Sinusoidal Steady State Notes

ECE 3510

A.Stolp
2/27/00
rev, 2/2/06

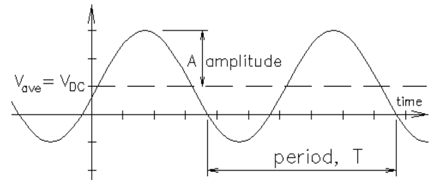
T = Period

f = frequency, cycles / second $\quad f = \frac{1}{T} = \frac{\omega}{2\cdot\pi}$

ω = radian frequency, radians/sec $\quad \omega = 2\cdot\pi\cdot f$

A = amplitude

DC = average



$y(t) = A\cdot\cos(\omega\cdot t + \phi)$

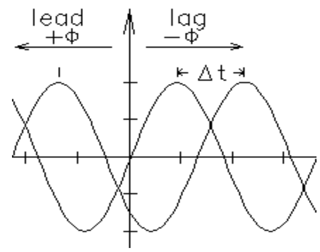
Phasor

voltage: $v(t) = V_p\cdot\cos(\omega\cdot t + \phi) \qquad \mathbf{V}(\omega) = V_p\cdot e^{j\cdot\phi}$

current: $i(t) = I_p\cdot\cos(\omega\cdot t + \phi) \qquad \mathbf{I}(\omega) = I_p\cdot e^{j\cdot\phi}$

Phase: $\phi = \frac{\Delta t}{T}\cdot 360\cdot\text{deg}$ or: $\phi = \frac{\Delta t}{T}\cdot 2\cdot\pi\cdot\text{rad}$

Phase:



Adding and subtracting Sinusoidal AC voltages or currents

Two sinusoidal voltages: $v_1(t) = 5\cdot V\cdot\cos(\omega\cdot t + 36.87\cdot\text{deg})$ and $v_2(t) = 3.162\cdot V\cdot\cos(\omega\cdot t - 18.44\cdot\text{deg})$

a) using phasor notation, find $v_3 = v_1 - v_2$

$\mathbf{V_1} := 5\cdot V\cdot e^{j\cdot(36.87\cdot\text{deg})}$

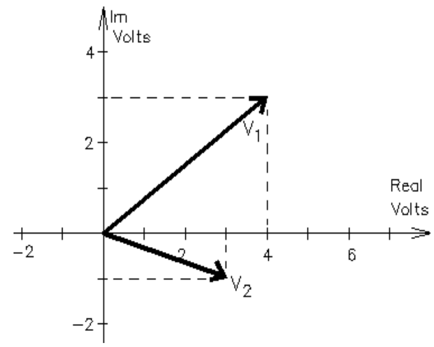
$5\cdot V\cdot\cos(36.87\cdot\text{deg}) = 4\cdot V$

$5\cdot V\cdot\sin(36.87\cdot\text{deg}) = 3\cdot V \qquad \mathbf{V_1} = 4 + 3j\ \cdot V$

$\mathbf{V_2} := 3.162\cdot V\cdot e^{j\cdot(-18.44\cdot\text{deg})}$

$3.162\cdot V\cdot\cos(-18.44\cdot\text{deg}) = 3\cdot V$

$3.162\cdot V\cdot\sin(-18.44\cdot\text{deg}) = -1\cdot V \qquad \mathbf{V_2} = 3 - j\ \cdot V$



Subtract real parts: $4\cdot V - 3\cdot V = 1\cdot V \qquad \mathbf{V_3} := \mathbf{V_1} - \mathbf{V_2}$

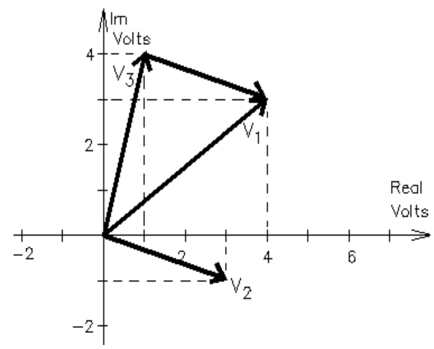
Subtract imaginary parts: $3\cdot V - -1\cdot V = 4\cdot V \qquad \mathbf{V_3} = 1 + 4j\ \cdot V$

$v_1(t) - v_2(t) = (1 + 4\cdot j)\cdot V$

Magnitude: $\sqrt{(1\cdot V)^2 + (4\cdot V)^2} = 4.123\cdot V$ OR: $\left|\mathbf{V_3}\right| = 4.123\cdot V$

Angle: $\text{atan}\left(\frac{4\cdot V}{1\cdot V}\right) = 75.96\cdot\text{deg} \qquad \arg\left(\mathbf{V_3}\right) = 75.96\cdot\text{deg}$

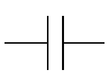
So: $v_3(t) = v_1(t) - v_2(t) = 4.123\cdot V\cdot\cos(\omega\cdot t + 75.96\cdot\text{deg})\cdot V$



Phasor analysis with impedances, For steady-state sinusoidal response ONLY

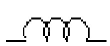
AC impedance

Capacitor



$i_C = C\cdot\frac{d}{dt}v_C \qquad v_C = \frac{1}{C}\cdot\int i_C(t)\,dt \qquad \mathbf{Z_C} = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C} \qquad \mathbf{V_C}(j\omega) = \frac{1}{j\cdot\omega\cdot C}\cdot\mathbf{I}(j\omega)$

