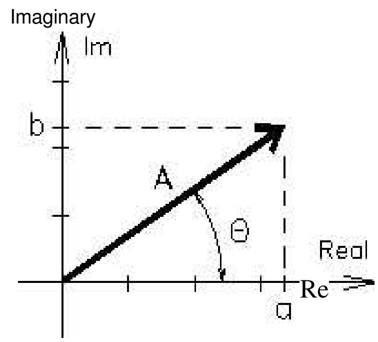
# Complex Numbers

ECE 3600




$j = \sqrt{-1}$ the imaginary number

**Rectangular Form** $\mathbf{A} = a + b \cdot j$

$\mathrm{Re}(\mathbf{A}) = a \qquad \mathrm{Im}(\mathbf{A}) = b$

**Polar Form** $\mathbf{A} = A \cdot e^{j \cdot \theta}$

$\mathrm{Re}(\mathbf{A}) = A \cdot \cos(\theta) \qquad \mathrm{Im}(\mathbf{A}) = A \cdot \sin(\theta)$

**Conversions** $A = |\mathbf{A}| = \sqrt{a^2 + b^2} \qquad \theta = \arg(\mathbf{A}) = \mathrm{atan}\left(\frac{b}{a}\right)$

$a = A \cdot \cos(\theta) \qquad b = A \cdot \sin(\theta)$

$\mathbf{A} = A \cdot e^{j \cdot \theta} = A \cdot \cos(\theta) + A \cdot \sin(\theta) \cdot j \qquad \mathbf{A} = a + b \cdot j = \left(\sqrt{a^2 + b^2}\right) \cdot e^{j \cdot \mathrm{atan}\left(\frac{b}{a}\right)}$

**Special Cases** $j := \sqrt{-1} = e^{j \cdot 90 \cdot deg} \qquad \frac{1}{j} = -j = e^{-j \cdot 90 \cdot deg} \qquad e^{j \cdot 0 \cdot deg} = 1 \qquad e^{-j \cdot 180 \cdot deg} = e^{-j \cdot 180 \cdot deg} = -1$

$j \cdot e^{j \cdot \theta} = e^{j \cdot (\theta + 90 \cdot deg)}$

Define a 2nd number: rect: $\mathbf{D} = c + d \cdot j$ polar: $\mathbf{D} = D \cdot e^{j \cdot \phi}$

**Equality** $\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

**Addition and Subtraction**

$\mathbf{A} + \mathbf{D} = (a + b \cdot j) + (c + d \cdot j) = (a + c) + (b + d) \cdot j$

$\mathbf{A} - \mathbf{D} = (a + b \cdot j) - (c + d \cdot j) = (a - c) + (b - d) \cdot j$

Convert polars to rectangular form first

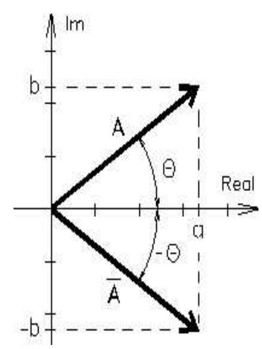
**Multiplication and Division** $\mathbf{A} \cdot \mathbf{D} = (a + b \cdot j) \cdot (c + d \cdot j) = (a \cdot c - b \cdot d) + (b \cdot c + a \cdot d) \cdot j$

Rectangular: $\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b \cdot j}{c + d \cdot j} = \frac{a + b \cdot j}{c + d \cdot j} \cdot \frac{c - d \cdot j}{c - d \cdot j} = \frac{a \cdot c + b \cdot d}{c^2 + d^2} + \frac{b \cdot c - a \cdot d}{c^2 + d^2} \cdot j$

Polar: $\mathbf{A} \cdot \mathbf{D} = A \cdot e^{j \cdot \theta} \cdot D \cdot e^{j \cdot \phi} = A \cdot D \cdot e^{j \cdot (\theta + \phi)}$

$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A \cdot e^{j \cdot \theta}}{D \cdot e^{j \cdot \phi}} = \frac{A}{D} \cdot e^{j \cdot (\theta - \phi)}$

**Powers** $\mathbf{A}^n = A^n \cdot e^{j \cdot n \cdot \theta} = A^n \cdot \cos(n \cdot \theta) + A^n \cdot \sin(n \cdot \theta) \cdot j$ Convert rectangulars first, usually




**Conjugates**

| complex number | Conjugate | |
|---|---|---|
| $\mathbf{A} = a + b \cdot j$ | $\overline{\mathbf{A}} = a - b \cdot j$ | $\overline{\overline{\mathbf{A}}} = \mathbf{A}$ |
| $\mathbf{A} = A \cdot e^{j \cdot \theta}$ | $\overline{\mathbf{A}} = A \cdot e^{-j \cdot \theta}$ | |
| $\mathbf{F} = \frac{3 + 5 \cdot j}{(2 - 6 \cdot j) \cdot e^{j \cdot 40 \cdot deg}}$ | $\overline{\mathbf{F}} = \frac{3 - 5 \cdot j}{(2 + 6 \cdot j) \cdot e^{-j \cdot 40 \cdot deg}}$ | |

**Euler's equation** $e^{j \cdot \alpha} = \cos(\alpha) + j \cdot \sin(\alpha)$ OR: $\cos(\alpha) = \frac{e^{j \cdot \alpha} + e^{-j \cdot \alpha}}{2} \qquad \sin(\alpha) = \frac{e^{j \cdot \alpha} - e^{-j \cdot \alpha}}{2 \cdot j}$

$e^{j \cdot (\omega \cdot t + \theta)} = \cos(\omega \cdot t + \theta) + j \cdot \sin(\omega \cdot t + \theta)$

$\mathrm{Re}\left[e^{j \cdot (\omega \cdot t + \theta)}\right] = \cos(\omega \cdot t + \theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega \cdot t + \theta)$ by $e^{j \cdot \theta}$

**Calculus** Remember, when we write $e^{j \cdot \theta}$, we really mean $e^{j \cdot (\omega \cdot t + \theta)}$

$$\frac{d}{dt}\mathbf{A} = \frac{d}{dt}\left(A \cdot e^{j \cdot \theta}\right) = j \cdot \omega \cdot A \cdot e^{j \cdot \theta} = \omega \cdot A \cdot e^{j \cdot (\theta + 90 \cdot deg)}$$

$$\int \mathbf{A}\, dt = \int A \cdot e^{j \cdot \theta}\, dt = \frac{1}{j \cdot \omega} \cdot A \cdot e^{j \cdot \theta} = \frac{1}{\omega} \cdot A \cdot e^{j \cdot (\theta - 90 \cdot deg)}$$

# Review of Phasors

ECE 3600

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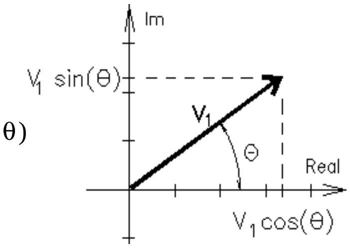
9/3/08
rev,

For steady-state sinusoidal response ONLY

## Phasors




Time domain

$$v(t) = \sqrt{2}\cdot V_1\cdot\cos(377\cdot t + \theta)$$

Phasor, frequency domain (RMS)

$$\mathbf{V_1} = V_1\cdot e^{j\cdot\theta} = V_1 \underline{/\theta} = V_1\cdot\cos(\theta) + j\cdot V_1\cdot\sin(\theta)$$

