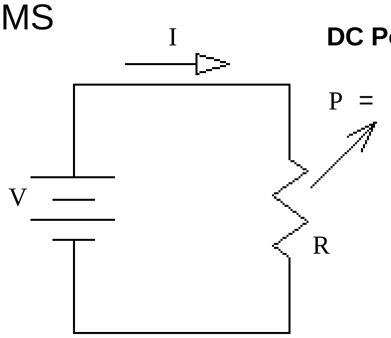
A. Stolp
3/31/09,
1/14/24

RMS

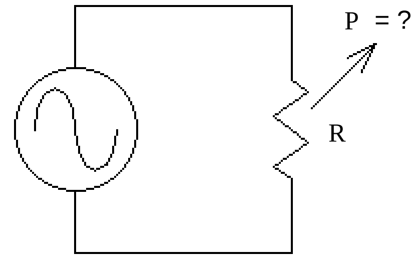
DC Power

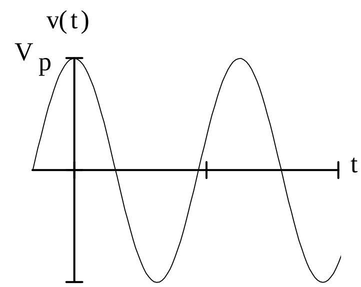


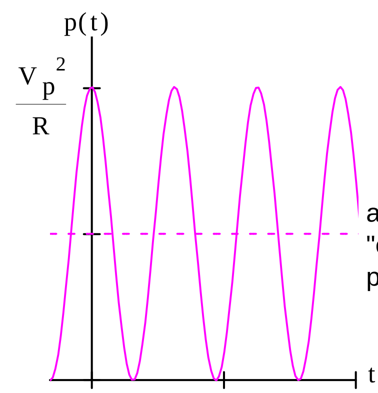
$$P = V \cdot I = \frac{V^2}{R} = I^2 \cdot R$$

AC Power

$$v(t) = V_p \cdot \cos(\omega \cdot t)$$







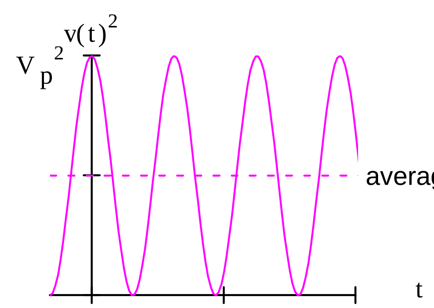
average or "effective" power = $\dfrac{\left(\dfrac{V_p^{\,2}}{R}\right)}{2}$

Couldn't we define an "effective" voltage that would allow us to use the same relationships for AC power as used for DC power?

$$P_{ave} = \frac{\left(\dfrac{V_p^{\,2}}{R}\right)}{2} = \frac{\left(\dfrac{V_p^{\,2}}{2}\right)}{R} = \frac{\left(\dfrac{V_p}{\sqrt{2}}\right)^2}{R}$$

$$V_{eff} = \sqrt{\left(\frac{V_p}{\sqrt{2}}\right)^2} = \frac{V_p}{\sqrt{2}} = V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2\, dt}$$

Root — Mean (average) — Square



average = $\dfrac{V_p^{\,2}}{2}$

RMS **R**oot of the **M**ean of the **S**quare

Use RMS in power calculations

Sinusoids

$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2\, dt} = \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t)\right)^2 dt} = \sqrt{\frac{1}{T} \cdot \int_0^T V_p^{\,2} \cdot \left(\frac{1}{2} + \frac{1}{2} \cdot \cos(2 \cdot \omega \cdot t)\right) dt}$$

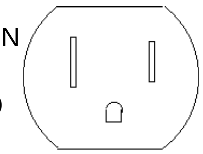
$$= \frac{V_p}{\sqrt{2}} \cdot \sqrt{\frac{1}{T} \cdot \int_0^T (1)\, dt + \frac{1}{T} \cdot \int_0^T \cos(2 \cdot \omega \cdot t)\, dt} = \frac{V_p}{\sqrt{2}} \cdot \sqrt{1 + 0}$$

Common household power

$f = 60 \cdot Hz$

$\omega = 377 \cdot \frac{rad}{sec}$

$T = 16.67 \cdot ms$

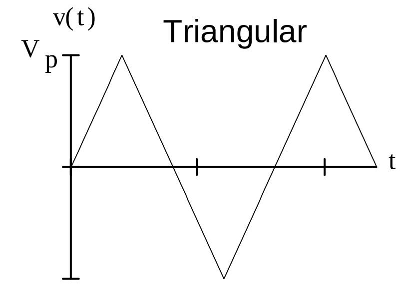


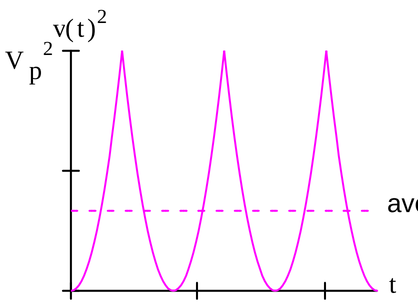
$V_{rms} := 120 \cdot V$

$V_p = V_{rms} \cdot \sqrt{2} = 170 \cdot V$

What about other wave shapes??

Triangular

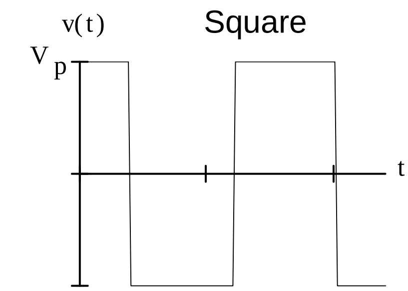


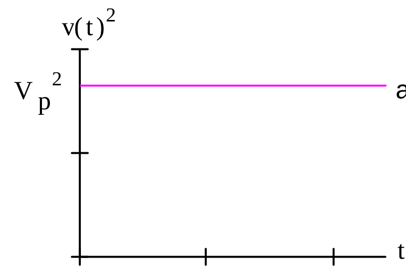


$\text{average} = \frac{V_p^2}{3}$

$V_{rms} = \frac{V_p}{\sqrt{3}}$

Square

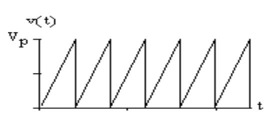


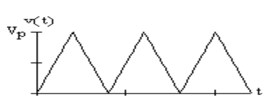


$\text{average} = V_p^2$

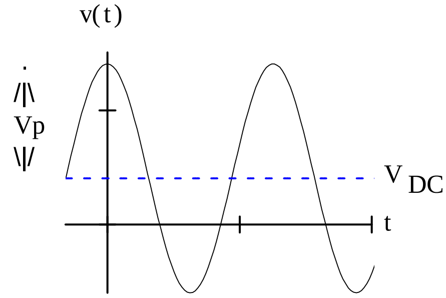
$V_{rms} = V_p$ Same for DC

Works for all types of triangular and sawtooth waveforms





How about AC + DC ?



