Single phase power pulses at 120 Hz . This is not suitable for motors or generators over about 5 hp .

## 2-Phase Power

Two-phase power is constant as long as the two loads are balanced. But, the return current is larger than either load current.


## 3-Phase Power



## Voltages



## Powers



Three phase power is constant as long as the three loads are balanced.


If loads are balanced, ground return current will be zero. If the loads are close to balanced the relatively small return current can be carried by the earth ground.


## Basics

Single phase power pulses at 120 Hz . This is not good for motors or generators over about 5 hp .

Three phase power is constant as long as the three loads are balanced.

Three lines are needed to transmit 3-phase power. If loads are balanced, ground return current will be zero.

The 3 lines coming into your house are NOT 3-phase. They are +120 V , Gnd, -120 V . (The two 120 s are 1800 out-of-phase, allowing for 240 V connections)
3 -phase outlets have 4 connections


## Wye connection:

Connect each load or generator phase between a line and ground.
A $\quad \mathbf{I}_{\mathrm{A}} \longrightarrow$


$$
\begin{gathered}
\left|\mathbf{v}_{\mathbf{A N}}\right|=\left|\mathbf{v}_{\mathbf{B N}}\right|=\left|\mathbf{v}_{\mathbf{C N}}\right|=\mathrm{V}_{\mathrm{LN}}=\frac{\mathrm{v}_{\mathrm{LL}}}{\sqrt{3}}=\frac{\mathrm{v}_{\mathrm{L}}}{\sqrt{3}} \\
\left|\mathbf{I}_{\mathbf{A}}\right|=\left|\mathbf{I}_{\mathbf{B}}\right|=\left|\mathbf{I}_{\mathbf{C}}\right|=\mathrm{I}_{\mathrm{L}}=\sqrt{3} \cdot \mathrm{I}_{\mathrm{LL}}
\end{gathered}
$$

Delta connection:
Connect each load or generator phase between two lines.

To get equivalent line currents with equivalent voltages:

$$
\mathbf{Z}_{\mathbf{Y}}=\frac{\mathbf{Z}_{\Delta}}{3}
$$

$$
\mathbf{z}_{\Delta}=3 \cdot \mathbf{Z}_{\mathbf{y}}
$$

Wye, Y, connection:
Connect each load or generator phase between a line and ground.


Delta, $\Delta$, connection:
Connect each load or generator phase between two lines.

$\mathrm{V}_{\mathrm{LL}}=\sqrt{3} \cdot \mathrm{~V}_{\mathrm{LN}} \quad \mathrm{I}_{\mathrm{LL}}=\frac{\mathrm{I}_{\mathrm{L}}}{\sqrt{3}}$

$$
=3 \cdot \mathrm{~V}_{\mathrm{LL}} \cdot \mathrm{I}_{\mathrm{LL}} \quad=\sqrt{3} \cdot \mathrm{~V}_{\mathrm{LL}} \cdot \mathrm{I}_{\mathrm{L}}
$$

$$
\mathrm{P}_{3 \phi}=3 \cdot \mathrm{P}_{1 \phi}=3 \cdot \mathrm{~V}_{\mathrm{LN}} \cdot \mathrm{I} \mathrm{~L} \cdot \mathrm{pf}=3 \cdot \mathrm{~V}_{\mathrm{LL}} \cdot \mathrm{I}_{\mathrm{LL}} \cdot \mathrm{pf}=\sqrt{3} \cdot \mathrm{~V}_{\mathrm{LL}} \cdot \mathrm{I}_{\mathrm{L}} \cdot \mathrm{pf}=\mathrm{S}_{3 \phi} \cdot \mathrm{pf} \underset{\mathrm{pf}=\cos (\theta)}{ }
$$

Reactive power:

$$
\mathrm{Q}_{3 \phi}=3 \cdot \mathrm{Q}_{1 \phi}=3 \cdot \mathrm{~V}_{\mathrm{LN}^{\mathrm{I}}} \cdot \sin (\theta) \text { etc } \ldots=\sqrt{\left(\left|\mathbf{S}_{3 \phi}\right|\right)^{2}-\mathrm{P}_{3 \phi}{ }^{2}}
$$