## Impedances,

**AC impedance**

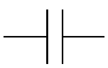
Inductor

$$v_L = L\cdot\frac{d}{dt}i_L = L\cdot\frac{d}{dt}I_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot L\cdot\left[I_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

$$\mathbf{V_L}(\omega) = j\cdot\omega\cdot L\cdot\mathbf{I}(\omega)$$

$$\mathbf{Z_L} = j\cdot\omega\cdot L$$

Capacitor



$$i_C = C\cdot\frac{d}{dt}v_C = C\cdot\frac{d}{dt}V_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot C\cdot\left[V_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

$$\mathbf{I_C}(\omega) = j\cdot\omega\cdot C\cdot\mathbf{V}(\omega) \qquad \mathbf{V_C}(\omega) = \frac{1}{j\cdot\omega\cdot C}\cdot\mathbf{I}(\omega)$$

$$\mathbf{Z_C} = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C}$$

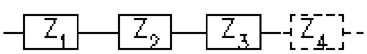
Resistor

$$v_R = i_R\cdot R \qquad \mathbf{V_R}(\omega) = R\cdot\mathbf{I}(\omega)$$

$$\mathbf{Z_R} = R$$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

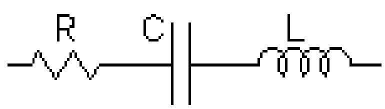
**series:**




$$\mathbf{Z_{eq}} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots$$

$$f := 60\cdot Hz \qquad \omega := 2\cdot\pi\cdot f \qquad \omega = 377\cdot\frac{rad}{sec}$$

Example:




$$R := 20\cdot\Omega \qquad C := 60\cdot\mu F \qquad L := 80\cdot mH$$

$$j\cdot\omega\cdot L = 30.159j\cdot\Omega$$

$$\frac{1}{j\cdot\omega\cdot C} = -44.21j\cdot\Omega$$

$$\mathbf{Z_{eq}} := R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L = 20\cdot\Omega - 44.21\cdot j\cdot\Omega + 30.16\cdot j\cdot\Omega = 20 - 14.05j\cdot\Omega$$

$$\sqrt{(20\cdot\Omega)^2 + (14.05\cdot\Omega)^2} = 24.44\cdot\Omega \qquad \operatorname{atan}\left(\frac{-14.05\cdot\Omega}{20\cdot\Omega}\right) = -35.09\cdot deg$$

$$\mathbf{Z_{eq}} = 24.44\Omega \underline{/-35.1}^o$$

If: $\mathbf{V} := 120\cdot V\cdot e^{j\cdot 0\cdot deg}$ $\quad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z_{eq}}} = \frac{120\cdot V}{24.44\cdot\Omega} = 4.91\cdot A \quad \underline{/}\ 0 - -35.1 = 35.1\ deg$

$$4.91\cdot\cos(35.1\cdot deg) = 4.017 \qquad 4.91\cdot\sin(35.1\cdot deg) = 2.823 \qquad \mathbf{I} = 4.017 + 2.822j\cdot A$$

slight roundoff error

**Voltage divider:**

$$V_{Zn} = V_{total}\cdot\frac{Z_n}{Z_1+Z_2+Z_3+\ldots}$$

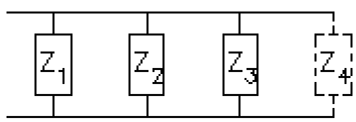
Eg: $$V_C := V\cdot\frac{\frac{1}{j\cdot\omega\cdot C}}{Z_{eq}} = 120\cdot V\cdot e^{j\cdot 0\cdot deg}\cdot\frac{44.21\cdot e^{-j\cdot 90\cdot deg}\cdot\Omega}{24.44\cdot e^{-j\cdot 35.1\cdot deg}\cdot\Omega}$$

$$120\cdot V\cdot\frac{44.21\cdot\Omega}{24.44\cdot\Omega} = 217.07\bullet V \qquad \angle\ 0+-90--35.1 = -54.9\ \ deg$$

$$V_C = 217.1V\ \underline{/-54.9}^o \qquad V_C = 124.771-177.604j\ \bullet V$$

$$217.1\cdot\cos(-54.9\cdot deg) = 124.8 \qquad 217.1\cdot\sin(-54.9\cdot deg) = -177.6$$

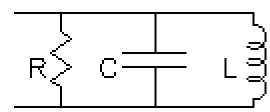
**parallel:**




$$Z_{eq} = \frac{1}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}+\ldots}$$

Example:

$$f := 60\cdot Hz \qquad \omega := 2\cdot\pi\cdot f \qquad \omega = 377\bullet\frac{rad}{sec}$$




$$L := 80\cdot mH$$

$$j\cdot\omega\cdot L = 30.159j\bullet\Omega$$

$$R := 20\cdot\Omega \qquad C := 60\cdot\mu F \qquad \frac{1}{\omega\cdot L} = 3.316\bullet 10^{-2}\ \bullet\frac{1}{\Omega}$$

$$\frac{1}{j\cdot\omega\cdot C} = -44.21j\bullet\Omega \qquad \omega\cdot C = 2.262\bullet 10^{-2}\ \bullet\frac{1}{\Omega}$$

$$Z_{eq} := \frac{1}{\frac{1}{R}+\frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)}+\frac{1}{j\cdot\omega\cdot L}} = \frac{1}{\frac{1}{R}+j\cdot\omega\cdot C-\frac{j}{\omega\cdot L}} = \frac{1}{\frac{1}{20\cdot\Omega}+2.262\cdot 10^{-2}\cdot j\cdot\frac{1}{\Omega}-3.316\cdot 10^{-2}\cdot j\cdot\frac{1}{\Omega}} = \frac{1}{\left(5\cdot 10^{-2}-1.054\cdot 10^{-2}\cdot j\right)\cdot\frac{1}{\Omega}}$$

$$= \frac{1}{\left(5\cdot 10^{-2}-1.054\cdot 10^{-2}\cdot j\right)\cdot\frac{1}{\Omega}}\cdot\frac{5\cdot 10^{-2}+1.054\cdot 10^{-2}\cdot j}{\left(5\cdot 10^{-2}+1.054\cdot 10^{-2}\cdot j\right)} = 19.149+4.037j\ \bullet\Omega$$

$$\sqrt{\left(5\cdot 10^{-2}\cdot\frac{1}{\Omega}\right)^2+\left(1.054\cdot 10^{-2}\cdot\frac{1}{\Omega}\right)^2} = 5.11\bullet 10^{-2}\ \bullet\frac{1}{\Omega} \qquad \operatorname{atan}\left(\frac{-1.054\cdot 10^{-2}\cdot\Omega}{5\cdot 10^{-2}\cdot\Omega}\right) = -11.9\bullet deg$$

$$\frac{1}{5.11\cdot 10^{-2}\cdot\frac{1}{\Omega}} = 19.569\bullet\Omega \qquad \angle\ 0--11.9 = 11.9\ \ deg \qquad Z_{eq} = 19.57\Omega\ \underline{/11.9}^o$$

If: $$V := 120\cdot V\cdot e^{j\cdot 0\cdot deg} \qquad I := \frac{V}{Z_{eq}} = \frac{120\cdot V}{19.57\cdot\Omega} = 6.132\bullet A \qquad \angle\ 0-11.9 = -11.9\ \ deg$$

$$6.132\cdot\cos(-11.9\cdot deg) = 6 \qquad 6.132\cdot\sin(-11.9\cdot deg) = -1.264 \qquad I = 6-1.265j\ \bullet A$$

slight roundoff error

**Current divider:**

$$I_{Zn} = I_{total}\cdot\frac{\frac{1}{Z_n}}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}+\ldots}$$

Eg: $$I_L := I\cdot\frac{\frac{1}{j\cdot\omega\cdot L}}{\frac{1}{R}+j\cdot\omega\cdot C+\frac{1}{j\cdot\omega\cdot L}} = I\cdot\frac{\left(\frac{1}{j\cdot\omega\cdot L}\right)}{\left(\frac{1}{Z_{eq}}\right)} = I\cdot\frac{Z_{eq}}{j\cdot\omega\cdot L}$$

$$= 6.132\cdot A\cdot e^{j\cdot -11.9\cdot deg}\cdot\frac{19.57\cdot e^{j\cdot 11.9\cdot deg}\cdot\Omega}{30.159\cdot e^{j\cdot 90\cdot deg}\cdot\Omega}$$

$$I_L = 6.132\cdot A\cdot\frac{19.57\cdot\Omega}{30.159\cdot\Omega} = 3.979\bullet A \qquad \angle\ -11.9+11.9-90 = -90\ \ deg \qquad I_L = -3.979\bullet 10^3 j\ \bullet mA$$