$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2 \, dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t) + V_{DC}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left[\left(V_p \cdot \cos(\omega \cdot t)\right)^2 + 2 \cdot \left(V_p \cdot \cos(\omega \cdot t)\right) \cdot V_{DC} + V_{DC}^2\right] dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t)\right)^2 dt + \frac{1}{T} \cdot \int_0^T 2 \cdot \left(V_p \cdot \cos(\omega \cdot t)\right) \cdot V_{DC} \, dt + \frac{1}{T} \cdot \int_0^T V_{DC}^2 \, dt}$$

--- zero over one period ---

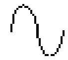
$$= \sqrt{V_{rmsAC}^2 + 0 + V_{DC}^2} \qquad = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$$

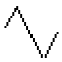
For any sum of waveforms or harmonics:

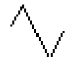
$$V_{rms} = \sqrt{V_{rms1}^2 + V_{rms2}^2 + V_{rms3}^2 + V_{rms3}^2} \quad . . . \text{etc}$$

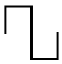
rectified average $V_{ra} = \frac{1}{T}\cdot\int_0^T |v(t)|\,dt$

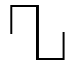
 sinusoid: $V_{rms} = \frac{V_p}{\sqrt{2}}$ $I_{rms} = \frac{I_p}{\sqrt{2}}$

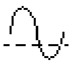
 $V_{ra} = \frac{2}{\pi}\cdot V_p$ $I_{ra} = \frac{2}{\pi}\cdot I_p$

 triangular: $V_{rms} = \frac{V_p}{\sqrt{3}}$ $I_{rms} = \frac{I_p}{\sqrt{3}}$

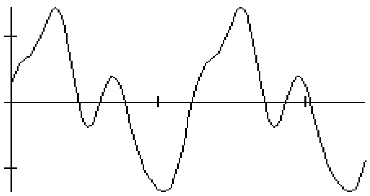
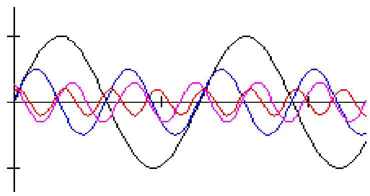
 $V_{ra} = \frac{1}{2}\cdot V_p$ $I_{ra} = \frac{1}{2}\cdot I_p$

 square: $V_{rms} = V_p$ $I_{rms} = I_p$

 $V_{ra} = V_{rms} = V_p$ $I_{ra} = I_{rms} = I_p$

 waveform + DC $V_{rms} = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$

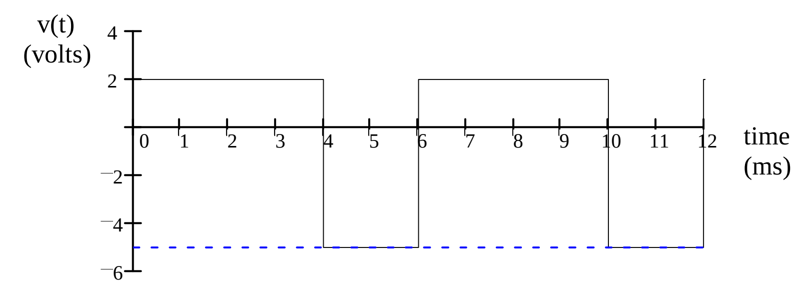
Most AC meters don't measure true RMS. Instead, they measure V_{ra}, display $1.11V_{ra}$, and call it RMS. That works for sine waves but not for any other waveform.

 = 

$V_{rms} = \sqrt{V_{rms1}^2 + V_{rms2}^2 + V_{rms3}^2 + V_{rms3}^2}$ etc...

Some waveforms don't fall into these forms, then you have to perform the math from scratch

For waveform shown

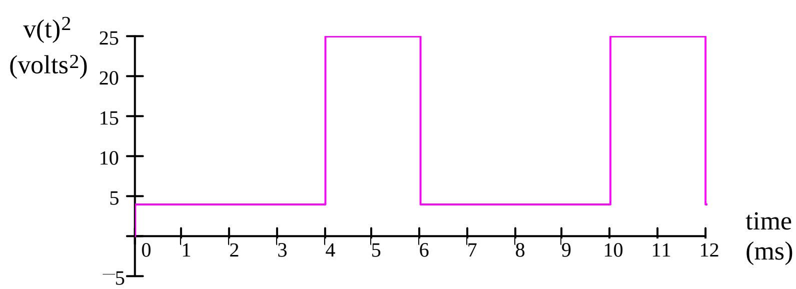


The average DC (V_{DC}) value

$$\frac{2\cdot V\cdot(4\cdot ms) + (-5\cdot V)\cdot(2\cdot ms)}{6\cdot ms} = -0.333\cdot V$$

The RMS (effective) value

Graphical way



$$\frac{4\cdot V^2\cdot(4\cdot ms) + 25\cdot V^2\cdot(2\cdot ms)}{6\cdot ms} = 11\cdot V^2$$

$V_{RMS} := \sqrt{11\cdot V^2}$ $V_{RMS} = 3.32\cdot V$

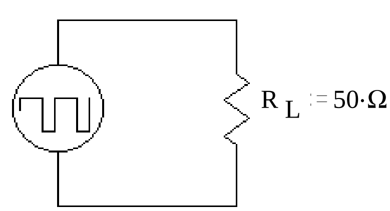
OR...

$$V_{RMS} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\,dt}$$

$$= \sqrt{\frac{1}{6\cdot ms}\cdot\left[\int_{0\cdot ms}^{4\cdot ms}(2\cdot V)^2\,dt + \int_{4\cdot ms}^{6\cdot ms}(-5\cdot V)^2\,dt\right]} = \sqrt{\frac{1}{6\cdot ms}\cdot\left[4\cdot ms\cdot(2\cdot V)^2 + 2\cdot ms\cdot(-5\cdot V)^2\right]} = 3.32\cdot V$$

The voltage is hooked to a resistor, as shown, for 6 seconds.

