ECE 3510 Step Responses

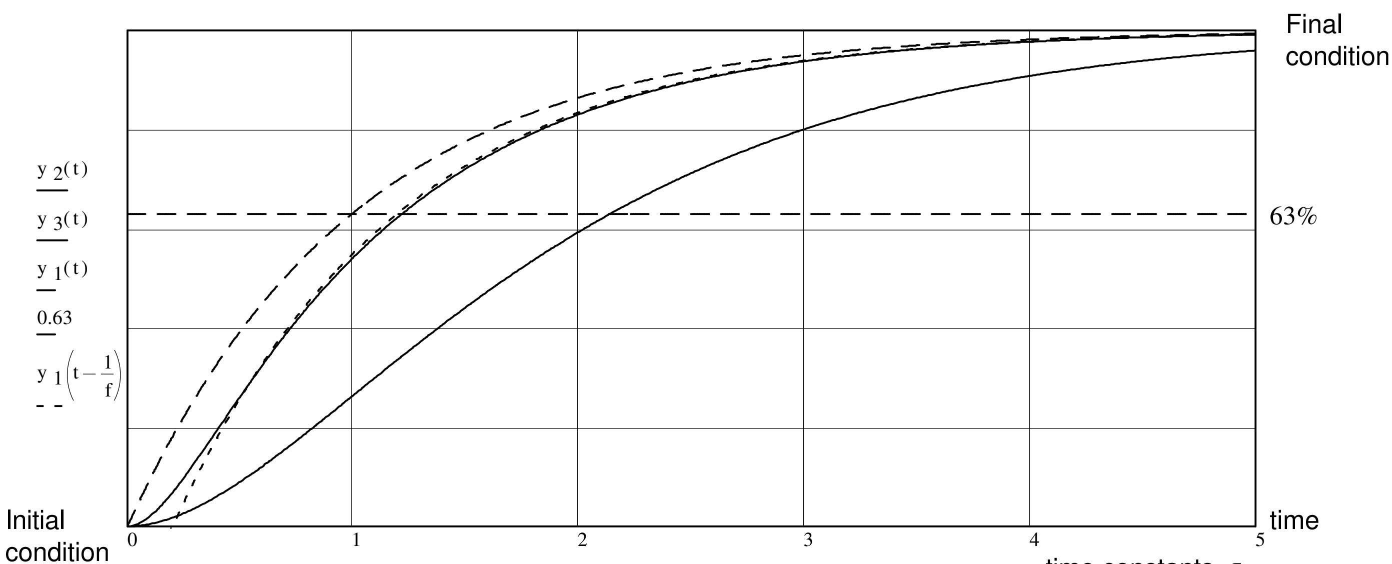
$H(s) = \frac{k}{s+a}\cdot a$ $\quad a := 1 \quad k := 1 \quad y_1(t) := a\cdot\left(\frac{k}{a} - \frac{k}{a}\cdot e^{-a\cdot t}\right)$

$H(s) = \frac{k}{(s+a)^2}\cdot a^2$ $\quad a := 1 \quad k := 1 \quad y_2(t) := a^2\cdot\left(\frac{k}{a^2} - \frac{k}{a^2}\cdot e^{-a\cdot t} - \frac{k}{a}\cdot t\cdot e^{-a\cdot t}\right)$

$H(s) = \frac{k}{(s+a_1)\cdot(s+a_2)}\cdot(a_1\cdot a_2)$

Notice that I modified the responses with an a, a^2, and a_1a_2 to make the steps the same size in each case.

$a_1 := a \quad f := 5 \quad a_2 := f\cdot a_1 \quad y_3(t) := a_1\cdot a_2\cdot\left[\frac{k}{a_1\cdot a_2} + \frac{k}{a_1\cdot(a_1 - a_2)}\cdot e^{-a_1\cdot t} + \frac{k}{a_2\cdot(a_2 - a_1)}\cdot e^{-a_2\cdot t}\right]$



Rising Curves

Some Important Features:

1) The poles closest to the jω axis are the **dominant** poles.

2) Poles to the left of the dominant poles may introduce and effect that looks like **time delay**.

3) Conversely, the effects of a time delay (non-linear) can sometimes be modeled by an extra pole (linear) to the left of the dominant poles.