Duh... $$\frac{V}{j\cdot\omega\cdot L} = -3.979\bullet 10^3 j\ \bullet mA$$

# ECE 3600 Phasor Examples

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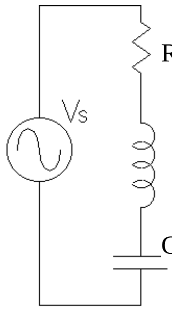
**Ex. 1** Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2\cdot kHz$



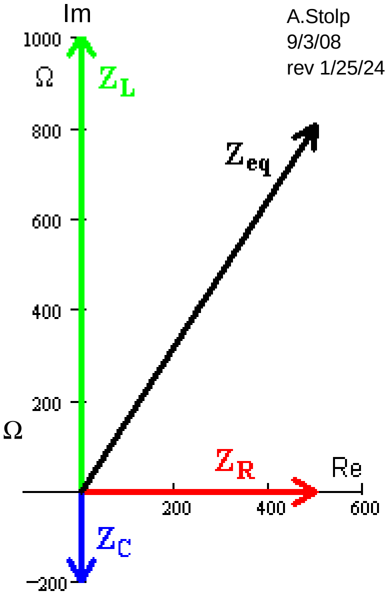

$R := 500\cdot\Omega$ $\quad \omega := 2\cdot\pi\cdot f$ $\quad \omega = 12566\cdot\frac{rad}{sec}$

$L := 80\cdot mH$ $\quad \mathbf{Z_L} := j\cdot\omega\cdot L$ $\quad \mathbf{Z_L} = 1.005j\cdot k\Omega$

$C := 0.4\cdot\mu F$ $\quad \mathbf{Z_C} := \frac{1}{j\cdot\omega\cdot C}$ $\quad \mathbf{Z_C} = -0.199j\cdot k\Omega$

$\mathbf{Z_{eq}} := R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}$ $\quad \mathbf{Z_{eq}} = (500 + 806.366j)\cdot\Omega$

$\sqrt{500^2 + 806^2} = 948.491$ $\quad \operatorname{atan}\left(\frac{806}{500}\right) = 58.187\cdot deg$ $\quad \mathbf{Z_{eq}} = 948.5\Omega\ \underline{/\ 58.2^o}$




find the current: $\mathbf{I} := \frac{6\cdot V\cdot e^{j\cdot 0}}{\mathbf{Z_{eq}}}$ $\quad$ magnitude: $\frac{6\cdot V}{948.5\cdot\Omega} = 6.326\cdot mA$ $\quad$ angle: $0\cdot deg - 58.2\cdot deg = -58.2\cdot deg$

$\mathbf{I} = 6.326mA\ \underline{/-58.2^o}$

| | <u>find the magnitude</u> | <u>find the angle</u> | |
|---|---|---|---|
| $\mathbf{V_R} := \mathbf{I}\cdot R$ | $6.326\cdot mA\cdot 500\cdot\Omega = 3.163\cdot V$ | $-58.2\cdot deg + 0\cdot deg = -58.2\cdot deg$ | $\mathbf{V_R}$ = 3.163V /-58.2º |
| $\mathbf{V_L} := \mathbf{I}\cdot\mathbf{Z_L}$ | $6.326\cdot mA\cdot 1005\cdot\Omega = 6.358\cdot V$ | $-58.2\cdot deg + 90\cdot deg = 31.8\cdot deg$ | $\mathbf{V_L}$ = 6.358V / 31.8º |
| $\mathbf{V_C} := \mathbf{I}\cdot\mathbf{Z_C}$ | $6.326\cdot mA\cdot(-199)\cdot\Omega = -1.259\cdot V$ | $-58.2\cdot deg + (90)\cdot deg = 31.8\cdot deg$ | $\mathbf{V_C}$ = -1.259V / 31.8º |
| OR: | $6.326\cdot mA\cdot(199)\cdot\Omega = 1.259\cdot V$ | $-58.2\cdot deg + (-90)\cdot deg = -148.2\cdot deg$ | $\mathbf{V_C}$ = 1.259V /-148.2º |

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\frac{1}{j\cdot\omega\cdot C}}{R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) + j\cdot\omega\cdot L\cdot(j\cdot\omega\cdot C) + 1}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) - \omega^2\cdot L\cdot C + 1}\cdot 6\cdot V$$

$$= \frac{1}{\left(1 - \omega^2\cdot L\cdot C\right) + j\cdot\omega\cdot R\cdot C}\cdot 6\cdot V \qquad \left(1 - \omega^2\cdot L\cdot C\right) = -4.053 \qquad j\cdot\omega\cdot R\cdot C = 2.513j$$

$$= \frac{6\cdot V}{-4.053 + 2.513\cdot j}\cdot\frac{(-4.053 - 2.513\cdot j)}{(-4.053 - 2.513\cdot j)} = \frac{6\cdot V\cdot(-4.053 - 2.513\cdot j)}{(-4.053)^2 + 2.513^2}$$

$6\cdot V\cdot(-4.053 - 2.513\cdot j) = (-24.318 - 15.078j)\cdot V$

$(-4.053)^2 + 2.513^2 = 22.742$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078\cdot j}{22.742}\right)\cdot V = (-1.069 - 0.663j)\cdot V$$

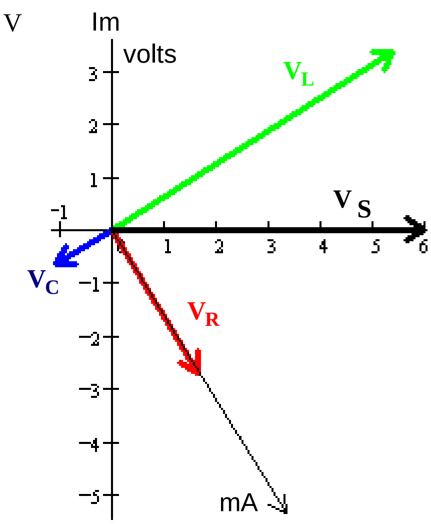
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

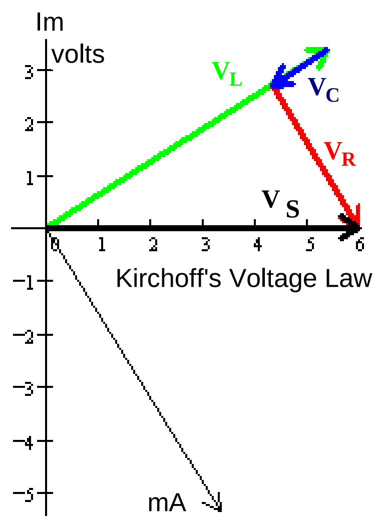
angle: $\operatorname{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81\cdot deg$

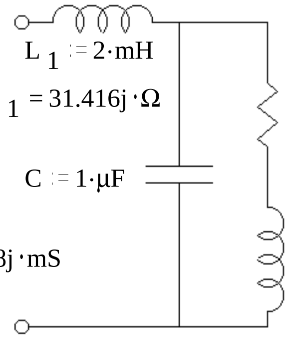
but this is actually in the third quadrant, so modify your calculator's results:

$31.81\cdot deg - 180\cdot deg = -148.19\cdot deg$

= 1.258V /-148.2º





$L_1 := 2\cdot mH$ $\quad j\cdot\omega\cdot L_1 = 31.416j\cdot\Omega$ $\quad R := 200\cdot\Omega$ $\quad C := 1\cdot\mu F$ $\quad L_2 := 8\cdot mH$ $\quad j\cdot\omega\cdot L_2 = 125.664j\cdot\Omega$

