ECE 2210 / 00   Phasor Examples

1. Add the sinusoidal voltages $v_1(t) = 4.5 \cdot V \cdot \cos(\omega \cdot t - 30 \cdot \deg)$

and $v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 15 \cdot \deg)$

using phasor notation, draw a phasor diagram of the
three phasors, then convert back to time domain form.

$v_1(t) = 4.5 \cdot V \cdot \cos(\omega \cdot t - 30 \cdot \deg)$

$V_1(\omega) = 4.5V \underline{/-30^o}$ or: $V_1(\omega) = 4.5 \cdot V \cdot e^{-j \cdot 30 \cdot \deg}$

and

$v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 15 \cdot \deg)$

$V_2(\omega) = 3.2V \underline{/15^o}$ or: $V_2(\omega) = 3.2 \cdot V \cdot e^{j \cdot 15 \cdot \deg}$

I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

$V_1 = 4.5V \underline{/-30^o}$ or: $V_1 := 4.5 \cdot V \cdot e^{-j \cdot 30 \cdot \deg}$ $V_2 = 3.2V \underline{/15^o}$ or: $V_2 := 3.2 \cdot V \cdot e^{j \cdot 15 \cdot \deg}$

$4.5 \cdot V \cdot \cos(-30 \cdot \deg) = 3.897 \cdot V$ $4.5 \cdot V \cdot \sin(-30 \cdot \deg) = -2.25 \cdot V$ $V_1 = 3.897 - 2.25j \cdot V$ \
                                                                                                              } add
$3.2 \cdot V \cdot \cos(15 \cdot \deg) = 3.091 \cdot V$ $3.2 \cdot V \cdot \sin(15 \cdot \deg) = 0.828 \cdot V$ $V_2 = 3.091 + 0.828j \cdot V$ /

Add real parts: $3.897 + 3.091 = 6.988$ $V_3 := V_1 + V_2$

Add imaginary parts: $-2.25 + 0.828 = -1.422$ $V_3 = 6.988 - 1.422j \cdot V$ sum

Change V_3 back to polar coordinates:

$\sqrt{6.988^2 + 1.422^2} = 7.131$ $\text{atan}\left(\dfrac{-1.422}{6.988}\right) = -11.502 \cdot \deg$

OR, in Mathcad notation (you'll see these in future solutions):

$|V_3| = 7.131 \cdot V$ $\arg(V_3) = -11.5 \cdot \deg$

Change V_3 back to the time domain:

$v_3(t) = v_1(t) + v_2(t) = 7.13 \cdot \cos(\omega \cdot t - 11.5 \cdot \deg) \cdot V$

drawing of the
phasor diagram
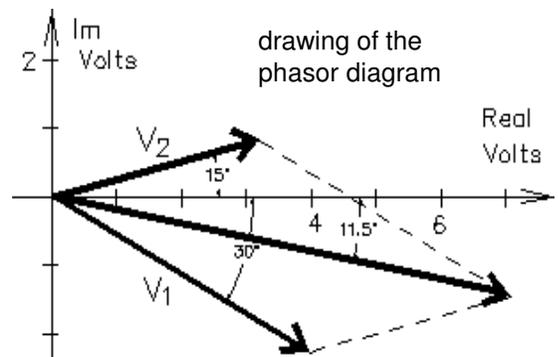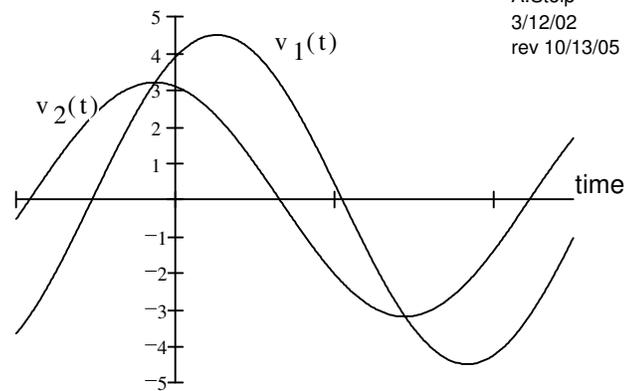
2. Two sinusoidal voltages: $v_1(t) = 5 \cdot V \cdot \cos(\omega \cdot t + 36.87 \cdot \deg)$ and $v_2(t) = 3.162 \cdot V \cdot \cos(\omega \cdot t - 18.44 \cdot \deg)$

a) using phasor notation, find $v_3 = v_1 - v_2$

$V_1 := 5 \cdot V \cdot e^{j \cdot (36.87 \cdot \deg)}$ $5 \cdot V \cdot \cos(36.87 \cdot \deg) = 4 \cdot V$