Cautions about "L" subscripts:
$\mathrm{I}_{\mathrm{L}}$ is always the line current, same as would flow in a Y-connected device.
$\mathrm{V}_{\mathrm{L}}$ is always the line-to-line voltage, same as across a $\Delta$-connected device.
When a single phase is taken from a 3-phase panel, then the line voltage $\left(\mathrm{V}_{\mathrm{L}}\right)$ of that single phase is the line-to-neutral voltage of the 3-phase input to that panel, so the value of $\mathrm{V}_{\mathrm{L}}$ changes in the panel (isn't that nice?).
$Z_{L}$ could be the load impedance, either $Y$-connected or $\Delta$-connected, or it could be the line impedance-the impedance in the line itself, between the source and the load.

Cautions about " $\phi$ " or "ph" subscripts:
In our book: $\quad \mathrm{V}_{\phi}=$ the voltage across a single phase of a source or load and depends on the connection of that load, $\mathrm{V}_{\mathrm{LN}}$ for Y -connected devices and $\mathrm{V}_{\mathrm{LL}}$ for $\Delta$-connected devices.
$I_{\phi} \quad$ Also depends on connection.
In some books: $\quad \mathrm{V}_{\phi}=\mathrm{V}_{\mathrm{ph}}=\mathrm{V}_{\mathrm{LN}} \quad \mathrm{I}_{\phi}=\mathrm{I}_{\mathrm{ph}}=$ current in a Y-connection <= DON'T USE in this class
Phase sequences:


Common usage: $\quad V_{L}=V_{L L}=$ "line voltage" = line-to-line voltage An unspecified voltage or a "line" voltage must always be assumed to be line-to-line,


Our Approach Only works if system is Balanced (Always so in our class, until we see faults)

1) Change all $\Delta$-connected loads to equivalent $Y$-connected loads $Z_{Y}=\frac{Z_{\Delta}}{3}$

Delta, $\Delta$

$\qquad$

Wye, Y

2) Find all voltages as $v_{L N}$, especially $V_{L N}=\frac{V_{L}}{\sqrt{3}}$
3) Change all power numbers to $1 \phi$.


$$
\begin{aligned}
& \mathrm{I}_{\mathrm{L}} \longrightarrow \\
& =3 \mathrm{x} \\
& \mathrm{P}_{1 \phi}=\frac{\mathrm{P}_{3 \phi}}{3} \quad \mathrm{Q}_{1 \phi}=\frac{\mathrm{Q}_{3 \phi}}{3} \\
& \mathrm{~S}_{\mathbf{1 \phi}}=\frac{\mathrm{S}_{\mathbf{3} \boldsymbol{\phi}}}{3} \\
& S_{1 \phi}=\left|\mathbf{s}_{1 \phi}\right|=\frac{S_{3 \phi}}{3}
\end{aligned}
$$

4) Solve the remaining single-phase problem.
5) Return to "line" voltages and $3 \phi$ powers, as necessary.

$$
\mathrm{v}_{\mathrm{L}}=\sqrt{3} \cdot \mathrm{v}_{\mathrm{LN}} \quad \mathrm{P}_{3 \phi}=3 \cdot \mathrm{P}_{1 \phi}, \quad \begin{aligned}
\mathrm{Q}_{3 \phi} & =3 \cdot \mathrm{Q}_{1 \phi} \\
\left|\mathbf{s}_{3 \phi}\right| & =3 \cdot\left|\mathbf{s}_{\mathbf{1 \phi}}\right| \\
\mathbf{s}_{\mathbf{3 \phi}} & =3 \cdot \mathbf{s}_{\mathbf{1 \phi}}
\end{aligned}
$$

In rare cases, you may also need:

$$
\mathrm{I}_{\Delta}=\mathrm{I}_{\mathrm{LL}}=\frac{\mathrm{I}_{\mathrm{L}}}{\sqrt{3}}
$$

and: $\quad \mathbf{Z}_{\Delta}=3 \cdot \mathbf{Z}_{\mathbf{Y}}$

## ECE 3600 3-Phase Examples

Ex. 1 A Y-connected load is connected to 208-V, 3-phase. It draws 1.2 kW of power at a power factor of $75 \%$, leading.