**Ex. 2** a) Find $\mathbf{Z_{eq}}$. $\quad f := 2.5\cdot kHz \quad \omega := 2\cdot\pi\cdot f \quad \omega = 15708\cdot\frac{rad}{sec}$

$$\mathbf{Z_{eq}} = j\cdot\omega\cdot L_1 + \cfrac{1}{\cfrac{1}{R + j\cdot\omega\cdot L_2} + \cfrac{1}{\cfrac{1}{j\cdot\omega\cdot C}}} = j\cdot\omega\cdot L_1 + \cfrac{1}{\cfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}$$

$$j\cdot\omega\cdot C = 15.708j\cdot mS$$

$$\mathbf{Z_{eq}} := j\cdot\omega\cdot L_1 + \cfrac{1}{\cfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C} = 31.416\cdot j\cdot\Omega + \cfrac{1}{\cfrac{1}{(200 + 125.664\cdot j)\cdot\Omega} + 15.708\cdot j\cdot mS}$$

$$= 31.416\cdot j\cdot\Omega + \frac{1}{(3.585 - 2.252\cdot j + 15.708\cdot j)\cdot mS} = 31.416\cdot j\cdot\Omega + (18.487 - 69.391\cdot j)\cdot\Omega = 18.487 - 37.975j\cdot\Omega$$

$$|\mathbf{Z_{eq}}| = 42.238\cdot\Omega \qquad \arg(\mathbf{Z_{eq}}) = -64.043\cdot deg$$

b) $\mathbf{V_{in}} := 12\cdot V\cdot e^{j\cdot 20\cdot deg}$ Find $\mathbf{I_{L1}}$, $\mathbf{V_C}$ $\quad \mathbf{I_{L1}} := \frac{\mathbf{V_{in}}}{\mathbf{Z_{eq}}} \quad \frac{12\cdot V}{42.24\cdot\Omega} = 284.1\cdot mA \quad 20\cdot deg - (-64.04)\cdot deg = 84.04\cdot deg$

$$\mathbf{I_{L1}} = 284.1mA\ \underline{/\ 84.04^o} = 284.1\cdot mA\cdot e^{j\cdot 84.04\cdot deg} \qquad \mathbf{I_{L1}} = 29.485 + 282.569j\cdot mA$$

$$\mathbf{V_C} := \mathbf{I_{L1}}\cdot(18.486 - 69.384\cdot j)\cdot\Omega \qquad 284.1\cdot mA\cdot\sqrt{18.486^2 + 69.384^2}\cdot\Omega = 20.4\cdot V \qquad 84.04\cdot deg + \operatorname{atan}\left(\frac{-69.384}{18.486}\right) = 8.959\cdot deg$$

$$\mathbf{V_C} = 20.4V\ \underline{/\ 8.96^o}$$

To find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\cfrac{1}{\cfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}{j\cdot\omega\cdot L_1 + \cfrac{1}{\cfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}\cdot\mathbf{V_{in}} = \frac{1}{j\cdot\omega\cdot L_1\cdot\left(\cfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C\right) + 1}\cdot\mathbf{V_{in}} \qquad \mathbf{V_C} = 20.153 + 3.178j\cdot V$$

You could then use another voltage divider to find $\mathbf{V_R}$ or $\mathbf{V_{L2}}$.

c) Find $\mathbf{I_{L2}}$

$$\mathbf{I_{L2}} := \frac{\mathbf{V_C}}{R + j\cdot\omega\cdot L_2} = \frac{20.4\cdot V\cdot e^{j\cdot 8.96\cdot deg}}{236.202\cdot\Omega\cdot e^{j\cdot 32.142\cdot deg}} = \frac{20.4\cdot V}{236.202\cdot\Omega}\ \underline{/\ 8.96 - 32.142^o} = 86.4mA\ \underline{/ -23.18}$$

Or, directly by Current divider:

$$\mathbf{I_{L2}} := \frac{\cfrac{1}{R + j\cdot\omega\cdot L_2}}{j\cdot\omega\cdot C + \cfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{1}{j\cdot\omega\cdot C\cdot(R + j\cdot\omega\cdot L_2) + 1}\cdot\mathbf{I_{L1}} = 79.404 - 34.001j\cdot mA$$

d) How about $\mathbf{I_C}$?

$$\mathbf{I_C} := \frac{\mathbf{V_C}}{\left(\cfrac{1}{j\cdot\omega\cdot C}\right)} = \mathbf{V_C}\cdot j\cdot\omega\cdot C = 20.4V\ \underline{/\ 8.96^o}\ \ 15.708mS\ \underline{/\ 90^o} = 320mA\ \underline{/\ 98.96^o}$$

Or, directly by Current divider:

$$\mathbf{I_C} := \frac{j\cdot\omega\cdot C}{j\cdot\omega\cdot C + \cfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}}$$

This current is greater than the input current. What's going on?

The angle between $\mathbf{I_C}$ & $\mathbf{I_{L2}}$ is big enough that they somewhat cancel each other out, partial resonance.

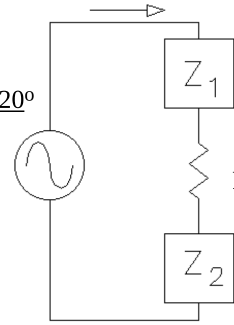
Check Kirchoff's Current Law: $\quad \mathbf{I_C} + \mathbf{I_{L2}} = 29.485 + 282.569j\cdot mA \quad = \quad \mathbf{I_{L1}} = 29.485 + 282.569j\cdot mA$

$i(t) = 25 \cdot mA \cdot \cos\left(377 \cdot \frac{rad}{sec} \cdot t + 10 \cdot deg\right)$

**Ex. 3** a) Find $\mathbf{Z_2}$.

$\mathbf{I} := 25 \cdot mA \cdot e^{j \cdot 10 \cdot deg}$

$\mathbf{V_{in}} := 10 \cdot V \cdot e^{-j \cdot 20 \cdot deg}$




$\mathbf{V_{in}}$ = 10V /-20º

$\mathbf{Z_1} := (80 - 60 \cdot j) \cdot \Omega$

$R := 50 \cdot \Omega$

$\mathbf{Z_2}$ = ?

$\mathbf{Z_T} := \frac{\mathbf{V_{in}}}{\mathbf{I}} = \frac{10 \cdot V}{25 \cdot mA}$ /-20 - 10º = 400Ω /-30º

$\mathbf{Z_T} = 346.41 - 200j \ \cdot \Omega$

$\mathbf{Z_2} := \mathbf{Z_T} - R - \mathbf{Z_1} = (346.41 - 200 \cdot j) \cdot \Omega - 50 \cdot \Omega - (80 - 60 \cdot j) \cdot \Omega = 216.41 - 140j \ \cdot \Omega$

b) Circle 1: i) The source current leads the source voltage <--- answer, because 10º > -20º.

ii) The source voltage leads the source current

**Ex. 4** a) The impedance $\mathbf{Z_1}$ (above) is made of two components in series. What are they and what are their values?

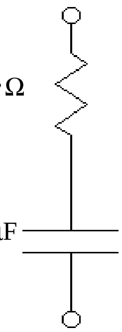
$\mathbf{Z_1} = 80 - 60j \ \cdot \Omega \qquad \omega := 377 \cdot \frac{rad}{sec}$

Must have a resistor because there is a real part.

$R := \mathrm{Re}(\mathbf{Z_1}) \qquad R = 80 \cdot \Omega$

Must have a capacitor because the imaginary part is negative.

$\mathrm{Im}(\mathbf{Z_1}) = -60 \cdot \Omega = \frac{-1}{\omega \cdot C} \qquad C := \frac{-1}{\omega \cdot \mathrm{Im}(\mathbf{Z_1})} \qquad C = 44.21 \cdot \mu F$



b) The impedance $\mathbf{Z_1}$ is made of two components in parallel. What are they and what are their values?

$\mathbf{Z_1} = 80 - 60j \ \cdot \Omega$

Must have a resistor because there is a real part.

Must have a capacitor because the imaginary part is negative.