The energy is transferred to the resistor during that 6 seconds:



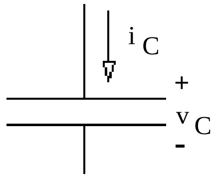
$P_L := \frac{V_{RMS}^2}{R_L}$ $P_L = 0.22\cdot W$

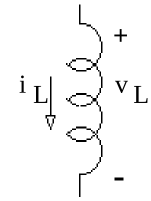
$W_L := P_L\cdot 6\cdot sec$ $W_L = 1.32\cdot joule$ All converted to heat

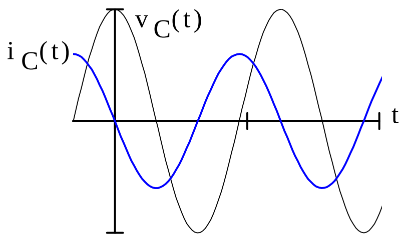
Use RMS in power calculations

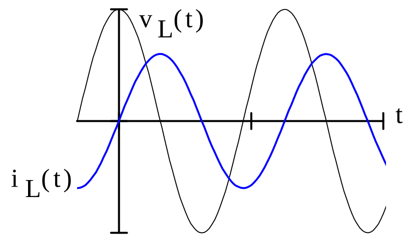
$$P = I_{Rrms}^{2} \cdot R = \frac{V_{Rrms}^{2}}{R} \quad \text{for Resistors ONLY ! !}$$

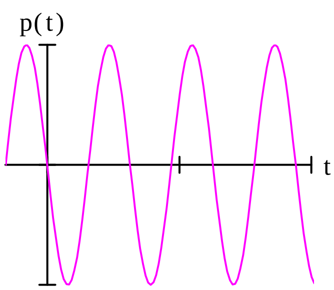
Capacitors and Inductors

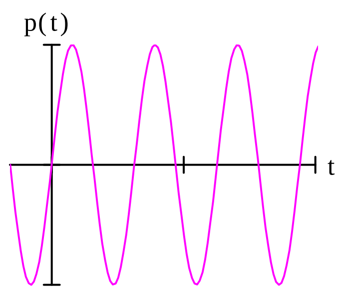












Average power is ZERO $P = 0$

Average power is ZERO $P = 0$

Capacitors and Inductors DO NOT dissipate (real) average power.

Reactive power is negative

$$Q_C = -I_{Crms} \cdot V_{Crms}$$

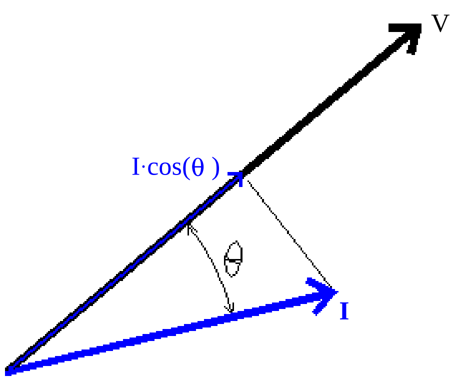
$$= -I_{Crms}^{2} \cdot \frac{1}{\omega \cdot C} = -V_{Crms}^{2} \cdot \omega \cdot C$$

Reactive power is positive

$$Q_L = I_{Lrms} \cdot V_{Lrms}$$

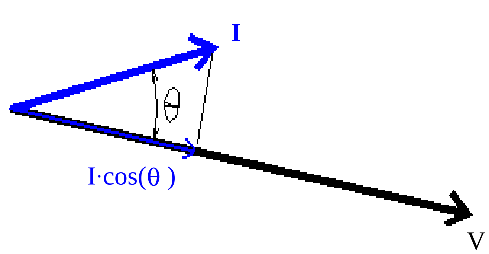
$$= I_{Lrms}^{2} \cdot \omega \cdot L = \frac{V_{Lrms}^{2}}{\omega \cdot L}$$

If current and voltage are not in phase, only the in-phase part of the current matters for the power-- DOT PRODUCT



"Lagging" power

Inductor dominates



"Leading" Power

Capacitor dominates

BOLD is a complex number

Real Power

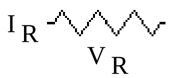
$$P = V \cdot I \cdot \cos(\theta) = I^2 \cdot |\mathbf{Z}| \cdot \cos(\theta) = \frac{V^2}{|\mathbf{Z}|} \cdot \cos(\theta)$$

$$P = \text{"Real" Power (average)} = V \cdot I \cdot pf = I^2 \cdot |\mathbf{Z}| \cdot pf = \frac{V^2}{|\mathbf{Z}|} \cdot pf$$

units: watts, kW, MW, etc.

$pf = \cos(\theta)$ = power factor

otherwise....



for resistors only part that uses real average power

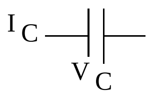
$$P = I_R^2 \cdot R = \frac{V_R^2}{R}$$

Reactive Power

$$Q = \text{Reactive "power"} = V \cdot I \cdot \sin(\theta)$$

units: VAR, kVAR, etc. "volt-amp-reactive"

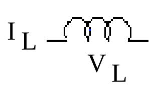
otherwise....



capacitors -> - Q

$$Q_C = I_C^2 \cdot X_C = \frac{V_C^2}{X_C}$$

$X_C = -\frac{1}{\omega \cdot C}$ and is a negative number



inductors -> + Q

$$Q_L = I_L^2 \cdot X_L = \frac{V_L^2}{X_L}$$

$X_L = \omega \cdot L$ and is a positive number

Complex and Apparent Power

$$\mathbf{S} = \text{Complex "power"} = P + jQ = VI \angle \theta = \mathbf{V} \cdot \overline{\mathbf{I}} = I^2 \cdot \mathbf{Z}$$

(complex conjugate)

units: VA, kVA, etc. "volt-amp"

NOT $\mathbf{V} \cdot \mathbf{I}$ **NOR** $\frac{\mathbf{V}^2}{\mathbf{Z}}$

$$S = \text{Apparent "power"} = |\mathbf{S}| = \sqrt{P^2 + Q^2} = V \cdot I$$

units: VA, kVA, etc. "volt-amp"

