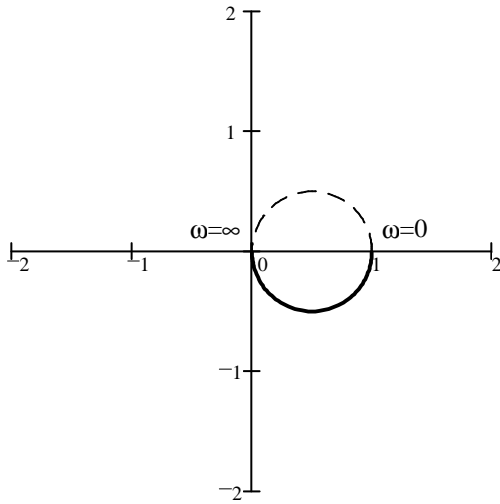
Z = CL poles in RHP
(must be zero (or ≤ 0)
if closed-loop stable)

ECE 3510 Nyquist Examples

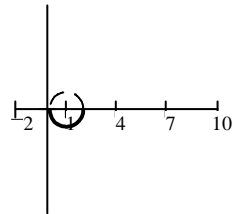
Example 1, Bodson p.119

$$G(s) := \frac{1}{(s+1)}$$

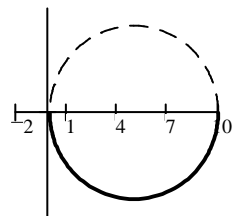
$$\frac{1}{j\omega+1} = \frac{1}{\sqrt{\omega^2+1^2}} - \text{atan}\left(\frac{\omega}{1}\right)$$



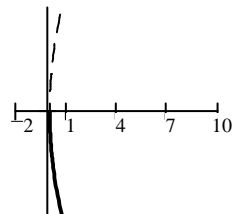
$$k \cdot G(s) = k \cdot \frac{1}{(s+1)} \quad k := 2$$



$$k := 10$$



$$k := 50$$

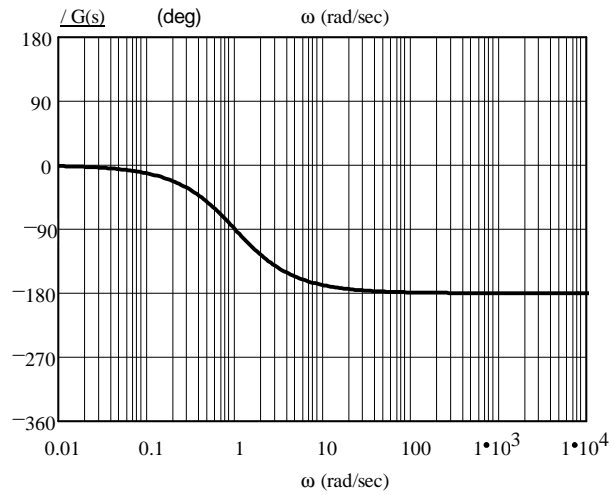
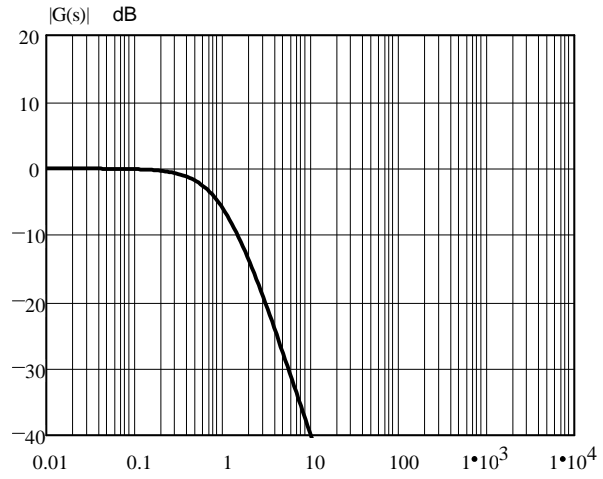
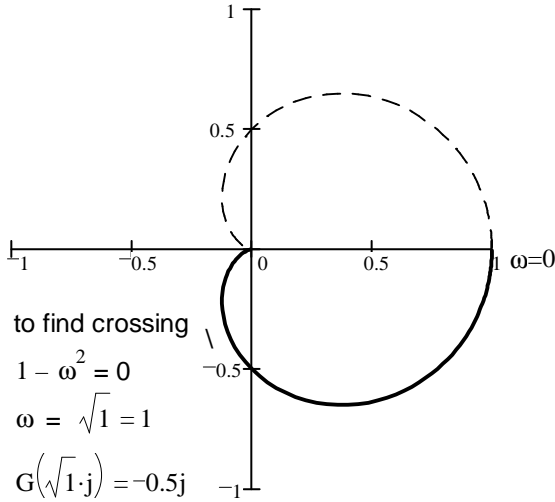