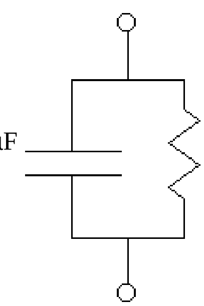
$$\mathbf{Z_1} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C} \qquad \frac{1}{\mathbf{Z_1}} = \frac{1}{(80 - 60 \cdot j) \cdot \Omega} \cdot \left(\frac{80 + 60 \cdot j}{80 + 60 \cdot j}\right) = \frac{80 + 60 \cdot j}{80^2 + 60^2} = \frac{80 + 60 \cdot j}{10{,}000} \cdot \frac{1}{\Omega}$$

$80^2 + 60^2 = 10000$

$$\frac{1}{\mathbf{Z_1}} = 8 + 6j \ \cdot mS = 0.008 + 0.006 \cdot j \ \frac{1}{\Omega} = \frac{1}{R} + j \cdot \omega \cdot C$$

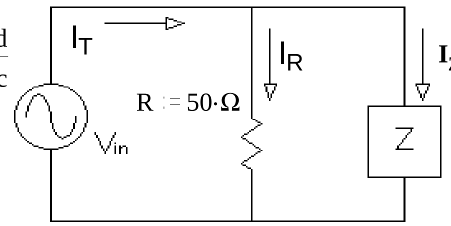
$\frac{1}{R} = 0.008 \cdot \frac{1}{\Omega} \qquad R := \frac{1}{0.008 \cdot \frac{1}{\Omega}} \qquad R = 125 \cdot \Omega$

$\omega \cdot C = 0.006 \cdot \frac{1}{\Omega} \qquad C := \frac{0.006 \cdot \frac{1}{\Omega}}{\omega} \qquad C = 15.915 \cdot \mu F$



$R = 125 \cdot \Omega$

$\omega := 1000 \cdot \frac{rad}{sec}$




$\mathbf{I_Z} = 100\ \underline{/\ 0^o}$ mA

$\mathbf{Z} := (40 + 60 \cdot j) \cdot \Omega$

**Ex. 5** a) Find $\mathbf{V_{in}}$ in polar form.

$\mathbf{I_Z} := 100 \cdot mA$    $\mathbf{Z} := (40 + 60 \cdot j) \cdot \Omega$

$\mathbf{V_{in}} := \mathbf{I_Z} \cdot \mathbf{Z}$    $\mathbf{V_{in}} = (4 + 6j) \cdot V$

$\sqrt{4^2 + 6^2} = 7.211$    $\mathrm{atan}\left(\frac{6}{4}\right) = 56.31 \cdot deg$    $\mathbf{V_{in}} = 7.21V\ \underline{/ \text{-}56.3^o}$

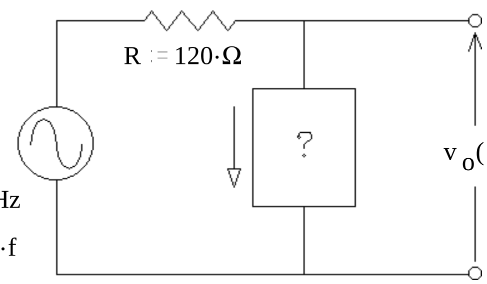
b) Find $\mathbf{I_T}$ in polar form.    $\mathbf{I_R} := \frac{\mathbf{V_{in}}}{R} = \frac{(4 + 6 \cdot j) \cdot V}{50 \cdot \Omega} = \frac{4 \cdot V}{50 \cdot \Omega} + \frac{6 \cdot j \cdot V}{50 \cdot \Omega} = (80 + 120j) \cdot mA$

$\mathbf{I_T} := \mathbf{I_R} + \mathbf{I_Z} = (80 + 120 \cdot j) \cdot mA + 100 \cdot mA = (180 + 120j) \cdot mA$

$|\mathbf{I_T}| = 216.3 \cdot mA$    $\arg(\mathbf{I_T}) = 33.69 \cdot deg$    $\mathbf{I_T} = 216.3mA\ \underline{/\ 33.7^o}$

c) Circle 1: i) $\mathbf{I_T}$ leads $\mathbf{V_{in}}$    ii) $\mathbf{V_{in}}$ leads $\mathbf{I_T}$    answer ii), $56.3^o > 33.7^o$

**Ex. 6** You need to design a circuit in which the the "output" voltage leads the input voltage ($v_S(t)$) by 30º of phase.




$f := 1 \cdot kHz$

$\omega := 2 \cdot \pi \cdot f$

$\omega = 6.283 \cdot 10^3 \cdot \frac{rad}{sec}$

a) What should go in the box: R, L, C?

$$\mathbf{V_o} = \frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}} \cdot \mathbf{V_S}$$

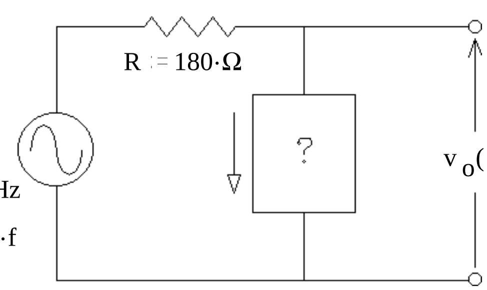
angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is 30º.

This can only happen if the angle of $\mathbf{Z_{box}}$ is positive, so $\mathbf{Z_{box}}$ is a inductor

b) Find its value. $\mathbf{V_o} = \quad \mathbf{V_o} = \frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L} \cdot \mathbf{V_S}$    angle: $\frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L}$ is $90 - \mathrm{atan}\left(\frac{\omega \cdot L}{R}\right) = 30^o$ so $\mathrm{atan}\left(\frac{\omega \cdot L}{R}\right) = 60^o$.

$\frac{\omega \cdot L}{R} = \tan(60 \cdot deg) = 1.732$    $L := \frac{R \cdot 1.732}{\omega}$    $L = 33.1 \cdot mH$

**Ex. 7** You need to design a circuit in which the the "output" voltage lags the input voltage ($v_S(t)$) by 40º of phase.




$f := 1 \cdot kHz$

$\omega := 2 \cdot \pi \cdot f$

a) What should go in the box: R, L, C?

$$\mathbf{V_o} = \frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}} \cdot \mathbf{V_S}$$

angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is -40º.

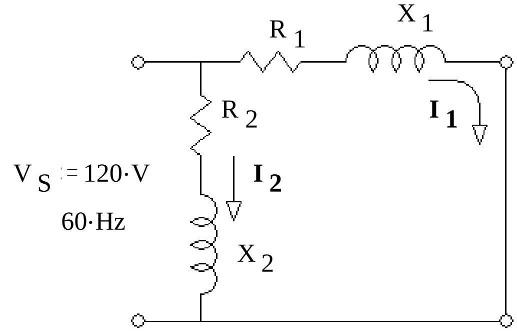
This can only happen if the angle of $\mathbf{Z_{box}}$ is negative, so $\mathbf{Z_{box}}$ is a capacitor

b) Find its value.

$$\mathbf{V_o} = \frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}} \cdot \mathbf{V_S} \quad \text{angle: } \frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}} \text{ is } -90 - \mathrm{atan}\left(\frac{-\frac{1}{\omega \cdot C}}{R}\right) = -90 - \mathrm{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right) \text{ so } \mathrm{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right) = \text{-}50^o$$

$-\frac{1}{\omega \cdot C \cdot R} = \tan(-50 \cdot deg) = -1.192$    $C := \frac{1}{\omega \cdot R \cdot 1.192}$    $C = 0.742 \cdot \mu F$

**Ex. 8** The magnitudes of $\mathbf{I_1}$ and $\mathbf{I_2}$ are 3A and 2A. They lag the supply voltage by 20° and 30°. respectively.




a) Find the values of $R_1$, $R_2$, $X_1$ and $X_2$.

$$\mathbf{Z_1} := \frac{120 \cdot V}{3 \cdot A \cdot e^{-j \cdot 20 \cdot deg}} \qquad \mathbf{Z_1} = 37.588 + 13.681j \ \cdot\Omega$$