Power factor

$pf = \cos(\theta)$ = power factor (sometimes expressed in %) $0 \le pf \le 1$

θ is the **phase angle** between the voltage and the current or the phase angle of the impedance. $\theta = \theta_Z$

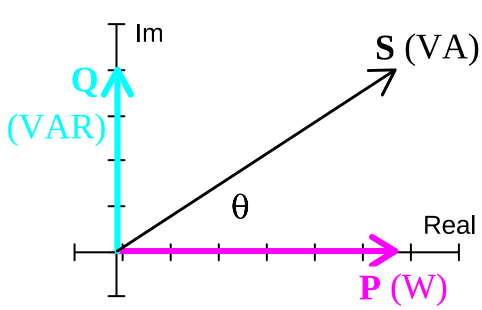
$\theta < 0$ Load is "Capacitive", power factor is "leading". This condition is very rare

$\theta > 0$ Load is "Inductive", power factor is "lagging". This condition is so common you can assume any power factor given is lagging unless specified otherwise. Transformers and motors make most loads inductive.

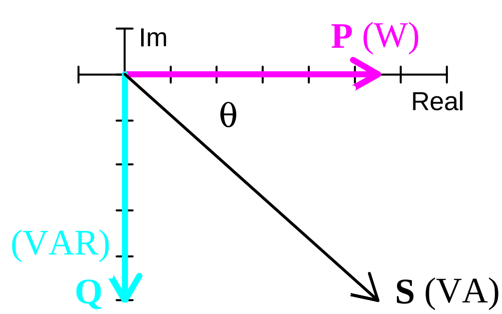
Industrial users are charged for the reactive power that they use, so for them, power factor < 1 is a bad thing.

Power factor < 1 is also bad for the power company. To deliver the same power to the load, they have more line current (and thus more line losses).

Power factors are "corrected" by adding capacitors (or capacitve loads) in parallel with the inductive loads which cause the problems. (In the rare case that the load is capacitive, the pf would be corrected by an inductor.)



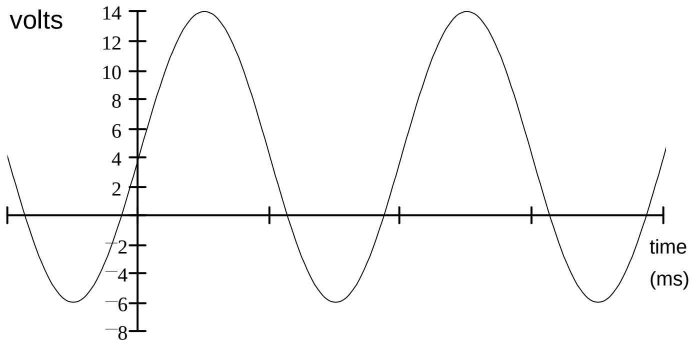
"Lagging" power



"Leading" Power

A. Stolp
1/14/24

Ex. 1 Find the DC and RMS of the following waveform



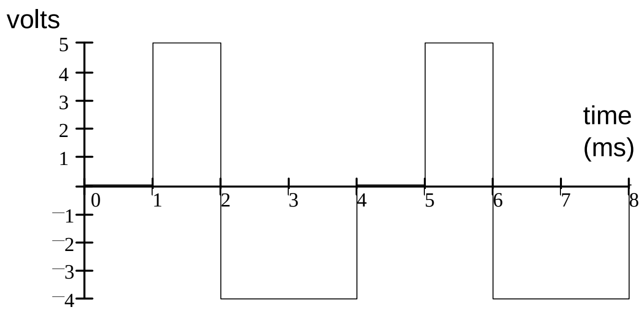
$$V_{DC} := \frac{14\cdot V + -6\cdot V}{2} \qquad V_{DC} = 4\cdot V$$

$$V_{pp} := 14\cdot V - -6\cdot V \qquad V_{pp} = 20\cdot V$$

$$V_{RMS} := \sqrt{\left(\frac{V_{pp}}{2\cdot\sqrt{2}}\right)^2 + V_{DC}{}^2}$$

$$V_{RMS} = 8.12\cdot V$$

Ex. 2 Find the DC, rectified average and RMS of the following waveform



$$V_{DC} = \frac{0\cdot V\cdot(1\cdot ms) + 5\cdot V\cdot(1\cdot ms) + (-4\cdot V)\cdot(2\cdot ms)}{4\cdot ms} = -0.75\cdot V$$

$$V_{RA} = \frac{0\cdot V\cdot(1\cdot ms) + 5\cdot V\cdot(1\cdot ms) + \left|-4\cdot V\right|\cdot(2\cdot ms)}{4\cdot ms} = 3.25\cdot V$$

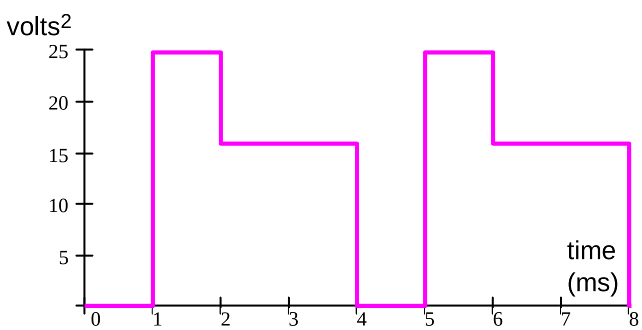
Rectified Average

$$V_{RMS} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\, dt}$$

Square

Root

Mean (average)



RMS (effective) value

Graphical way

$$\frac{(0\cdot V)^2\cdot(1\cdot ms) + (5\cdot V)^2\cdot(1\cdot ms) + (-4\cdot V)^2\cdot(2\cdot ms)}{4\cdot ms} = 14.25\cdot V^2$$

$$V_{RMS} = \sqrt{14.25\cdot V^2} = 3.77\cdot V$$

AC Power Examples

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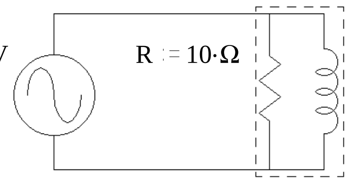
11/06/02
8/28/20
rev 1/14/24

Ex. 1 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

$\mathbf{V}_{in} := 110 \cdot V$ $\quad \omega := 377 \cdot \frac{rad}{sec}$ $\quad R := 10 \cdot \Omega$ $\quad L := 25 \cdot mH$



$$\mathbf{Z} := \frac{1}{\left(\frac{1}{R} + \frac{1}{j \cdot \omega \cdot L}\right)} = \frac{1}{0.1458 \cdot \frac{1}{\Omega} \cdot e^{-j \cdot 46.7 \cdot deg}}$$