$$
\mathrm{P}_{3 \phi}:=1.2 \cdot \mathrm{~kW} \quad \mathrm{pf}:=0.75
$$

a) Find the apparent power and the reactive power.
$S_{3 \phi}:=\frac{\mathrm{P}_{3 \phi}}{\mathrm{pf}}$
$S_{3 \phi}=1.6 \cdot \mathrm{kVA}$
$\mathrm{Q}_{3 \phi}:=-\sqrt{\mathrm{S}_{3 \phi}{ }^{2}-\mathrm{P}_{3 \phi}{ }^{2}}$
$\mathrm{Q}_{3 \phi}=-1.058 \cdot \mathrm{kVAR}$
Negative because the power factor is leading.
b) Find the line current

Our Approach 1) Change all $\Delta$-connected loads to equivalent Y -connected loads $\mathbf{Z}_{\mathbf{Y}}=\frac{\mathbf{Z}_{\Delta}}{3}$ NOT NEEDED
2) Find all voltages as $\mathrm{V}_{\mathrm{LN}}:=\frac{208 \cdot \mathrm{~V}}{\sqrt{3}} \quad \mathrm{~V}_{\mathrm{LN}}=120.089 \cdot \mathrm{~V}$
3) Change all power numbers to $1 \phi . \quad P_{1 \phi}:=\frac{P_{3 \phi}}{3} \quad P_{1 \phi}=400 \cdot W$
$S_{1 \phi}:=\frac{S_{3 \phi}}{3} \quad S_{1 \phi}=533.333 \cdot \mathrm{VA}$
$\mathrm{Q}_{1 \phi}:=\frac{\mathrm{Q}_{3 \phi}}{3} \quad \mathrm{Q}_{1 \phi}=-352.767 \cdot \mathrm{VAR}$

c) Find the values of the load components, assuming they are connected in series.

The components must be a resistor and a capacitor because there is some real power and the power factor is leading.
assume $\omega=377 \cdot \frac{\mathrm{rad}}{\mathrm{sec}}$

d) Find the values of the load components, assuming they are connected in parallel.

Still a resistor and a capacitor.

$$
\mathrm{R}_{\mathrm{Lp}}:=\frac{\mathrm{V}_{\mathrm{LN}}{ }^{2}}{\mathrm{P}_{1 \phi}} \mathrm{R}_{\mathrm{Lp}}\left\langle\begin{array}{ll}
\mathrm{R}_{\mathrm{Lp}}=36.053 \cdot \Omega
\end{array}\right.
$$

e) Correct the power factor with Y-connected components. Need inductors

$$
\mathrm{Q}_{1 \phi \text { Ind }}:=-\mathrm{Q}_{1 \phi}=\frac{\mathrm{v}_{\phi}{ }^{2}}{\omega \cdot \mathrm{~L}_{\mathrm{Y}}} \quad \mathrm{~L}_{\mathrm{Y}}:=\frac{\mathrm{V}_{\phi}{ }^{2}}{\omega \cdot \mathrm{Q}_{1 \phi}} \quad \mathrm{~L}_{\mathrm{Y}}=325.3 \cdot \mathrm{mH}
$$

f) Correct the power factor with $\Delta$-connected components.

$$
\begin{aligned}
& \mathrm{L}_{\Delta}:=\frac{\left(\sqrt{3} \cdot \mathrm{~V}_{\phi}\right)^{2}}{\omega \cdot \mathrm{Q}_{1 \phi}} \quad \mathrm{~L}_{\Delta}=975.9 \cdot \mathrm{mH} \\
& \text { OR } \omega \cdot \mathrm{L}_{\Delta}=\mathbf{Z}_{\Delta}=3 \cdot \mathbf{Z}_{\mathbf{y}}=3 \cdot \omega \cdot \mathrm{~L}_{\mathrm{Y}} \quad 3 \cdot \mathrm{~L}_{\mathrm{Y}}=975.9 \cdot \mathrm{mH}
\end{aligned}
$$

Ex. 2 From F08, exam 1, Find the following:
a) The line current that would be measured by an ammeter.

$$
\mathrm{V}_{\mathrm{LL}}:=480 \cdot \mathrm{~V} \quad \mathbf{Z}_{\Delta}:=(30+12 \cdot \mathrm{j}) \cdot \Omega
$$