This plot never circles -1, no matter what the gain, so $N = 0$, no matter what k is.

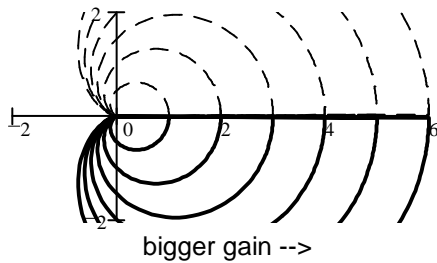
ECE 3510 Nyquist Examples p.2

$$G(s) := \frac{1}{(s+1)^2} = \frac{1}{s^2 + 2s + 1} = \frac{1}{(j\omega)^2 + 2(j\omega) + 1} = \frac{1}{(1-\omega^2) + 2j\omega} \cdot \frac{(1-\omega^2) - 2j\omega}{(1-\omega^2) - 2j\omega} = \frac{(1-\omega^2) - 2j\omega}{(1-\omega^2)^2 + (2j\omega)^2}$$

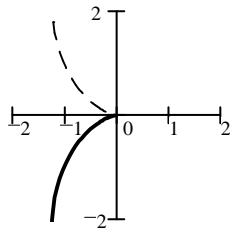
$$\frac{1}{(j\omega + 1)^2} = \frac{1}{\omega^2 + 1^2} - 2 \cdot \text{atan}\left(\frac{\omega}{1}\right)$$



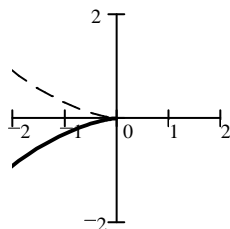
$$k \cdot G(s) = k \cdot \frac{1}{(s+1)^2}$$