$\mathbf{Z} = 4.704 + 4.991j \;\cdot \Omega$ $\quad |\mathbf{Z}| = 6.859 \cdot \Omega$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 46.7 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.686$

$\mathbf{I} := \frac{\mathbf{V}_{in}}{\mathbf{Z}}$ $\quad \mathbf{I} = 11 - 11.671j \;\cdot A$ $\quad |\mathbf{I}| = 16.038 \cdot A$ $\quad \arg(\mathbf{I}) = -46.7 \cdot deg$

$P := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 1.21 \cdot kW$

$Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 1.284 \cdot kVAR$ $\quad$ OR... $\quad Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sqrt{1^2 - pf^2}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := \mathbf{V}_{in} \cdot \overline{\mathbf{I}}$ $\quad$ OR.. $\quad \mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 1.21 + 1.284j \;\cdot kVA$ $\quad S := \sqrt{\mathrm{Re}(\mathbf{S})^2 + \mathrm{Im}(\mathbf{S})^2} = |\mathbf{S}| = 1.764 \cdot kVA$

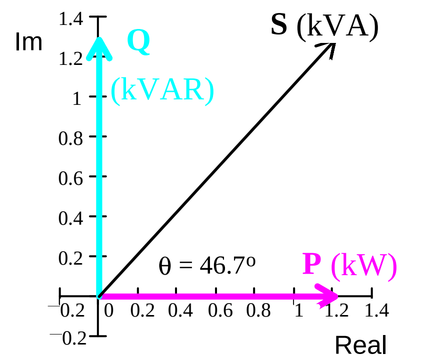
$\mathrm{atan}\left(\frac{\mathrm{Im}(\mathbf{S})}{\mathrm{Re}(\mathbf{S})}\right) = 46.696 \cdot deg$ same as θ $\quad \mathbf{S} = 1.764kVA \;\underline{/46.7^o}$

OR, since we know that the voltage across each element of the load is V_{in} ...
Real power is dissipated only by resistors

$P := \frac{(|\mathbf{V}_{in}|)^2}{R}$ $\quad P = 1.21 \cdot kW$ $\quad Q := \frac{(|\mathbf{V}_{in}|)^2}{\omega \cdot L}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$

$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = 1.764 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.686$



What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

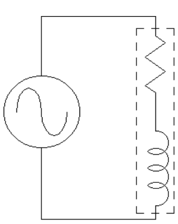
$\mathrm{Im}(\mathbf{I}) = -11.671 \cdot A$ $\quad X_C := \frac{\mathbf{V}_{in}}{\mathrm{Im}(\mathbf{I})}$ $\quad X_C = -9.425 \cdot \Omega = \frac{-1}{\omega \cdot C}$

$\frac{1}{|X_C| \cdot \omega} = 281 \cdot \mu F$ $\quad$ OR.. $\quad \omega = \frac{1}{\sqrt{L \cdot C}}$ $\quad C := \frac{1}{L \cdot \omega^2}$ $\quad C = 281 \cdot \mu F$

Ex. 2 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

Series R & L

$\mathbf{V}_{in} := 110 \cdot V$ $\quad \omega := 377 \cdot \frac{rad}{sec}$



$R := 10 \cdot \Omega$ $\quad$ load $\quad L := 25 \cdot mH$

$\mathbf{Z} := R + j \cdot \omega \cdot L$

$\mathbf{Z} = 10 + 9.425j \;\cdot \Omega$ $\quad |\mathbf{Z}| = 13.742 \cdot \Omega$

$\theta := \arg(\mathbf{Z})$ $\quad \theta = 43.304 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.728$

$\mathbf{I} := \frac{\mathbf{V}_{in}}{\mathbf{Z}}$ $\quad \mathbf{I} = 5.825 - 5.49j \;\cdot A$

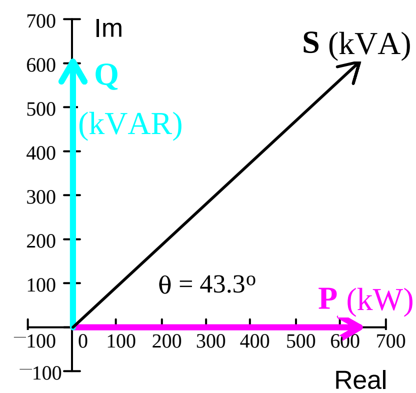
$|\mathbf{I}| = 8.005 \cdot A$ $\quad \arg(\mathbf{I}) = -43.304 \cdot deg$

$P := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 0.641 \cdot kW$

$Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 0.604 \cdot kVAR$

$\mathbf{S} := \mathbf{V}_{in} \cdot \overline{\mathbf{I}}$ $\quad \mathbf{S} = 0.641 + 0.604j \;\cdot kVA$

$|\mathbf{S}| = 0.881 \cdot kVA$ $\quad \arg(\mathbf{S}) = 43.304 \cdot deg$ $\quad \mathbf{S} = 881VA\underline{/43.3^o}$



OR, if we first find the magnitude of the current which flows through each element of the load...

$$|\mathbf{I}| = \frac{\mathbf{V_{in}}}{\sqrt{R^2 + (\omega \cdot L)^2}} = 8.005 \cdot A$$

$P := (|\mathbf{I}|)^2 \cdot R$ $\quad P = 0.641 \cdot kW$ $\quad Q := (|\mathbf{I}|)^2 \cdot (\omega \cdot L)$ $\quad Q = 0.604 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$ $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 0.881 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.728$

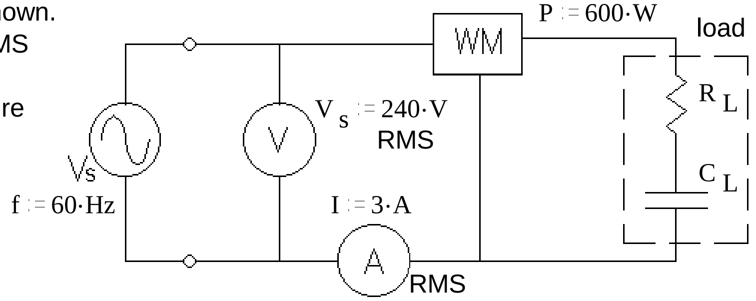
What value of C in parallel with R & L would make $pf = 1$ $(Q = 0)$?