$5 \cdot V \cdot \sin(36.87 \cdot \deg) = 3 \cdot V$ $V_1 = 4 + 3j \cdot V$

$V_2 := 3.162 \cdot V \cdot e^{j \cdot (-18.44 \cdot \deg)}$ $3.162 \cdot V \cdot \cos(-18.44 \cdot \deg) = 3 \cdot V$

$3.162 \cdot V \cdot \sin(-18.44 \cdot \deg) = -1 \cdot V$ $V_2 = 3 - j \cdot V$

Subtract real parts: $4 \cdot V - 3 \cdot V = 1 \cdot V$ $V_3 := V_1 - V_2$

Subtract imaginary parts: $3 \cdot V - -1 \cdot V = 4 \cdot V$ $V_3 = 1 + 4j \cdot V$

$v_1(t) - v_2(t) = (1 + 4 \cdot j) \cdot V$

Magnitude: $\sqrt{(1 \cdot V)^2 + (4 \cdot V)^2} = 4.123 \cdot V$ $|V_3| = 4.123 \cdot V$

OR:

Angle: $\text{atan}\left(\dfrac{4 \cdot V}{1 \cdot V}\right) = 75.96 \cdot \deg$ $\arg(V_3) = 75.96 \cdot \deg$

So: $v_3(t) = v_1(t) - v_2(t) = 4.123 \cdot V \cdot \cos(\omega \cdot t + 75.96 \cdot \deg) \cdot V$

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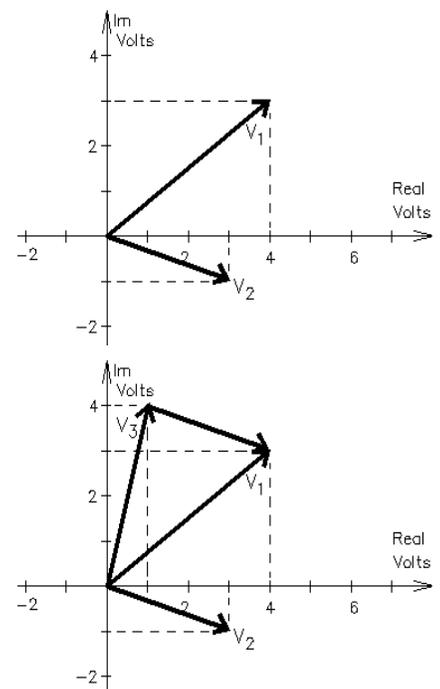
3 Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2 \cdot kHz$

$\mathbf{V}(j\omega)$

$6 \cdot V \cdot e^{j0}$

$f := 2 \cdot kHz$ V_S
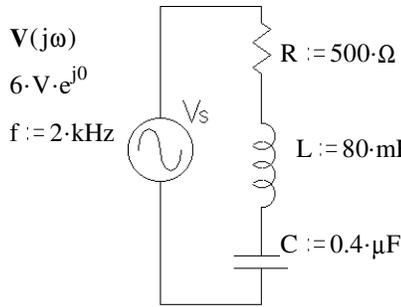
$R := 500 \cdot \Omega$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 1.257 \cdot 10^4 \cdot \dfrac{rad}{sec}$

$L := 80 \cdot mH$ $\mathbf{Z_L} := j \cdot \omega \cdot L$ $\mathbf{Z_L} = 1.005j \cdot k\Omega$

$C := 0.4 \cdot \mu F$ $\mathbf{Z_C} := \dfrac{1}{j \cdot \omega \cdot C}$ $\mathbf{Z_C} = -0.199j \cdot k\Omega$

$\mathbf{Z_{eq}} := R + j \cdot \omega \cdot L + \dfrac{1}{j \cdot \omega \cdot C}$ $\mathbf{Z_{eq}} = 500 + 806.366j \cdot \Omega$