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Complex Poles (The underdamped response)

$$H(s) = \frac{k}{s^2 + 2\cdot a\cdot s + a^2 + b^2} = \frac{k}{\omega_n^2}\cdot\frac{\omega_n^2}{s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^2}$$

$\omega_n^2 = a^2 + b^2$ $\qquad$ ω_n = natural frequency

$\zeta\cdot\omega_n = a$

DC gain

$$H(0) = \frac{k}{a^2 + b^2} = \frac{k}{\omega_n^2}$$

$\zeta = \dfrac{a}{\omega_n} = \dfrac{a}{\sqrt{a^2 + b^2}}$ = damping factor

$= \sin(\theta)$

Note: H(0) is left out of the plots below plots below to make them all end at 1.

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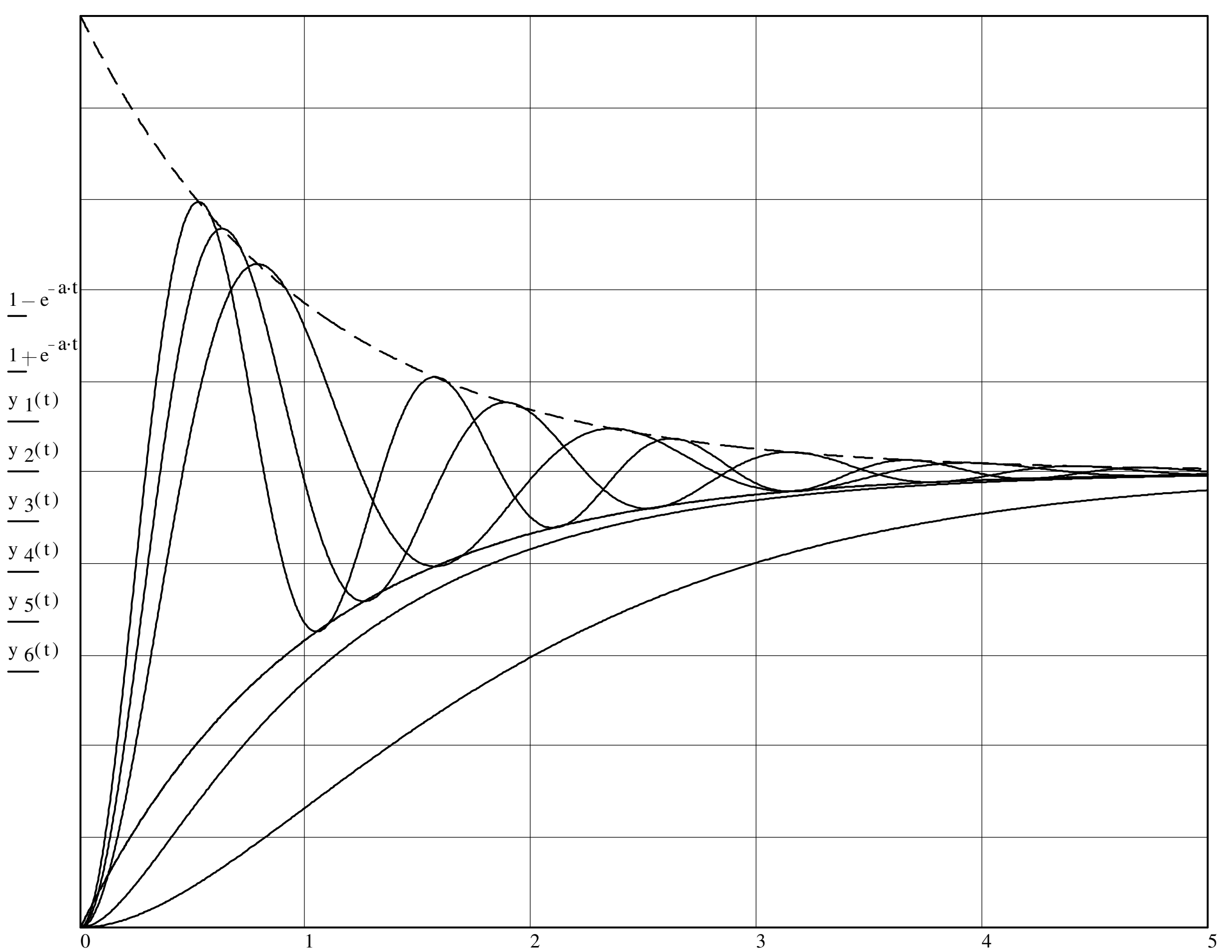
$$y(t) = x_m\cdot H(0)\cdot\left(1 - e^{-a\cdot t}\cdot\cos(b\cdot t) - \frac{a}{b}\cdot e^{-a\cdot t}\cdot\sin(b\cdot t)\right)$$



Underdamped Curves