k := 10

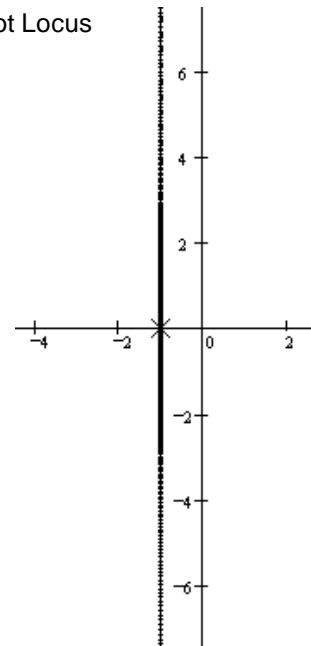


k := 50



N = 0, no matter what k is
But ringing gets worse with k

Root Locus

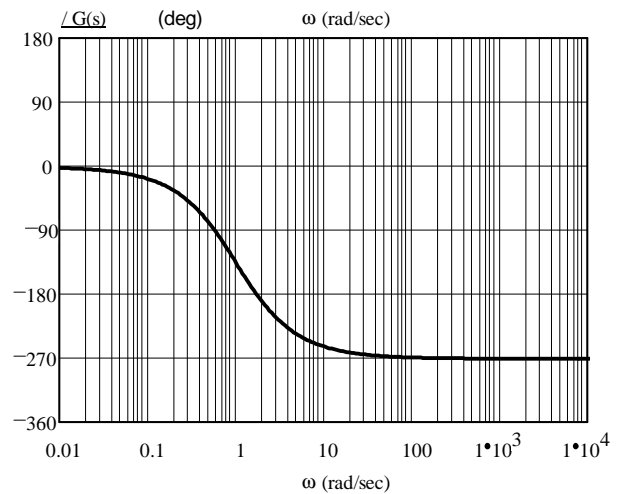
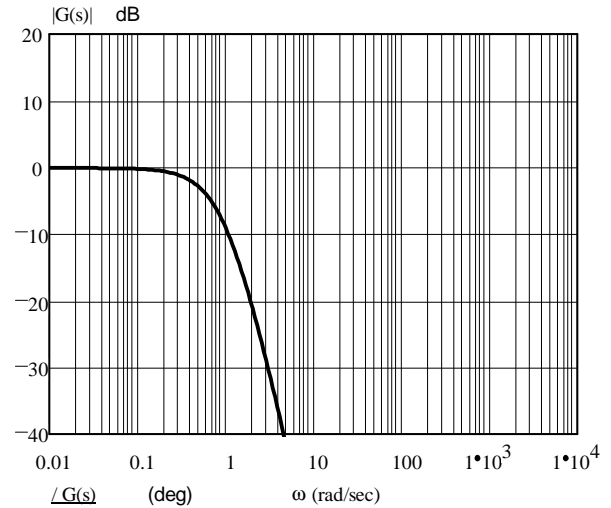
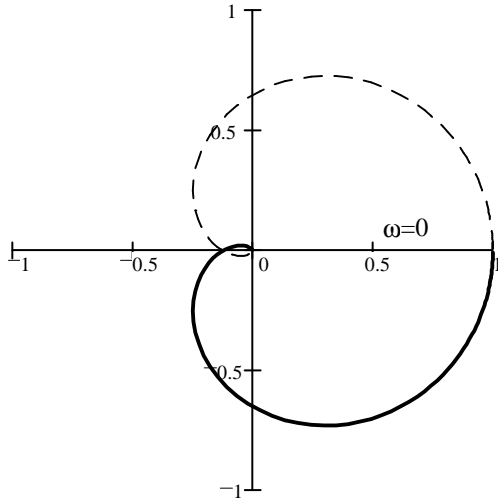


Example 2, Bodson p.119.

ECE 3510 Nyquist Examples p.3

$$G(s) := \frac{1}{(s+1)^3}$$

$$\frac{1}{(j\omega+1)^3} = \frac{1}{\left(\sqrt{\omega^2+1^2}\right)^3} - 3 \cdot \text{atan}\left(\frac{\omega}{1}\right)$$

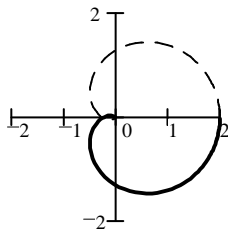


Find real-axis crossing (in left half plane)

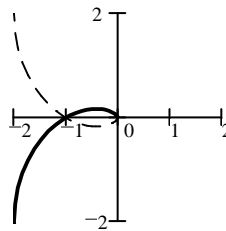
$$\frac{180}{3} = 60 \text{ deg} \quad \tan(60 \cdot \text{deg}) = \sqrt{3} = \frac{\omega}{1} = \omega$$

$$\frac{1}{\left(\sqrt{\omega^2+1^2}\right)^3} = \frac{1}{\left[\sqrt{(\sqrt{3})^2+1^2}\right]^3} = \frac{1}{8}$$

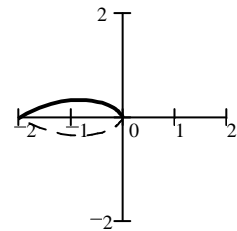
$$k \cdot \frac{1}{(s+1)^3} \quad k := 2$$