$\sqrt{500^2 + 806^2} = 948.491$ $atan\left(\dfrac{806}{500}\right) = 58.187 \cdot deg$ $\mathbf{Z_{eq}} = 948.5\Omega \underline{/\,58.2^\circ}$
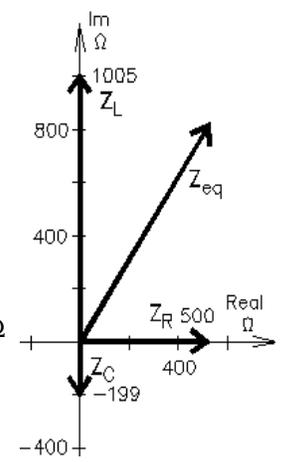
find the current: $\mathbf{I} := \dfrac{6 \cdot V \cdot e^{j0}}{\mathbf{Z_{eq}}}$ magnitude: $\dfrac{6 \cdot V}{948.5 \cdot \Omega} = 6.326 \cdot mA$ angle: $0 \cdot deg - 58.2 \cdot deg = -58.2 \cdot deg$

$\mathbf{I} = 6.326mA \underline{/\text{-}58.2^\circ}$

	find the magnitude	find the angle	
$\mathbf{V_R} := \mathbf{I} \cdot R$	$6.326 \cdot mA \cdot 500 \cdot \Omega = 3.163 \cdot V$	$-58.2 \cdot deg + 0 \cdot deg = -58.2 \cdot deg$	$\mathbf{V_R} = 3.163V \underline{/\text{-}58.2^\circ}$
$\mathbf{V_L} := \mathbf{I} \cdot \mathbf{Z_L}$	$6.326 \cdot mA \cdot 1005 \cdot \Omega = 6.358 \cdot V$	$-58.2 \cdot deg + 90 \cdot deg = 31.8 \cdot deg$	$\mathbf{V_L} = 6.358V \underline{/\,31.8^\circ}$
$\mathbf{V_C} := \mathbf{I} \cdot \mathbf{Z_C}$	$6.326 \cdot mA \cdot (-199) \cdot \Omega = -1.259 \cdot V$	$-58.2 \cdot deg + (90) \cdot deg = 31.8 \cdot deg$	$\mathbf{V_C} = -1.259V \underline{/\,31.8^\circ}$
OR:	$6.326 \cdot mA \cdot (199) \cdot \Omega = 1.259 \cdot V$	$-58.2 \cdot deg + (-90) \cdot deg = -148.2 \cdot deg$	$\mathbf{V_C} = 1.259V \underline{/\text{-}148.2^\circ}$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$\mathbf{V_C} := \dfrac{\dfrac{1}{j \cdot \omega \cdot C}}{R + j \cdot \omega \cdot L + \dfrac{1}{j \cdot \omega \cdot C}} \cdot 6 \cdot V \quad = \dfrac{1}{R \cdot (j \cdot \omega \cdot C) + j \cdot \omega \cdot L \cdot (j \cdot \omega \cdot C) + 1} \cdot 6 \cdot V \quad = \dfrac{1}{R \cdot (j \cdot \omega \cdot C) - \omega^2 \cdot L \cdot C + 1} \cdot 6 \cdot V$

$= \dfrac{1}{\left(1 - \omega^2 \cdot L \cdot C\right) + j \cdot \omega \cdot R \cdot C} \cdot 6 \cdot V$ $\left(1 - \omega^2 \cdot L \cdot C\right) = -4.053$ $j \cdot \omega \cdot R \cdot C = 2.513j$

$= \dfrac{6 \cdot V}{-4.053 + 2.513j} \cdot \dfrac{(-4.053 - 2.513 \cdot j \,)}{(-4.053 - 2.513 \cdot j \,)} = \dfrac{6 \cdot V \cdot (-4.053 - 2.513 \cdot j \,)}{(-4.053)^2 + 2.513^2}$

$6 \cdot V \cdot (-4.053 - 2.513 \cdot j \,) = -24.318 - 15.078j \cdot V$

$(-4.053)^2 + 2.513^2 = 22.742$

$= \left(\dfrac{-24.318}{22.742} - \dfrac{15.078 \cdot j}{22.742}\right) \cdot V = -1.069 - 0.663j \cdot V$

magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

angle: $atan\left(\dfrac{-0.663}{-1.069}\right) = 31.81 \cdot deg$

but this is actually in the third quadrant,
so modify your calculator's results:

$31.81 \cdot deg - 180 \cdot deg = -148.19 \cdot deg$

$= 1.258V \underline{/\text{-}148.2^\circ}$
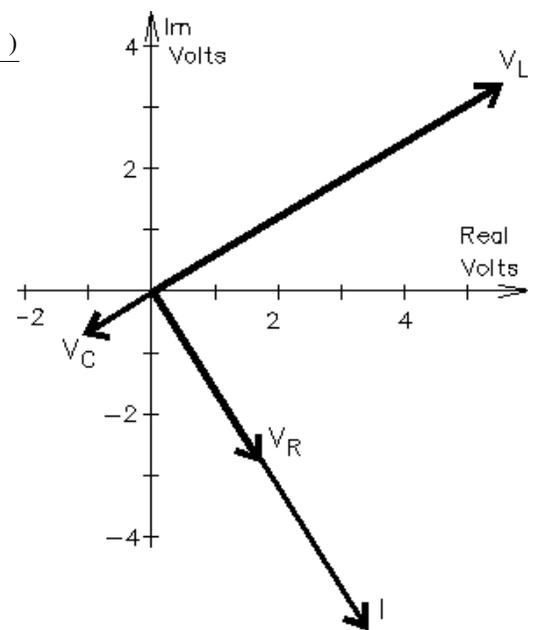
4. a) Find $\mathbf{Z_{eq}}$. $f := 2.5 \cdot kHz$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 1.571 \cdot 10^4 \cdot \dfrac{rad}{sec}$

$L_1 := 2 \cdot mH$ $j \cdot \omega \cdot L_1 = 25.133j \cdot \Omega$

$R := 200 \cdot \Omega$

$C := 1 \cdot \mu F$

$L_2 := 8 \cdot mH$
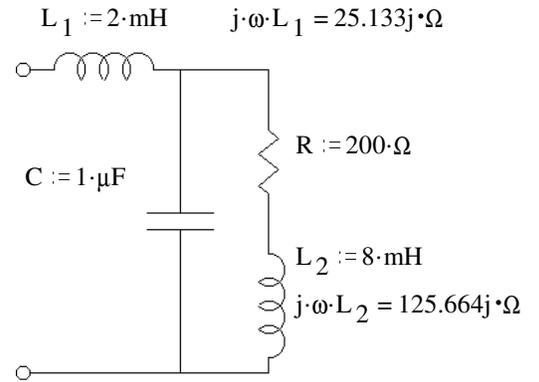
$j \cdot \omega \cdot L_2 = 125.664j \cdot \Omega$

Left branch

$$\mathbf{Z_l} := \frac{1}{j \cdot \omega \cdot C} \qquad\qquad \mathbf{Z_l} = -63.662j \cdot \Omega$$

Right branch

$$\mathbf{Z_r} := j \cdot \omega \cdot L_2 + R \qquad \mathbf{Z_r} = 200 + 125.664j \cdot \Omega$$

$$\mathbf{Z_{eq}} := j \cdot \omega \cdot L_1 + \cfrac{1}{\cfrac{1}{R + j \cdot \omega \cdot L_2} + \cfrac{1}{\cfrac{1}{j \cdot \omega \cdot C}}} \qquad = j \cdot \omega \cdot L_1 + \cfrac{1}{\cfrac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C} \qquad = j \cdot \omega \cdot L_1 + \cfrac{R + j \cdot \omega \cdot L_2}{1 + j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}$$

Sometimes it's worth simplifying a little before puttiing in numbers.

$$= j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

$$\mathbf{Z_{eq}} = 31.416 j \cdot \Omega + \frac{(200 + 125.664 \cdot j) \cdot \Omega}{-0.974 + 3.142 \cdot j} \cdot \left(\frac{-0.974 - 3.142 \cdot j}{-0.974 - 3.142 \cdot j} \right) \qquad = 25.133 j \cdot \Omega + \frac{(200 + 125.664 \cdot j) \cdot (-0.974 - 3.142 \cdot j)}{0.974^2 + 3.142^2}$$

$$= 31.416 j \cdot \Omega + \frac{((200 \cdot (-0.974) - 125.664 \cdot (-3.142)) + (125.664 \cdot (-0.974) - 200 \cdot 3.142) \cdot j) \cdot \Omega}{0.974^2 + 3.142^2}$$

$$= 31.416 j \cdot \Omega + \frac{(200.036288 - 750.796736 \cdot j) \cdot \Omega}{10.82084} \qquad = 31.416 j \cdot \Omega + 18.486 \cdot \Omega - 69.384 \cdot j \cdot \Omega = 18.486 - 37.968j \cdot \Omega$$

$$\sqrt{18.49^2 + 37.97^2} = 42.233 \qquad atan\left(\frac{-37.97}{18.49} \right) = -64.036 \cdot deg \qquad \mathbf{Z_{eq}} = 42.24\Omega \underline{/-64.04^o}$$