$Q = 603.9 \cdot VAR$ $\quad$ so we need: $\quad Q_C := -Q$ $\quad Q_C = -603.9 \cdot VAR = \frac{V_{in}^2}{X_C}$

$X_C := \frac{\mathbf{V_{in}}^2}{Q_C}$ $\quad X_C = -20.035 \cdot \Omega = \frac{-1}{\omega \cdot C}$ $\quad C := \frac{1}{|X_C| \cdot \omega}$ $\quad C = 132 \cdot \mu F$

Check: $\frac{1}{\frac{1}{R + j \cdot \omega \cdot L} + j \cdot \omega \cdot C} = 18.883 \cdot \Omega$ $\quad$ No j term, so $\theta = 0^o$

Ex. 3 R, & C together are the load in the circuit shown. The RMS voltmeter measures 240 V, the RMS ammeter measures 3 A, and the wattmeter measures 600 W. Find the following: Be sure to show the correct units for each value.



a) The value of the load resistor. R_L = ?

$P = I^2 \cdot R_L$

$R_L := \frac{P}{I^2}$ $\quad R_L = 66.7 \cdot \Omega$

b) The apparent power. $|\mathbf{S}|$ = ? $\quad S := V_s \cdot I$ $\quad S = 720 \cdot VA$

c) The reactive power. Q = ? $\quad Q := -\sqrt{S^2 - P^2}$ $\quad Q = -398 \cdot VAR$

d) The complex power. $\mathbf{S}$ = ? $\quad \mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 600 - 398i \cdot VA$

e) The power factor. pf = ? $\quad pf := \frac{P}{V_s \cdot I}$ $\quad pf = 0.833$

f) The power factor is leading or lagging? $\quad$ leading (load is capacitive, Q is negative)

g) The two components of the load are in a box which cannot be opened. Add (draw it) another component to the circuit above which can correct the power factor (make $pf = 1$). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

$f = 60 \cdot Hz$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad \omega = 376.991 \cdot \frac{rad}{sec}$

Add an inductor in parallel with load

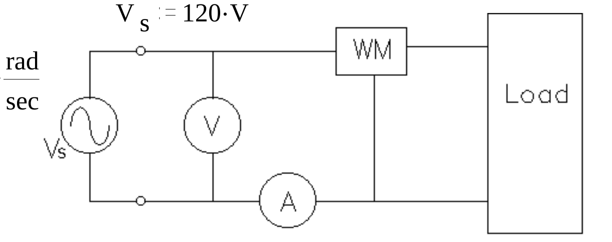
$Q = -398 \cdot VAR$ $\quad$ so we need: $\quad Q_L := -Q$ $\quad Q_L = 398 \cdot VAR = \frac{V_s^2}{X_L}$

$X_L := \frac{V_s^2}{Q_L}$ $\quad X_L = 144.725 \cdot \Omega = \omega \cdot L$ $\quad L := \frac{|X_L|}{\omega}$ $\quad L = 384 \cdot mH$

Ex. 4 For the 60 Hz load shown in the figure, the RMS voltmeter measures 120 V. The phasor diagram for the power is also shown. Find the following:

$V_s := 120 \cdot V$

$\omega := 377 \cdot \frac{rad}{sec}$



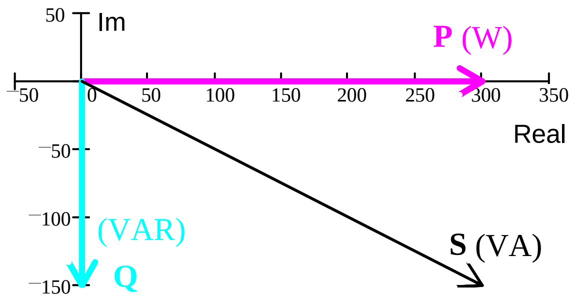
a) The complex power. **S** = ?

$P := 300 \cdot W$ $\quad Q := -150 \cdot VA$

$\mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = (300 - 150j) \cdot VA$

b) The apparent power. |**S**| = ? $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 335.4 \cdot VA$

c) The power factor. pf = ? $\quad pf := \frac{P}{|\mathbf{S}|}$ $\quad pf = 0.894$



d) The item marked "WM" in the figure is a wattmeter, what does it read? (give a number) $\quad P = 300 \cdot W$

e) The item marked "A" in the figure is an RMS ammeter, what does it read? (give a number)

$I := \frac{|\mathbf{S}|}{V_s}$ $\quad I = 2.795 \cdot A$ $\quad I = 2.8 \cdot A$

f) The power factor is leading or lagging? leading (Q is negative)

g) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

Add an inductor in parallel with load

$Q = -150 \cdot VAR$ need: $Q_L := -Q$ $\quad Q_L = 150 \cdot VAR \quad = \frac{{V_s}^2}{\omega \cdot L}$ $\quad L := \frac{{V_s}^2}{\omega \cdot Q_L}$ $\quad L = 255 \cdot mH$

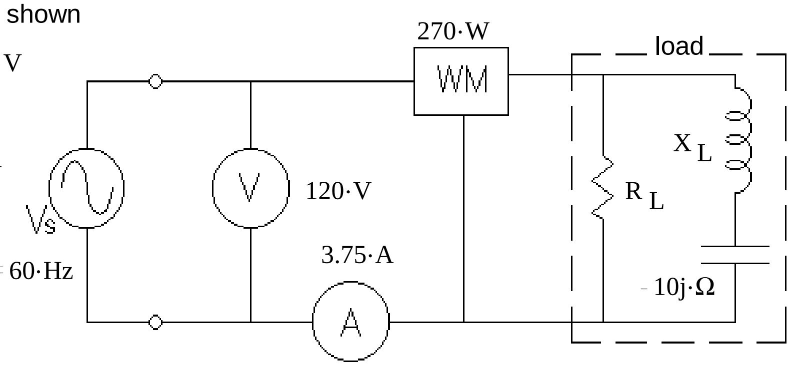
Ex. 5 R, L, & C together are the load in the circuit shown

The RMS voltmeter measures 120 V. $\quad V_s := 120 \cdot V$

The wattmeter measures 270 W. $\quad P := 270 \cdot W$

The RMS ammeter measures 3.75 A. $\quad I := 3.75 \cdot A$

$f := 60 \cdot Hz$



Find the following: Be sure to show the correct units for each value.

a) The value of the load resistor. R_L = ?