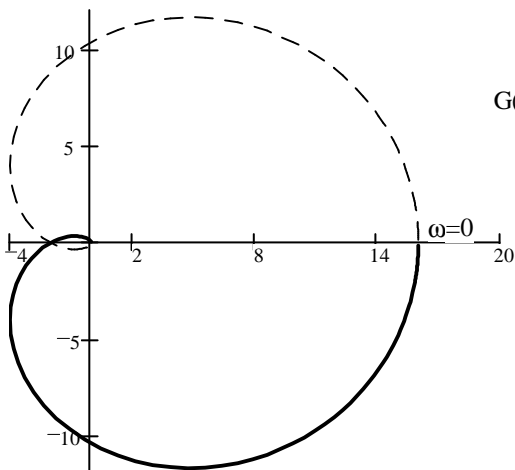
$$k := 8$$



$$k := 16$$



$$k := 16$$



$$G(s) = k \cdot \frac{1}{(s+1)^3}$$

$$P := 0$$

$$N := 2$$

$$Z := N + P$$

$$Z = 2$$

2 closed-loop poles in RHP

Closed-loop UNSTABLE

Gain Margins

For: $G(s) = \frac{1}{(s+1)^3}$ Real-axis crossing (in left half plane) is at $1/8$
 Gain can be 8 times bigger before $N = 2$ GM := 8

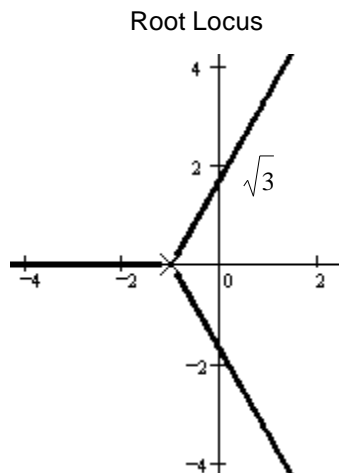
For: $G(s) = \frac{2}{(s+1)^3}$ GM := 4

For: $G(s) = \frac{4}{(s+1)^3}$ GM := 4

For: $G(s) = \frac{20}{(s+1)^3}$ GM := $\frac{8}{20} = \frac{2}{5}$

Other ways to find the same maximum gain

$$G(s) = \frac{1}{(s+1)^3}$$



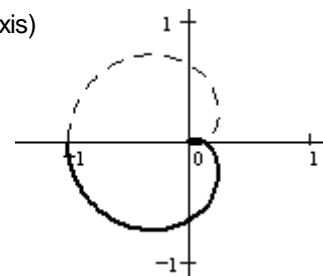
By Routh-Hurwitz:

Closed-loop denominator
 $(s+1)^3 + k = s^3 + 3s^2 + 3s + 1 + k$

s^3	1	3	
s^2	3	$1+k$	
s^1	$\frac{9 - 1 \cdot (1+k)}{3}$	$k < 8$	same result as previous page
s^0	$1+k$	$k > -1$	

Nyquist plot for negative k

(plot flips around $j\omega$ axis)