Our Approach

1) Change all $\Delta$-connected loads to equivalent Y -connected loads

$$
\mathbf{Z}_{\mathbf{Y}}:=\frac{\mathbf{Z}_{\Delta}}{3} \quad \mathbf{Z}_{\mathbf{Y}}=10+4 \mathrm{j} \cdot \Omega
$$

2) Find all voltages as $\mathrm{V}_{\mathrm{LN}} \quad \mathrm{V}_{\mathrm{LL}}=480 \cdot \mathrm{~V} \quad \mathrm{~V}_{\mathrm{LN}}:=\frac{\mathrm{V}_{\mathrm{LL}}}{\sqrt{3}} \quad \mathrm{~V}_{\mathrm{LN}}=277.128 \cdot \mathrm{~V}$
3) Change all power numbers to $1 \phi$. NOT NEEDED


All $\mathbf{Z}:=(30+12 \cdot \mathrm{j}) \cdot \Omega$

昷 $10 \cdot \Omega$

$$
\mathrm{I}_{\mathrm{L}}:=\frac{\mathrm{V}_{\mathrm{LN}}}{\left|\mathbf{Z}_{\mathbf{Y}}\right|}=\frac{277.128 \cdot \mathrm{~V}}{\sqrt{10^{2}+4^{2} \cdot \Omega}}=25.731 \cdot \mathrm{~A}
$$

$$
\mathrm{I}_{\mathrm{L}}=25.731 \cdot \mathrm{~A}
$$

b) The power consumed by the three-phase load.
c) The value of Y -connected impedances that would result in exactly the same line currents and same pf.

$$
\mathbf{Z}_{\mathbf{Y}}=10+4 \mathrm{j} \cdot \Omega
$$

d) The value of Y-connected capacitors that would correct the pf.

so we need:
$\mathrm{Q}_{\mathrm{C}}:=-\mathrm{Q}_{1 \phi} \quad \mathrm{Q}_{\mathrm{C}}=-2.65 \cdot \mathrm{kVAR}=-\frac{\mathrm{V}_{\mathrm{LN}}{ }^{2}}{\left(\frac{1}{\omega \cdot \mathrm{C}}\right)}=-\mathrm{V}_{\mathrm{LN}}{ }^{2} \cdot \omega \cdot \mathrm{C}$

$$
\mathrm{Q}_{\mathrm{C}}=-2.65 \cdot \mathrm{kVAR}=-\frac{\mathrm{V}_{\mathrm{LN}^{2}}}{\left(\frac{1}{\omega \cdot \mathrm{C}}\right)}=-\mathrm{V}_{\mathrm{LN}}{ }^{2} \cdot \omega \cdot \mathrm{C}
$$

$$
\mathrm{C}=91.5 \cdot \mu \mathrm{~F}
$$

## ECE 3600 3-Phase Examples p3

Ex. 3 For the three-phase delta-connected load in fig P1 .7, The line-to-line voltage and line current are:

$$
\mathbf{V}_{\mathbf{A B}}=480 \cdot \mathrm{~V} \quad \underline{0}^{\circ} \quad \mathbf{I}_{\mathbf{A}}=10 \mathrm{~A} /-40^{\circ}
$$

a) What is $\mathbf{V}_{\mathbf{C A}}$ ?

Normal phase angles


 $\underline{V}_{1 B}$ sequence
b) What is the phase current in the load?

$$
\mathrm{I}_{\mathrm{LL}}=\frac{\mathrm{I}_{\mathrm{L}}}{\sqrt{3}} \quad \frac{10 \cdot \mathrm{~A}}{\sqrt{3}}=5.774 \cdot \mathrm{~A}
$$

c) What is the time-average power into the load?

$$
\mathbf{V}_{\mathbf{A N}}:=\frac{480 \cdot \mathrm{~V}}{\sqrt{3}} \underline{\mathrm{l}-30^{\circ}} \text { Since } \quad \mathbf{I}_{\mathbf{A}}=10 \mathrm{~A} /-40^{\circ}
$$

I lags $\mathbf{V}$ by $10^{\circ}$
$\theta:=10 \cdot \mathrm{deg}$

$$
=10 \mathrm{~A} /-40^{\circ}
$$


$\mathrm{P}_{1 \phi}=(277.128 \cdot \mathrm{~V} \cdot 10 \cdot \mathrm{~A}) \cdot \cos (\theta)=2.729 \cdot \mathrm{~kW}$
$\mathrm{P}_{3 \phi}=3 \cdot(277.128 \cdot \mathrm{~V} \cdot 10 \cdot \mathrm{~A}) \cdot \cos (\theta)=8.188 \cdot \mathrm{~kW}$
d) What is the phase impedance?