$P = \frac{{V_s}^2}{R_L}$ $\quad R_L := \frac{{V_s}^2}{P}$ $\quad R_L = 53.3 \cdot \Omega$

b) The magnitude of the impedance of the load inductor (reactance) . $|\mathbf{Z_L}| = X_L$ = ?

$I_R := \frac{V_s}{R_L}$ $\quad I_R = 2.25 \cdot A$ $\quad I_L := \sqrt{I^2 - {I_R}^2}$ $\quad I_L = 3 \cdot A$ $\quad X := \frac{V_s}{I_L}$ $\quad X = 40 \cdot \Omega$

$X_C := -10 \cdot \Omega$ $\quad X_L := X - X_C$ $\quad X_L = 50 \cdot \Omega$

c) The reactive power. Q = ? $\quad Q := \sqrt{(V_s \cdot I)^2 - P^2}$ $\quad Q = 360 \cdot VAR$ $\quad$ positive, because the load is primarily inductive

d) The power factor is leading or lagging? lagging (load is inductive, Q is positive)

e) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

Add a capacitor in parallel with load

$f = 60 \cdot Hz \qquad \omega := 2 \cdot \pi \cdot f \qquad \omega = 376.991 \cdot \frac{rad}{sec}$

$Q = 360 \cdot VAR$ so we need: $Q_C := -Q$

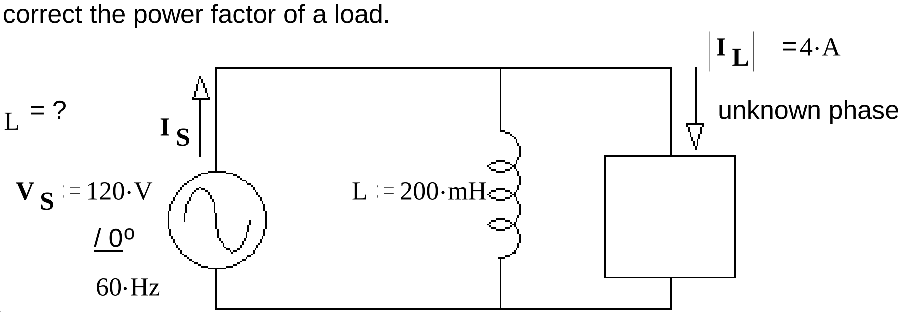
$$Q_C = -360 \cdot VAR = -\frac{V_s^2}{\frac{1}{\omega \cdot C}} = -\omega \cdot C \cdot V_s^2$$

$$C := \frac{Q_C}{-\omega \cdot V_s^2} \qquad C = 66.3 \cdot \mu F$$

Ex. 6 An inductor is used to completely correct the power factor of a load.

Find the following:

a) The power consumed by the load. $P_L = ?$



$I_L := 4 \cdot A \qquad \omega = 376.991 \cdot \frac{rad}{sec}$

$Q_L := \frac{\left(\left|\mathbf{V_S}\right|\right)^2}{\omega \cdot L} \qquad Q_L = 190.986 \cdot VAR \qquad Q_{load} := -Q_L$

$S_L := \left|\mathbf{V_S}\right| \cdot I_L \qquad S_L = 480 \cdot VA \qquad P_L := \sqrt{S_L^2 - Q_{load}^2} \qquad P_L = 440.4 \cdot W$

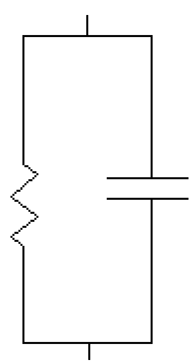
b) The power supplied by the source. $P_S = P_L = 440 \cdot W$

c) The source current (magnitude and phase). $\mathbf{I_S} := \frac{P_L}{\mathbf{V_S}} \qquad \mathbf{I_S} = 3.67 \cdot A \ \angle 0^{\circ}$

because the source sees a pf = 1

d) The load can be modeled as 2 parts in parallel. Draw the model and find the values of the parts.

$P = \frac{V^2}{R}$



$Q_C = V^2 \cdot (\omega \cdot C)$

$R_L := \frac{\left(\left|\mathbf{V_S}\right|\right)^2}{P_L} \qquad R_L = 32.7 \cdot \Omega$

$C_L := \frac{-Q_{load}}{\omega \left(\left|\mathbf{V_S}\right|\right)^2}$

$C_L = 35.181 \cdot \mu F$

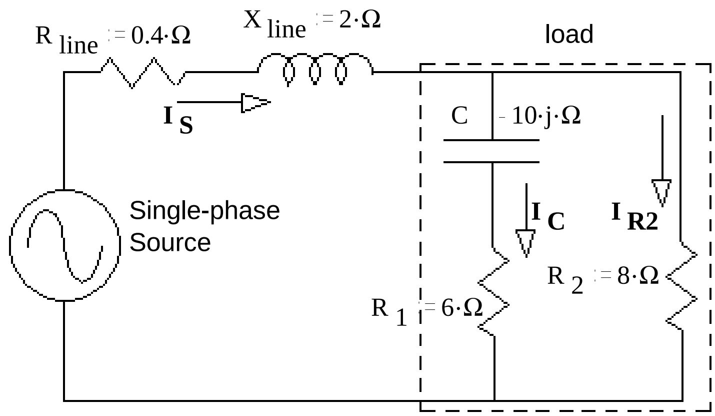
e) The inductor, L, is replaced with a 50 mH inductor.

circle one

i) The **new** source current $|\mathbf{I_S}|$ is **greater** than that calculated in part c). <-- Answer

ii) The **new** source current $|\mathbf{I_S}|$ is **the same** as that calculated in part c).

iii) The **new** source current $|\mathbf{I_S}|$ is **less** than that calculated in part c).