b) $\mathbf{V_{in}} := 12 \cdot V \cdot e^{j \cdot 20 \cdot deg}$ Find $\mathbf{I_{L1}}$, $\mathbf{V_C}$ $\mathbf{I_{L1}} := \dfrac{\mathbf{V_{in}}}{\mathbf{Z_{eq}}}$ $\dfrac{12 \cdot V}{42.24 \cdot \Omega} = 284.091 \cdot mA$ $20 \cdot deg - (-64.04) \cdot deg = 84.04 \cdot deg$

$$\mathbf{I_{L1}} = 284mA \underline{/ 84.04^o} \quad = \quad 284 \cdot mA \cdot e^{j \cdot 84.04 \cdot deg} \qquad \mathbf{I_{L1}} = 29.485 + 282.569j \cdot mA$$

$$\mathbf{V_C} := \mathbf{I_{L1}} \cdot (18.486 - 69.384 \cdot j) \cdot \Omega \qquad 284 \cdot mA \cdot \sqrt{18.486^2 + 69.384^2} \cdot \Omega = 20.392 \cdot V$$

$$84.04 \cdot deg + atan\left(\frac{-69.384}{18.486} \right) = 8.959 \cdot deg \qquad \mathbf{V_C} = 20.4V \underline{/ 8.96^o}$$

To find V_C directly:

You could then use another voltage divider to find $\mathbf{V_R}$ or $\mathbf{V_{L2}}$.

$$\mathbf{V_C} := \cfrac{\cfrac{1}{\cfrac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}}{j \cdot \omega \cdot L_1 + \cfrac{1}{\cfrac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}} \cdot \mathbf{V_{in}} \quad = \cfrac{1}{j \cdot \omega \cdot L_1 \cdot \left(\cfrac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C \right) + 1} \cdot \mathbf{V_{in}} \quad = \cfrac{1}{\cfrac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1}$$

$$= \cfrac{1}{\cfrac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot \mathbf{V_{in}} \qquad = \cfrac{1}{\cfrac{j \cdot \omega \cdot L_1 \cdot (R - j \cdot \omega \cdot L_2)}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot \mathbf{V_{in}}$$

$$= \cfrac{1}{\left[\cfrac{\omega^2 \cdot L_1 \cdot L_2}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1 \right] + j \cdot \cfrac{\omega \cdot L_1 \cdot R}{R^2 + (\omega \cdot L_2)^2}} \cdot \mathbf{V_{in}} \qquad = \cfrac{12 \cdot V \cdot e^{j \cdot 20 \cdot deg}}{0.58816 \cdot e^{j \cdot 11.039 \cdot deg}} \qquad = \cfrac{12 \cdot V}{0.58816} \underline{/ 20 - 11.039^o}$$

$$= 20.4V \underline{/ 8.96^o} \qquad Same$$

$$\mathbf{V_C} = 20.153 + 3.178j \cdot V$$

4. Continued   Let's find I_{L2}.

$$Z_r := j \cdot \omega \cdot L_2 + R$$

$$Z_r = 200 + 125.664j \ \cdot\Omega \qquad \sqrt{200^2 + 125.664^2} = 236.202 \qquad \text{atan}\left(\frac{125.664}{200}\right) = 32.142 \cdot deg$$

$$I_{L2} = \frac{20.4 \cdot V \cdot e^{j \cdot 8.96 \cdot deg}}{236.202 \cdot \Omega \cdot e^{j \cdot 32.142 \cdot deg}} = \frac{20.4 \cdot V}{236.202 \cdot \Omega} \ \underline{/8.96 - 32.142^o} \quad = 86.4mA \ \underline{/-23.18}$$

Or, directly by Current divider:

$$I_{L2} := \frac{\dfrac{1}{R + j \cdot \omega \cdot L_2}}{j \cdot \omega \cdot C + \dfrac{1}{R + j \cdot \omega \cdot L_2}} \cdot I_{L1} = \frac{1}{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2) + 1} \cdot I_{L1} = \frac{I_{L1}}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

denominator: $\sqrt{(1 - \omega^2 \cdot C \cdot L_2)^2 + (\omega \cdot C \cdot R)^2} = 3.289 \qquad \text{atan}\left(\dfrac{\omega \cdot C \cdot R}{1 - \omega^2 \cdot C \cdot L_2}\right) + 180 \cdot deg = 107.224 \cdot deg$

$$I_{L2} = \frac{284 \cdot mA \cdot e^{j \cdot 84.04 \cdot deg}}{3.289 \cdot e^{j \cdot 107.224 \cdot deg}} = \frac{284 \cdot mA}{3.289} \ \underline{/84.04 - 107.224^o} \quad = 86.4mA \ \underline{/-23.18}$$

$$I_{L2} = 79.404 - 34.001j \ \cdot mA$$

How about I_C? $\quad I_C := \dfrac{V_C}{\left(\dfrac{1}{j \cdot \omega \cdot C}\right)} = V_C \cdot j \cdot \omega \cdot C = 20.4V \ \underline{/8.96^o} \ 0.015708 \ \underline{/90^o} \ \dfrac{1}{\Omega} = 320mA \ \underline{/98.96^o}$