$N = 2$ for any k more negative than -1

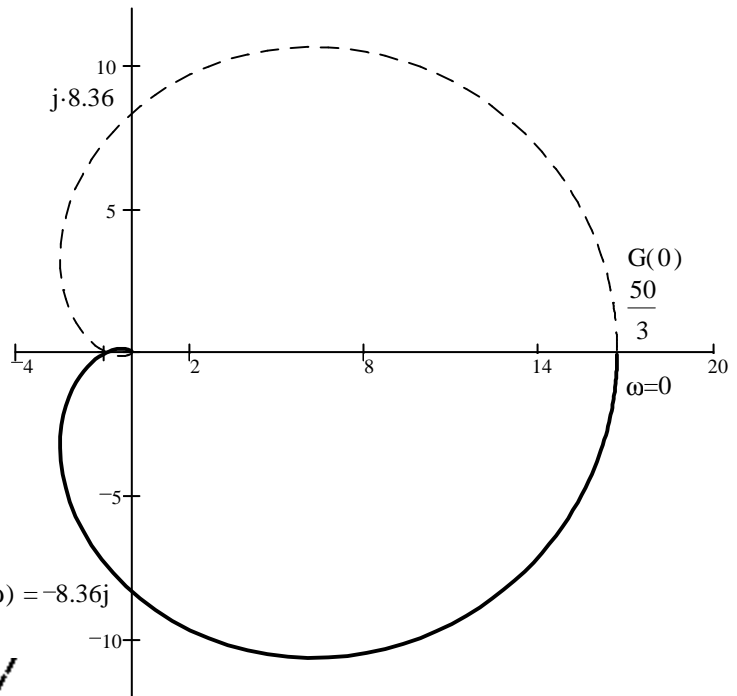
$$G(s) := \frac{100}{(s+10)} \cdot \frac{1}{(s+3)} \cdot \frac{5}{(s+1)} = \frac{500}{(s+10) \cdot (s+3) \cdot (s+1)}$$

$$G(0) = \frac{500}{10 \cdot 3 \cdot 1} = \frac{50}{3}$$

$$G(\omega) = \frac{500}{(j\omega+10) \cdot (j\omega+3) \cdot (j\omega+1)} = \frac{500}{[(-1) \cdot \omega^2 + 13j\omega + 30] \cdot (j\omega+1)} = \frac{500}{-\omega^3 j - \omega^2 + 13(-1) \cdot \omega^2 + 43j\omega + 30}$$

$$= \frac{500}{(-14\omega^2 + 30) + j(43\omega - \omega^3)} \cdot \frac{(-14\omega^2 + 30) - j(43\omega - \omega^3)}{(-14\omega^2 + 30) - j(43\omega - \omega^3)}$$

$$= \frac{(-14\omega^2 + 30) - j(43\omega - \omega^3)}{(-14\omega^2 + 30)^2 + (43\omega - \omega^3)^2}$$

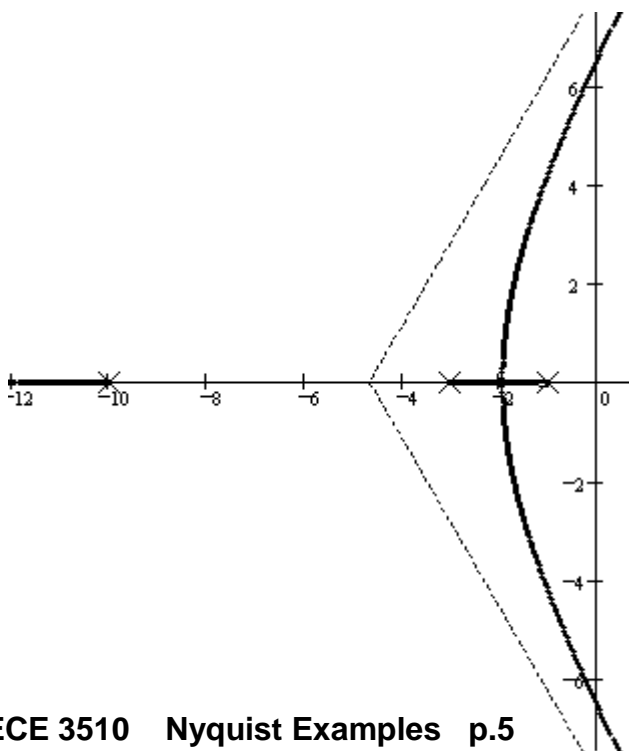


Real part goes to zero

$$-14\omega^2 + 30 = 0$$

$$\omega := \sqrt{\frac{30}{14}}$$

$$G(j\omega) = -8.36j$$



Imaginary part goes to zero

$$43\omega - \omega^3 = 0$$

$$\omega := \sqrt{43}$$

$$G(j\sqrt{43}) = -0.874$$

