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4/15/09

f(k)		F(z)
$f(k) = \frac{1}{2\cdot\pi\cdot j}\cdot\oint F(z)\cdot z^{k-1}\,dz$ integral around a closed path in the complex plane		$F(z) = \sum_{k=0}^{\infty} f(k)\cdot z^{-k}$
$\delta(k)$	impulse	1
$\delta(k-m)$	shifted impulse	$\frac{1}{z^m}$
$u(k)$	unit step	$\frac{z}{z-1}$

All the following are multipied by $u(k)$

k		$\frac{z}{(z-1)^2}$
k^2		$\frac{z\cdot(z+1)}{(z-1)^3}$
k^3		$\frac{z\cdot\left(z^2+4\cdot z+1\right)}{(z-1)^4}$
a^k	power series	$\frac{z}{z-a}$
$k\cdot a^k$		$\frac{a\cdot z}{(z-a)^2}$
$k^2\cdot a^k$		$\frac{a\cdot z\cdot(z+a)}{(z-a)^3}$
$k^3\cdot a^k$		$\frac{a\cdot z\cdot\left(z^2+4\cdot a\cdot z+a^2\right)}{(z-a)^4}$
$\cos\left(\Omega_o\cdot k\right)$	sinusoids	$\frac{z\left(z-\cos\left(\Omega_o\right)\right)}{z^2-2\cdot\cos\left(\Omega_o\right)\cdot z+1}$
$\sin\left(\Omega_o\cdot k\right)$		$\frac{z\cdot\sin\left(\Omega_o\right)}{z^2-2\cdot\cos\left(\Omega_o\right)\cdot z+1}$
$(\lvert p\rvert)^k\cdot\cos\left(\theta_p\cdot k\right)$		$\frac{z\cdot\left(z-\lvert p\rvert\cdot\cos\left(\theta_p\right)\right)}{z^2-2\cdot\lvert p\rvert\cdot\cos\left(\theta_p\right)\cdot z+(\lvert p\rvert)^2}$
$(\lvert p\rvert)^k\cdot\sin\left(\theta_p\cdot k\right)$		$\frac{z\cdot\left(\lvert p\rvert\cdot\sin\left(\theta_p\right)\right)}{z^2-2\cdot\lvert p\rvert\cdot\cos\left(\theta_p\right)\cdot z+(\lvert p\rvert)^2}$

ECE 3510

z - transforms

Good for inverse z-transforms

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F(z)	f(k)
	All the following are multipied by $u(k)$ unless specified otherwise
Poles at zero	
$\frac{A\cdot z}{z} = A$	$A\cdot\delta(k)$
$\frac{B\cdot z}{z^2} = \frac{B}{z}$	$B\cdot\delta(k-1)$
$\frac{C\cdot z}{z^3} = \frac{C}{z^2}$	$C\cdot\delta(k-2)$
$\frac{D\cdot z}{z^4} = \frac{D}{z^3}$	$D\cdot\delta(k-3)$
Poles on real axis (not at zero)	
$\frac{B\cdot z}{(z-p)}$	$B\cdot p^k$
$\frac{C\cdot z}{(z-p)^2}$	$C\cdot k\cdot p^{k-1}$
$\frac{D\cdot z}{(z-p)^3}$	$D\cdot\frac{1}{2}\cdot k\cdot(k-1)\cdot p^{k-2}$
$\frac{E\cdot z}{(z-p)^4}$	$E\cdot\frac{1}{6}\cdot k\cdot(k-1)\cdot(k-2)\cdot p^{k-3}$
Complex poles	
$\frac{B\cdot z}{(z-p)}+\frac{\overline{B}\cdot z}{\left(z-\overline{p}\right)}$	$2\cdot\|B\|\cdot(\|p\|)^k\cdot\cos\left(\theta_p\cdot k+\theta_B\right)$
$\frac{B\cdot z}{(z-p)^2}+\frac{\overline{B}\cdot z}{\left(z-\overline{p}\right)^2}$	$2\cdot\|B\|\cdot k\cdot(\|p\|)^{k-1}\cdot\cos\left[\theta_p\cdot(k-1)+\theta_B\right]$
$\frac{B\cdot z}{(z-p)^3}+\frac{\overline{B}\cdot z}{\left(z-\overline{p}\right)^3}$	$\|B\|\cdot k\cdot(k-1)\cdot(\|p\|)^{k-2}\cdot\cos\left[\theta_p\cdot(k-2)+\theta_B\right]$
$\frac{B\cdot z}{(z-p)^4}+\frac{\overline{B}\cdot z}{\left(z-\overline{p}\right)^4}$	$\frac{1}{3}\cdot\|B\|\cdot k\cdot(k-1)\cdot(k-2)\cdot(\|p\|)^{k-3}\cdot\cos\left[\theta_p\cdot(k-3)+\theta_B\right]$

where $B = |B|\cdot e^{j\cdot\theta_B}$ and $p = |p|\cdot e^{j\cdot\theta_p}$

if $B = C + D\cdot j$ and $p = q + r\cdot j$

then $|B| = \sqrt{C^2+D^2}$ and $|p| = \sqrt{q^2+r^2}$

$\theta_B = \operatorname{atan}\left(\frac{D}{C}\right)$ $\theta_p = \operatorname{atan}\left(\frac{r}{q}\right)$

z - transform Properties

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4/15/09

Operation	f(k)	F(z)	
	All the following are multipied by $u(k)$ unless specified otherwise		
Addition	$f(k) + g(k)$	$F(z) + G(z)$	
Scalar multiplication	$c\cdot f(k)$	$c\cdot F(z)$	
Linearity	$c\cdot f(k) + d\cdot g(k)$	$c\cdot F(z) + d\cdot G(z)$	
Right shift $m\geq 0$	$f(k-m)\cdot u(k-m)$	$\frac{1}{z^m}\cdot F(z) = z^{-m}\cdot F(z)$	
	$f(k-m)$	$\frac{1}{z^m}\cdot F(z) + \frac{1}{z^m}\cdot\sum_{k=1}^{m} f(-k)\cdot z^k$	
	$f(k-1)$	$z^{-1}\cdot F(z) + f(-1)$	
	$f(k-2)$	$z^{-2}\cdot F(z) + z^{-1}\cdot f(-1) + f(-2)$	
	$f(k-3)$	$z^{-3}\cdot F(z) + z^{-2}\cdot f(-1) + z^{-1}\cdot f(-2) + f(-3)$	
Left shift $m\geq 0$	$f(k+m)$	$z^m\cdot F(z) - z^m\cdot\sum_{k=0}^{m-1} f(k)\cdot z^{-k}$	
	$f(k+1)$	$z\cdot F(z) - z\cdot f(0)$	
	$f(k+2)$	$z^2\cdot F(z) - z^2\cdot f(0) - z\cdot f(1)$	
	$f(k+3)$	$z^3\cdot F(z) - z^3\cdot f(0) - z^2\cdot f(1) - z\cdot f(2)$	
Multiplication by p^k	$p^k\cdot f(k)$	$F\left(\frac{z}{p}\right)$	Frequency scaling
Multiplication by k	$k\cdot f(k)$	$-z\cdot\frac{d}{dz}F(z)$	Frequency differentiation
Time convolution	$f(k) * g(k)$	$F(z)\cdot G(z)$	
Initial value	$f(0)$	$\lim_{z\to\infty} F(z)$	
Final value	$f(\infty)$	$\lim_{z\to 1} (z-1)\cdot F(z)$ (all poles of $(z-1)F(z)$ inside unit circle)	

ECE 3510 Lines of Constant Damping in the z-plane

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