Or, directly by Current divider:

$$I_C := \frac{j \cdot \omega \cdot C}{j \cdot \omega \cdot C + \dfrac{1}{R + j \cdot \omega \cdot L_2}} \cdot I_{L1} = \frac{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2) + 1} \cdot I_{L1} = \frac{-\omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R} \cdot I_{L1}$$

numerator: $\sqrt{(\omega^2 \cdot C \cdot L_2)^2 + (\omega \cdot C \cdot R)^2} = 3.71 \qquad \text{atan}\left(\dfrac{\omega \cdot C \cdot R}{-\omega^2 \cdot C \cdot L_2}\right) + 180 \cdot deg = 122.142 \cdot deg$

denominator is the same as above. $\qquad\qquad\qquad\qquad\qquad$ Second quadrant

$$I_C = \frac{3.71 \cdot e^{j \cdot 122.14 \cdot deg}}{3.289 \cdot e^{j \cdot 107.224 \cdot deg}} \cdot 284 \cdot mA \cdot e^{j \cdot 84.04 \cdot deg} = \frac{3.71}{3.289} \cdot 284 \cdot mA \ \underline{/122.14 - 107.224 + 84.04^o} = 320mA \ \underline{/98.96^o}$$

This current is greater than the input current. What's going on?

The angle between I_C & I_{L2} is big enough that they somewhat cancel each other out.

Check Kirchoff's Current Law: $\quad I_C + I_{L2} = 29.485 + 282.569j \ \cdot mA \quad = I_{L1} = 29.485 + 282.569j \ \cdot mA$

5. a) Find V_{in} in polar form.

$I_Z := 100 \cdot mA \qquad Z := (80 - 60 \cdot j) \cdot \Omega$

$V_{in} := I_Z \cdot Z \qquad\qquad V_{in} = 8 - 6j \ \cdot V$

$\sqrt{8^2 + 6^2} = 10 \qquad \text{atan}\left(\dfrac{-6}{8}\right) = -36.87 \cdot deg$

$V_{in} = 10V \ \underline{/-36.9^o}$

$I_T \qquad\qquad I_R \qquad\qquad I_Z = 100 \ \underline{/0^o}$ mA

$R := 50 \cdot \Omega$

V_{in}

$Z \qquad Z := (80 - 60 \cdot j) \cdot \Omega$
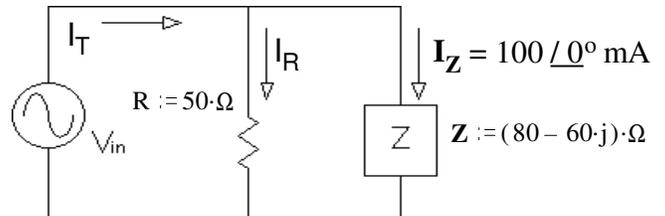
b) Find I_T in polar form. $\quad I_R := \dfrac{V_{in}}{R} = \dfrac{10 \cdot V}{50 \cdot \Omega} \ \underline{/-36.9^o} = \dfrac{10 \cdot V}{50 \cdot \Omega} \cdot \cos(-36.9 \cdot deg) + j \cdot \dfrac{10 \cdot V}{50 \cdot \Omega} \cdot \sin(-36.9 \cdot deg) = 160 - 120i \ \cdot mA$

$I_T := I_R + I_Z = (160 - 120 \cdot j) \cdot mA + 100 \cdot mA \qquad I_T = 260 - 120j \ \cdot mA$

$\sqrt{260^2 + 120^2} = 286.356 \qquad \text{atan}\left(\dfrac{-120}{260}\right) = -24.78 \cdot deg \qquad I_T = 286mA \ \underline{/-24.8^o}$

c) Circle 1:   i) The source current leads the source voltage $\qquad$ ii) The source voltage leads the source current

$\qquad\qquad$ answer i), -24.8^o > -36.9^o