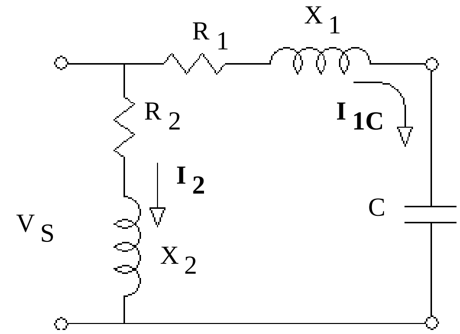
$$R_1 := Re(\mathbf{Z_1}) \qquad R_1 = 37.588 \cdot\Omega$$

$$X_1 := Im(\mathbf{Z_1}) \qquad X_1 = 13.681 \cdot\Omega$$

$$\mathbf{Z_2} := \frac{120 \cdot V}{2 \cdot A \cdot e^{-j \cdot 30 \cdot deg}} \qquad \mathbf{Z_2} = 51.962 + 30j \ \cdot\Omega \qquad R_2 := Re(\mathbf{Z_2}) \qquad R_2 = 51.962 \cdot\Omega \qquad X_2 := Im(\mathbf{Z_2}) \qquad X_2 = 30 \cdot\Omega$$

b) Add C to the circuit such that $\mathbf{I_{1C}}$ leads $\mathbf{I_2}$ by 90°. Find the value of C.

$\omega := 2 \cdot \pi \cdot 60 \cdot Hz$




$$\mathbf{I_{1C}} = \frac{120 \cdot V}{R_1 + j \cdot X_1 + j \cdot X_C}$$ needs to be at an angle of + 50°

So: $$atan\left(\frac{X_1 + X_C}{R_1}\right) = -50 \cdot deg$$

$$\frac{X_1 + X_C}{R_1} = tan(-50 \cdot deg)$$

$$X_C := R_1 \cdot tan(-60 \cdot deg) - X_1 \qquad X_C = -78.785 \cdot\Omega = \frac{-1}{\omega \cdot C} \qquad C := \frac{-1}{\omega \cdot X_C} \qquad C = 33.7 \cdot\mu F$$

c) Change C so that the magnitudes of $\mathbf{I_{1C}}$ and $\mathbf{I_2}$ are the same. Find the new C.

$$\left|\mathbf{I_{1C}}\right| = \left|\frac{120 \cdot V}{R_1 + j \cdot X_1 + j \cdot X_C}\right|$$ needs to be 2A. So: $$\left|R_1 + j \cdot X_1 + j \cdot X_C\right| = 60 \cdot\Omega$$

$$\sqrt{R_1^{\,2} + \left(X_1 + X_C\right)^2} = 60 \cdot\Omega$$

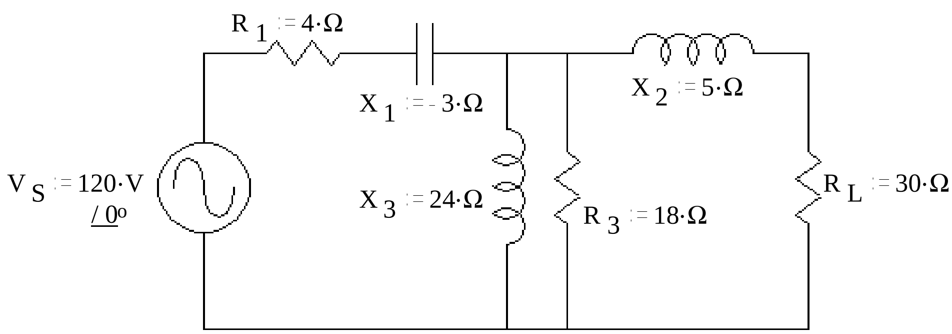
$$\left(X_1 + X_C\right) = \sqrt{(60 \cdot\Omega)^2 - R_1^{\,2}} = 46.767 \cdot\Omega$$

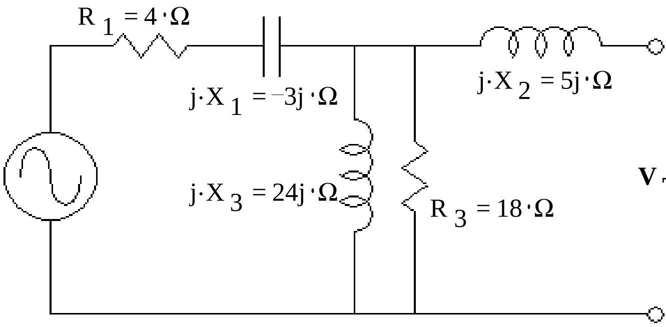
$$X_C := \sqrt{(60 \cdot\Omega)^2 - R_1^{\,2}} - X_1 \qquad X_C = 33.086 \cdot\Omega = \frac{-1}{\omega \cdot C}$$ NOT POSSIBLE

So: $$\left(X_1 + X_C\right) = -46.767 \cdot\Omega$$

And: $$X_C := -\sqrt{(60 \cdot\Omega)^2 - R_1^{\,2}} - X_1 \qquad X_C = -60.448 \cdot\Omega = \frac{-1}{\omega \cdot C} \qquad C := \frac{-1}{\omega \cdot X_C} \qquad C = 43.9 \cdot\mu F$$

**Ex. 9** a) In the circuit below $R_L$ is the load resistor. Find and draw the Thevenin equivalent of the rest of the circuit.



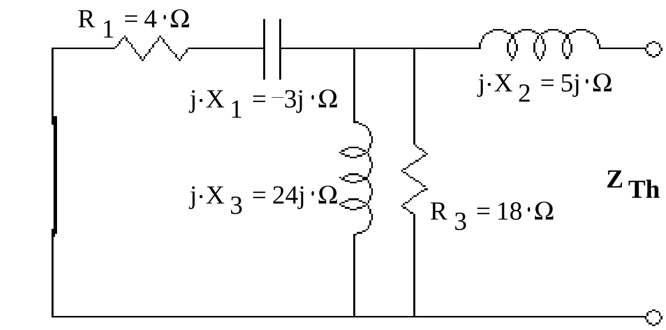



$$\mathbf{V_{Th}} := V_S\cdot\frac{\left(\dfrac{1}{\dfrac{1}{j\cdot X_3}+\dfrac{1}{R_3}}\right)}{R_1 + j\cdot X_1 + \left(\dfrac{1}{\dfrac{1}{j\cdot X_3}+\dfrac{1}{R_3}}\right)}$$

$\left|\mathbf{V_{Th}}\right| = 104.645\cdot V$

$\arg\left(\mathbf{V_{Th}}\right) = 16.899\cdot deg$

$\mathbf{V_{Th}} = 100.126 + 30.418j\ \cdot V$



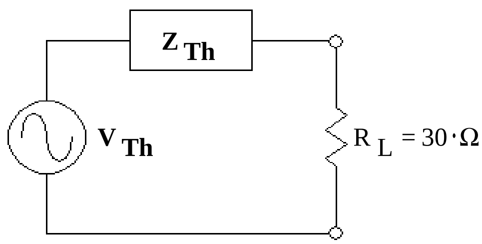

$$\mathbf{Z_{Th}} := \frac{1}{\dfrac{1}{R_1 + j\cdot X_1}+\dfrac{1}{j\cdot X_3}+\dfrac{1}{R_3}} + j\cdot X_2$$

$\left|\mathbf{Z_{Th}}\right| = 5.3962\cdot\Omega$

$\arg\left(\mathbf{Z_{Th}}\right) = 40.587\cdot deg$

$\mathbf{Z_{Th}} = 4.098 + 3.511j\ \cdot\Omega$

b) Use the Thevenin equivalent to find the current through the load resistor and the voltage across the load resistor.




$$\mathbf{I_{RL}} := \frac{\mathbf{V_{Th}}}{\mathbf{Z_{Th}} + R_L}$$

$\mathbf{I_{RL}} = 2.997 + 0.584j\ \cdot A$

$\left|\mathbf{I_{RL}}\right| = 3.053\cdot A$ $\quad\arg\left(\mathbf{I_{RL}}\right) = 11.02\cdot deg$

$\mathbf{V_{RL}} := \mathbf{I_{RL}}\cdot R_L$ $\quad\mathbf{V_{RL}} = 89.895 + 17.507j\ \cdot V$

$\left|\mathbf{V_{RL}}\right| = 91.584\cdot V$ $\quad\arg\left(\mathbf{V_{RL}}\right) = 11.02\cdot deg$

c) Find a replacement for $R_L$ in order to maximize the power delivered to $R_L$.