$$
\begin{aligned}
& \mathbf{Z}_{\mathbf{Y}}:=\frac{277.128 \cdot \mathrm{~V}}{10 \cdot \mathrm{~A}} \underline{I-30-(-40)^{0}} \\
& \mathbf{Z}_{\Delta}=3 \cdot \mathbf{Z} \mathbf{Y}=83.14 \cdot \Omega \quad \underline{/ 10}^{\mathrm{o}}
\end{aligned}
$$

$$
\mathbf{Z}_{\mathbf{Y}}=27.71 \cdot \Omega
$$$11^{\circ}$

Ex. 4 In the three-phase circuit shown in Fig. P1.9. find the following:
a) The line current that would be measured by an ammeter.

Direct way

$$
\begin{array}{ll}
\mathrm{V}_{\mathrm{LL}}:=600 \cdot \mathrm{~V} & \mathbf{Z}_{\boldsymbol{\Delta}}:=(20+11 \cdot \mathrm{j}) \cdot \Omega \\
\mathrm{I}_{\mathrm{AB}}:=\left|\frac{\mathrm{V}_{\mathrm{LL}}}{\mathbf{Z}_{\boldsymbol{\Delta}}}\right| & \mathrm{I}_{\mathrm{AB}}=26.286 \cdot \mathrm{~A} \\
\mathrm{I}_{\mathrm{A}}:=\sqrt{3} \cdot \mathrm{I}_{\mathrm{AB}} & \mathrm{I}_{\mathrm{A}}=45.53 \cdot \mathrm{~A}
\end{array}
$$


$3 \phi$ source
$A B C$ sequence

Our Approach

$$
\mathrm{V}_{\mathrm{LN}}:=\frac{600 \cdot \mathrm{~V}}{\sqrt{3}} \quad \mathrm{~V}_{\mathrm{LN}}=346.41 \cdot \mathrm{~V}
$$

${ }^{\mathrm{I}} \mathrm{L}$ $\qquad$


$$
\begin{aligned}
& \text { All } \underline{\mathbf{Z}} ' s=20+j 1 \mathrm{l} \Omega \\
& \\
& \quad \text { Figure } \mathbf{P} \mathbf{1 . 9}
\end{aligned}
$$

b) The power factor of the three-phase load. $\quad \theta:=\operatorname{atan}\left(\frac{11}{20}\right) \quad \theta=28.811 \cdot \operatorname{deg} \quad$ pf $\cos (\theta)=0.876$
c) The voltage that would be measured between $B$ and $D$ by a voltmeter.



Using $\mathbf{V}_{\mathbf{A}}$ as reference ( $0^{\circ}$ ):
$\mathbf{V}_{\mathbf{B C}}:=600 \cdot \mathrm{~V} \cdot \mathrm{e}^{\mathrm{j} \cdot 90 \cdot \mathrm{deg}}$
$\mathbf{I}_{\mathbf{C A}}=26.286 \cdot \mathrm{~A} \cdot \mathrm{e}^{\mathrm{j} \cdot(150-28.811) \cdot \operatorname{deg}}$
$\mathbf{V}_{\mathbf{C D}}{ }^{:=} \mathbf{I} \mathbf{C A}^{20 \cdot \Omega}$
$\mathbf{V}_{\mathbf{C D}}=-272.251+449.734 \mathrm{j} \cdot \mathrm{V}$
$\mathbf{V}_{\mathbf{B D}}:=\mathbf{V}_{\mathbf{B C}}+\mathbf{V}_{\mathbf{C D}} \quad \mathbf{V}_{\mathbf{B D}}=-272.251-150.266 \mathrm{j} \cdot \mathrm{V} \quad\left|\mathbf{V}_{\mathbf{B D}}\right|=311 \cdot \mathrm{~V}$
(must be the sum, NOT the difference, see the + and - signs on the drawing.)