Ex. 7 C, R_1, & R_2 together are the load (in dotted box). The reactive power used by the load is

$Q_{load} := -600\cdot VAR$ Find:

a) The real power used by the load. $P_{load} = ?$

$X_C := -10\cdot\Omega$

$|\mathbf{I_C}| = I_C := \sqrt{\dfrac{Q_{load}}{X_C}}$ $\qquad I_C = 7.746\cdot A$

$V_{load} := I_C\cdot\sqrt{R_1^{\,2} + X_C^{\,2}}$ $\qquad V_{load} = 90.333\cdot V$

$P_{load} := I_C^{\,2}\cdot R_1 + \dfrac{V_{load}^{\,2}}{R_2}$ $\qquad P_{load} = 1.38\cdot kW$

b) The apparent power of the load. $|\mathbf{S}| = S := \sqrt{P_{load}^{\,2} + Q_{load}^{\,2}}$ $\qquad S = 1.505\cdot kVA$

c) The power factor of the load. $pf := \dfrac{P_{load}}{S}$ $\qquad pf = 0.917$

d) This power factor is: i) leading ii) lagging Leading, capacitor

e) The voltage at the load (magnitude). $V_{load} = 90.333\cdot V$ found above

f) The magnitudes of the three currents. $|\mathbf{I_C}| = ?$ $\qquad |\mathbf{I_{R2}}| = ?$ $\qquad |\mathbf{I_S}| = ?$

$|\mathbf{I_C}| = I_C = 7.746\cdot A$ found above

$|\mathbf{I_{R2}}| = I_{R2} = \dfrac{V_{load}}{R_2} = 11.292\cdot A$

$|\mathbf{I_S}| = I_S := \dfrac{S}{V_{load}}$ $\qquad I_S = 16.658\cdot A$

g) The source voltage (magnitude). $V_S = ?$

$P_{Line} := I_S^{\,2}\cdot R_{line}$ $\qquad P_{Line} = 111\cdot W$

$Q_{Line} := I_S^{\,2}\cdot X_{line}$ $\qquad Q_{Line} = 555\cdot VAR$

$|\mathbf{S_S}| = S_S := \sqrt{\left(P_{load} + P_{Line}\right)^2 + \left(Q_{load} + Q_{Line}\right)^2}$ $\qquad S_S = 1.492\cdot kVA$

$V_S := \dfrac{S_S}{I_S}$ $\qquad V_S = 89.546\cdot V$

h) Is there something weird about this voltage? If so, what? V_S is less than V_{Load}

Why? Because the Q of the line partially cancels the Q of the load

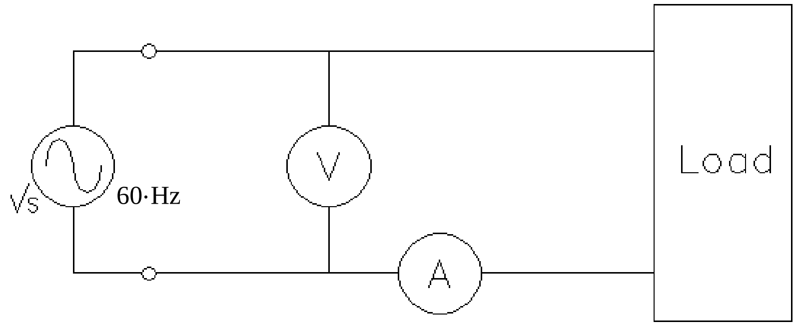
OR Partial resonance between the inductance in the line and the capacitance of the load.

i) The efficiency. $\eta = ?$

When asked for efficiency, assume the power used by R_{line} is a loss and P_{load} is the output power.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{load}}{P_{load} + P_{Line}} = 92.56\cdot\%$$

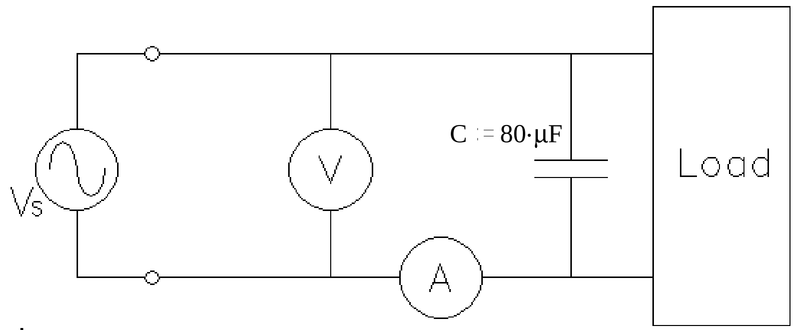
Ex. 8 In the circuit shown, the ideal voltmeter, V, reads 120V and ideal ammeter, A, reads 5A.



$S_{load} := 120\cdot V\cdot 5\cdot A \qquad S_{load} = 600\cdot VA$

a) You add a capacitor, C, and the ammeter reading changes to 5.3A. Find the following:

$P_{load} = ? \qquad Q_{load} = ?$



I_C is **NOT** 0.3A, That's <u>subtracting magnitudes</u>

$S_{load} := 120\cdot V\cdot 5\cdot A \qquad S_{load} = 600\cdot VA = \sqrt{P_{load}^2 + Q_{load}^2}$

OR $(600\cdot VA)^2 = P_{load}^2 + Q_{load}^2$

$P_{load}^2 = (600\cdot VA)^2 - Q_{load}^2$

$$Q_C := \frac{(120\cdot V)^2}{\left(-\frac{1}{\omega\cdot C}\right)} = -(120\cdot V)^2\cdot\omega\cdot C \qquad Q_C = -434.294\cdot VAR$$

With Capacitor:

$S_S := 120\cdot V\cdot 5.3\cdot A \qquad S_S = 636\cdot VA = \sqrt{P_{load}^2 + (Q_{load} + Q_C)^2}$

OR $(636\cdot VA)^2 = P_{load}^2 + (Q_{load} + Q_C)^2$

Substitute in $(636\cdot VA)^2 = \left[(600\cdot VA)^2 - Q_{load}^2\right] + (Q_{load} + Q_C)^2$

$= \left[(600\cdot VA)^2 - Q_{load}^2\right] + \left(Q_{load}^2 + 2\cdot Q_C\cdot Q_{load} + Q_C^2\right)$

$= (600\cdot VA)^2 + 2\cdot Q_C\cdot Q_{load} + Q_C^2$

$$Q_{load} := \frac{(636\cdot VA)^2 - (600\cdot VA)^2 - Q_C^2}{2\cdot Q_C} \qquad Q_{load} = 165.919\cdot VAR$$

$$P_{load} := \sqrt{S_{load}^2 - Q_{load}^2} \qquad P_{load} = 576.603\cdot W$$

Double Check: $S_S = \sqrt{P_{load}^2 + (Q_{load} + Q_C)^2} = 636\cdot VA$

The power factor was way over corrected by $C = 80\cdot\mu F$