$R_L := \left|\mathbf{Z_{Th}}\right|$

$R_L = 5.396\cdot\Omega$

d) Find the new current and voltage for the load resistor.

$$\mathbf{I_{RL}} := \frac{\mathbf{V_{Th}}}{\mathbf{Z_{Th}} + R_L}$$

$\mathbf{I_{RL}} = 10.32 - 0.612j\ \cdot A$ $\quad\left|\mathbf{I_{RL}}\right| = 10.338\cdot A$ $\quad\arg\left(\mathbf{I_{RL}}\right) = -3.395\cdot deg$

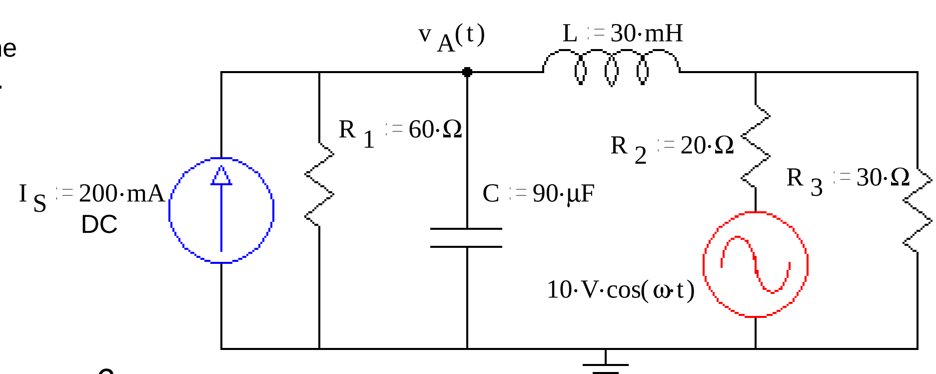
$\mathbf{V_{RL}} := \mathbf{I_{RL}}\cdot R_L$ $\quad\mathbf{V_{RL}} = 55.687 - 3.303j\ \cdot V$ $\quad\left|\mathbf{V_{RL}}\right| = 55.785\cdot V$ $\quad\arg\left(\mathbf{V_{RL}}\right) = -3.395\cdot deg$

**Ex. 10** The circuit shown has two sources. The current source is DC and the voltage source is 60Hz.

Using superposition, find the nodal voltage $v_A(t)$. Be sure to redraw the cicuit twice as part of your solution.

$v_A(t) = ?$

$\omega := 2\cdot\pi\cdot 60\cdot Hz$

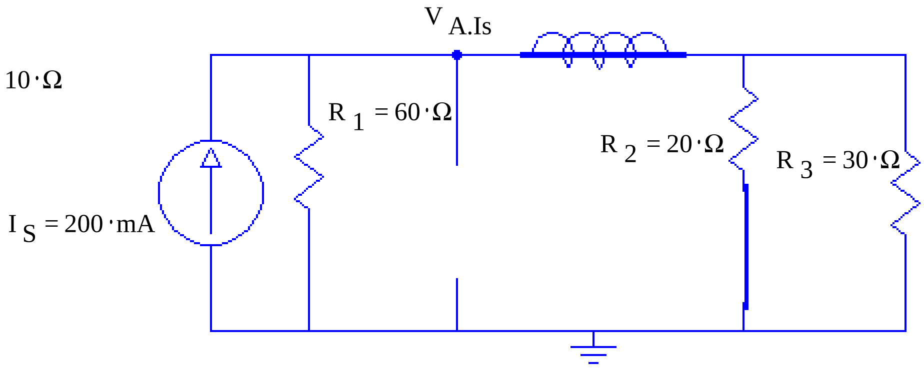


Eliminate voltage source

$$R_{eq} := \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} \qquad R_{eq} = 10\cdot\Omega$$

$$V_{A.Is} := I_S\cdot R_{eq}$$

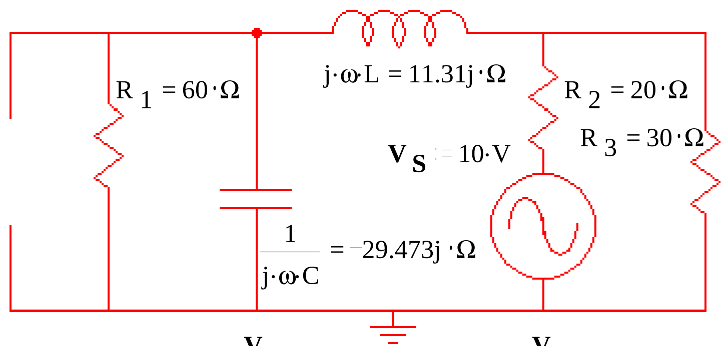
$$V_{A.Is} = 2\cdot V$$




Eliminate current source

Let's use nodal analysis

node A




$$\mathbf{I_L} = \mathbf{I_1} + \mathbf{I_C}$$

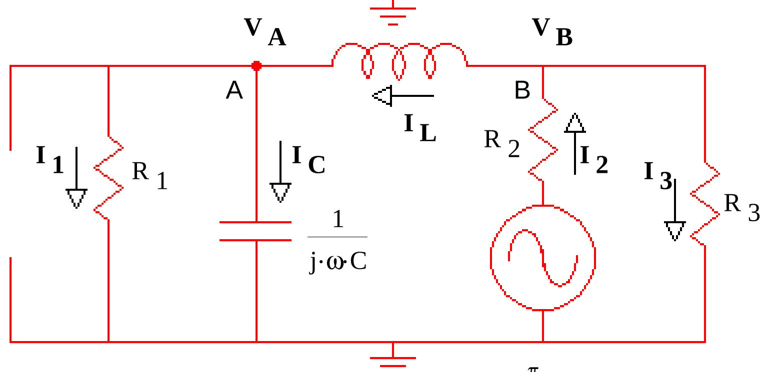
$$\frac{\mathbf{V_B}-\mathbf{V_A}}{j\cdot\omega\cdot L} = \frac{\mathbf{V_A}}{R_1}+\mathbf{V_A}\cdot j\cdot\omega\cdot C$$

$$\mathbf{V_B}-\mathbf{V_A} = \left(\frac{\mathbf{V_A}}{R_1}+\mathbf{V_A}\cdot j\cdot\omega\cdot C\right)\cdot(j\cdot\omega\cdot L)$$

$$j\cdot\omega\cdot L = 11.31j\cdot\Omega$$

$$\mathbf{V_B} = \left(\frac{\mathbf{V_A}}{R_1}+\mathbf{V_A}\cdot j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L+\mathbf{V_A}$$

$$j\cdot\omega\cdot C = 33.929j\cdot mS$$




node B

$$\mathbf{I_2} = \mathbf{I_L} + \mathbf{I_3}$$

$$\frac{\mathbf{V_S}-\mathbf{V_B}}{R_2} = \frac{\mathbf{V_B}-\mathbf{V_A}}{j\cdot\omega\cdot L}+\frac{\mathbf{V_B}}{R_3}$$

$$\frac{\mathbf{V_S}}{R_2}+\frac{\mathbf{V_A}}{j\cdot\omega\cdot L} = \mathbf{V_B}\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right) = \mathbf{V_B}\cdot(83.333-88.419j)\cdot mS = \mathbf{V_B}\cdot 121.5\cdot mS\cdot e^{-46.696\cdot\frac{\pi}{180}\cdot j}$$

$$\mathbf{V_B} = \frac{\mathbf{V_S}}{R_2\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right)}+\frac{\mathbf{V_A}}{j\cdot\omega\cdot L\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right)} = \left(\frac{\mathbf{V_A}}{R_1}+\mathbf{V_A}\cdot j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L+\mathbf{V_A}$$