Inductor



$v_L = L\cdot\frac{d}{dt}i_L \qquad i_L = \frac{1}{L}\cdot\int v_L(t)\,dt \qquad \mathbf{Z_L} = j\cdot\omega\cdot L \qquad \mathbf{V_L}(j\omega) = j\cdot\omega\cdot L\cdot\mathbf{I}(j\omega)$

Resistor

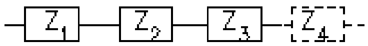
$v_R = i_R\cdot R \qquad i_R = \frac{v_R}{R} \qquad \mathbf{Z_R} = R \qquad \mathbf{V_R}(j\omega) = R\cdot\mathbf{I}(j\omega)$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

Sinusoidal Steady State Notes

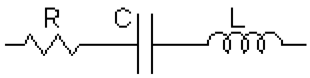
Impedances

series:



$$Z_{eq} = Z_1 + Z_2 + Z_3 + \ldots$$

Example:



$$Z_{eq} = R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L$$

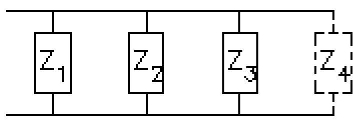
Voltage divider:

$$V_{Zn} = V_{total}\cdot\frac{Z_n}{Z_1 + Z_2 + Z_3 + \ldots}$$

If: $R := 200\cdot\Omega$ $C := 2\cdot\mu F$ $L := 16\cdot mH$ and $\omega := 4000\cdot\frac{rad}{sec}$

then $Z_{eq} = R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L = 200\cdot\Omega - 125\cdot j\cdot\Omega + 64\cdot j\cdot\Omega = 200 - 61j\ \cdot\Omega$

parallel:

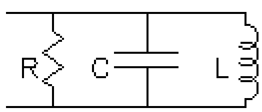


$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$$

Current divider:

$$I_{Zn} = I_{total}\cdot\frac{\frac{1}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$$

Example:



$$Z_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)} + \frac{1}{j\cdot\omega\cdot L}}$$

If: $R := 200\cdot\Omega$ $C := 2\cdot\mu F$ $L := 16\cdot mH$ and $\omega := 4000\cdot\frac{rad}{sec}$

then $Z_{eq} = \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C - \frac{j}{\omega\cdot L}} = \frac{1}{\frac{1}{200\cdot\Omega} + 8\cdot 10^{-3}\cdot j\cdot\frac{1}{\Omega} - 0.01563\cdot j\cdot\frac{1}{\Omega}}$

$= 60.1\cdot\Omega + 91.7\cdot\Omega = 109.6\Omega\ \underline{/56.7^{o}}$

$\sqrt{(60.1\cdot\Omega)^2 + (91.7\cdot\Omega)^2} = 109.6\cdot\Omega$ $\quad \operatorname{atan}\left(\frac{91.7\cdot\Omega}{60.1\cdot\Omega}\right) = 56.76\cdot deg$

Magnitude and Phase of transfer functions With steady-state sinusoidal inputs

$\omega := 2\cdot\frac{rad}{sec}$

$s := j\cdot\omega$

$$H(s) = \frac{2\cdot s^2 + \frac{5}{sec}\cdot s + \frac{20}{sec^2}}{s^2 + \frac{1}{sec}\cdot s + \frac{10}{sec^2}} = \frac{2\cdot(j\cdot\omega)^2 + \frac{5}{sec}\cdot(j\cdot\omega) + \frac{20}{sec^2}}{(j\cdot\omega)^2 + \frac{1}{sec}\cdot(j\cdot\omega) + \frac{10}{sec^2}} = \frac{12 + 10\cdot j}{6 + 2\cdot j}$$

$|H(j\cdot\omega)| = M = \frac{\sqrt{12^2 + 10^2}}{\sqrt{6^2 + 2^2}} = 2.47$ $\quad \underline{/H(j\omega)} = \operatorname{atan}\left(\frac{10}{12}\right) - \operatorname{atan}\left(\frac{2}{6}\right) = 21.37\cdot deg$

Expressing signals in the time domain The steady-state sinusoidal outputs

$f := 5\cdot Hz$

$\omega := 2\cdot\pi\cdot f$

$s := j\cdot\omega$

$$Y(s) = \frac{s^2 + \frac{20}{sec}\cdot s + \frac{1000}{sec^2}}{s^2 + \frac{10}{sec}\cdot s + \frac{800}{sec^2}} = \frac{(j\cdot\omega)^2 + \frac{20}{sec}\cdot(j\cdot\omega) + \frac{1000}{sec^2}}{(j\cdot\omega)^2 + \frac{10}{sec}\cdot(j\cdot\omega) + \frac{800}{sec^2}} = \frac{13.04 + 628.319\cdot j}{-186.96 + 314.159\cdot j} = 1.459 - 0.91j$$

$\omega = 31.42\cdot\frac{rad}{sec}$ $\quad y(t) = 1.46\cdot\cos\left(31.42\cdot\frac{rad}{sec}\cdot t\right) + 0.91\cdot\sin\left(31.42\cdot\frac{rad}{sec}\cdot t\right)$

note that the sine carries the opposite sign as the imaginary part.

$\sqrt{1.459^2 + 0.91^2} = 1.72$ $\quad \operatorname{atan}\left(\frac{-0.91}{1.459}\right) = -31.95\cdot deg$ $\quad y(t) = 1.72\cdot\cos\left(31.42\cdot\frac{rad}{sec}\cdot t - 32\cdot deg\right)$

Sinusoidal Steady State Notes