Equate to node A equation:

$$\frac{\mathbf{V_S}}{R_2\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right)} = \left(\frac{\mathbf{V_A}}{R_1}+\mathbf{V_A}\cdot j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L+\mathbf{V_A}-\frac{\mathbf{V_A}}{1+j\cdot\omega\cdot L\cdot\left(\frac{1}{R_3}+\frac{1}{R_2}\right)}$$

$$= \mathbf{V_A}\cdot\left[\left(\frac{1}{R_1}+j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L+1-\frac{1}{1+j\cdot\omega\cdot L\cdot\left(\frac{1}{R_3}+\frac{1}{R_2}\right)}\right]$$

$$1+j\cdot\omega\cdot L\cdot\left(\frac{1}{R_3}+\frac{1}{R_2}\right) = 1+0.942j$$

$$\left(\frac{1}{R_1}+j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L = -0.384+0.188j$$

$$\mathbf{V_A} := \frac{\mathbf{V_S}}{R_2\cdot\left(\frac{1}{j\cdot\omega\cdot L}+\frac{1}{R_3}+\frac{1}{R_2}\right)}\cdot\frac{1}{\left[\left(\frac{1}{R_1}+j\cdot\omega\cdot C\right)\cdot j\cdot\omega\cdot L+1-\frac{1}{1+j\cdot\omega\cdot L\cdot\left(\frac{1}{R_3}+\frac{1}{R_2}\right)}\right]}$$

$$\mathbf{V_A} = 4.796-3.5j\ \cdot V$$

$$\left|\mathbf{V_A}\right| = 5.938\cdot V \qquad \arg(\mathbf{V_A}) = -36.12\cdot deg$$

$$V_{A.Vs} = 5.938\cdot V\cdot\cos(377\cdot t-36.1\cdot deg)$$

$$= \frac{\mathbf{V_S}}{R_2\cdot 121.5\cdot mS\cdot e^{-j\cdot 46.696\cdot deg}\cdot\left[(-0.384+0.188j)+1-\frac{1}{1+0.942\cdot j}\right]}$$

Add the results $v_A(t) = 2\cdot V+5.938\cdot V\cdot\cos(377\cdot t-36.1\cdot deg)$

**Ex. 11** The circuit shown has two sources. The frequency of the current source is the third harmonic of the voltage source.

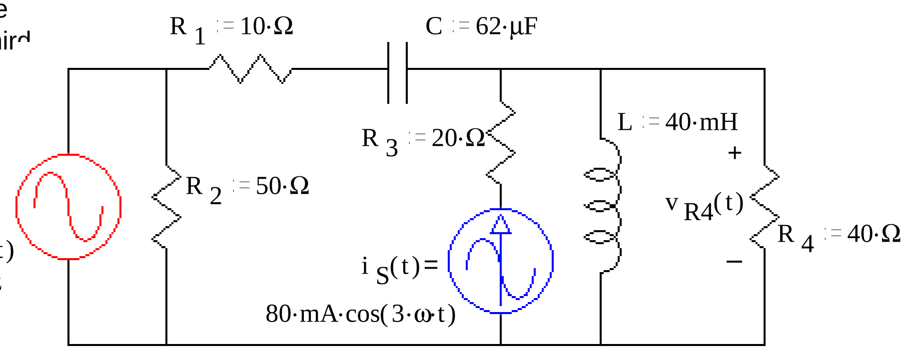
Using superposition, find the voltage across $R_4$. Be sure to redraw the cicuit twice as part of your solution.

$v_{R4}(t) = ?$

$v_S(t) := 10\cdot V\cdot\cos(\omega\cdot t)$

$f := 60\cdot Hz$

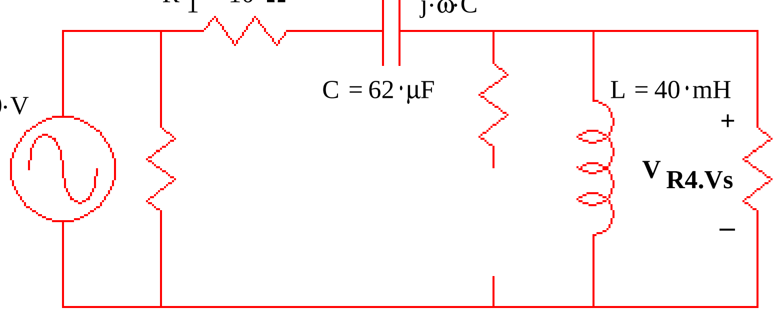
$\omega := 2\cdot\pi\cdot f$




Eliminate current source

$R_1 = 10\cdot\Omega$ $\quad \mathbf{Z_C} = \frac{1}{j\cdot\omega\cdot C} = -42.784j\cdot\Omega$

$\mathbf{V_S} := 10\cdot V$




$\mathbf{Z_L} = j\cdot\omega\cdot L = 15.08j\cdot\Omega$

$$\mathbf{V_{R4.Vs}} := \mathbf{V_S}\cdot\frac{\dfrac{1}{\left(\dfrac{1}{j\cdot\omega\cdot L}+\dfrac{1}{R_4}\right)}}{R_1+\dfrac{1}{j\cdot\omega\cdot C}+\dfrac{1}{\left(\dfrac{1}{j\cdot\omega\cdot L}+\dfrac{1}{R_4}\right)}}$$

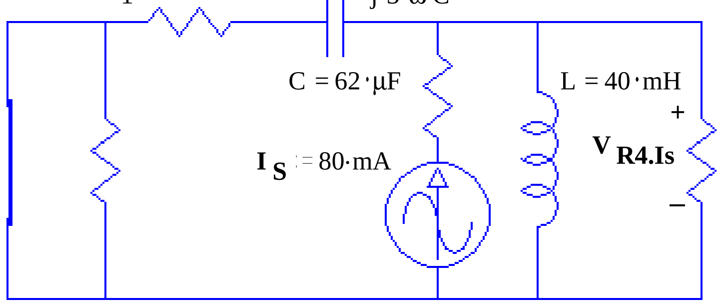
$\mathbf{V_{R4.Vs}} = -2.875 + 3.138j\cdot V$

$\left|\mathbf{V_{R4.Vs}}\right| = 4.256\cdot V \quad \arg\left(\mathbf{V_{R4.Vs}}\right) = 132.5\cdot deg$

$v_{R4.Vs}(t) := 4.256\cdot V\cdot\cos(\omega\cdot t + 132.5\cdot deg)$

Eliminate voltage source

$R_1 = 10\cdot\Omega$ $\quad \mathbf{Z_C} = \frac{1}{j\cdot 3\cdot\omega\cdot C} = -14.261j\cdot\Omega$




$\mathbf{Z_L} = j\cdot 3\cdot\omega\cdot L = 45.239j\cdot\Omega$

$$\mathbf{V_{R4.Is}} := \mathbf{I_S}\cdot\frac{1}{\left(\dfrac{1}{R_1+\dfrac{1}{j\cdot 3\cdot\omega\cdot C}}+\dfrac{1}{j\cdot 3\cdot\omega\cdot L}+\dfrac{1}{R_4}\right)}$$

$\mathbf{V_{R4.Is}} = 1.165 - 0.501j\cdot V$

$\left|\mathbf{V_{R4.Is}}\right| = 1.268\cdot V \quad \arg\left(\mathbf{V_{R4.Is}}\right) = -23.25\cdot deg$

$v_{R4.Is}(t) := 1.268\cdot V\cdot\cos(3\cdot\omega\cdot t - 23.25\cdot deg)$

**Add the results** $\quad v_{R4}(t) := 4.256\cdot V\cdot\cos(\omega\cdot t + 132.5\cdot deg) + 1.268\cdot V\cdot\cos(3\cdot\omega\cdot t - 23.25\cdot deg)$ $\quad t := 0, .2 .. 30$