Ex. 5 When all you have is impedances and an input voltage, it gets messy \& luckily, it's not a common problem.
Textbook problem 2-2. Figure P2-1 shows a three-phase power system with two loads. The $\Delta$-connected generator is producing a line voltage of 480 V , and the line impedance is $0.09+\mathrm{j} 0.16 \Omega$. Load 1 is Y -connected, with a phase impedance of $2.5 \Omega \underline{/ 36.87^{\circ}}$ and load 2 is $\Delta$-connected, with a phase impedance of $5 \Omega \underline{I}-20^{\circ}$.

a) What is the line voltage at the two loads?

$$
\mathbf{Z}_{\boldsymbol{\phi} 1}:=2.5 \cdot \mathrm{e}^{\mathrm{j} \cdot 36.87 \cdot \operatorname{deg}} \cdot \Omega
$$

$$
\mathbf{Z}_{\boldsymbol{\phi} 2}:=5 \cdot \mathrm{e}^{-\mathrm{j} \cdot 20 \cdot \operatorname{deg}} \cdot \Omega
$$

Find an equivalent Y -only circuit: $\quad \mathbf{Z}_{\text {line }}:=(0.09+0.16 \cdot \mathrm{j}) \cdot \Omega$

$\begin{array}{lll}\mathbf{V}_{\text {LNload }}:=\mathbf{I}_{\mathbf{L}} \cdot \mathbf{Z}_{\text {Yloads }} & \mathbf{V}_{\text {LNload }}=251.311-32.025 \mathrm{j} \cdot \mathrm{V} & \begin{array}{l}\left|\mathbf{V}_{\mathbf{L N l o a d}}\right|=253.343 \cdot \mathrm{~V} \\ \arg \left(\mathbf{V}_{\mathbf{L N l o a d}}\right)=-7.262 \cdot \mathrm{deg}\end{array} \\ \mathbf{V}_{\text {Lload }}:=\mathbf{V}_{\text {LNload }} \sqrt{3} & \mathbf{V}_{\text {Lload }}=435.283-55.47 \mathrm{j} \cdot \mathrm{V} & \left|\mathbf{V}_{\mathbf{L l o a d}}\right|=438.803 \cdot \mathrm{~V}\end{array}$
$\begin{array}{lll}\mathbf{V}_{\text {LNload }}:=\mathbf{I}_{\mathbf{L}} \cdot \mathbf{Z}_{\text {Yloads }} & \mathbf{V}_{\text {LNload }}=251.311-32.025 \mathrm{j} \cdot \mathrm{V} & \begin{array}{l}\left|\mathbf{V}_{\mathbf{L N l o a d}}\right|=253.343 \cdot \mathrm{~V} \\ \arg \left(\mathbf{V}_{\mathbf{L N l o a d}}\right)=-7.262 \cdot \mathrm{deg}\end{array} \\ \mathbf{V}_{\text {Lload }}:=\mathbf{V}_{\text {LNload }} \sqrt{3} & \mathbf{V}_{\text {Lload }}=435.283-55.47 \mathrm{j} \cdot \mathrm{V} & \left|\mathbf{V}_{\mathbf{L l o a d}}\right|=438.803 \cdot \mathrm{~V}\end{array}$

$$
\mathbf{Z}_{\text {Yloads }}:=\frac{1}{\frac{1}{\mathbf{Z}_{\boldsymbol{\phi} \mathbf{1}}}+\frac{1}{\mathbf{Z}_{\mathbf{Y} \mathbf{\phi} \mathbf{2}}}}
$$

$\mathbf{Z}_{\mathbf{Y t o t}}:=\mathbf{Z}_{\text {line }}{ }^{+} \mathbf{Z}_{\text {Yloads }}$
$\mathbf{I}_{\mathbf{L}}:=\frac{\mathbf{V}_{\mathbf{Y}}}{\mathbf{Z}_{\mathbf{Y t o t}}}$
$\mathbf{Z}_{\text {Yloads }}=1.13+0.044 \mathrm{j} \cdot \Omega$
$\left|\mathbf{Z}_{\text {Yloads }}\right|=1.131 \cdot \Omega$ $\arg \left(\mathbf{Z}_{\text {Yloads }}\right)=2.254 \cdot \mathrm{deg}$
$\left|\mathbf{Z}_{\mathbf{Y t o t}}\right|=1.237 \cdot \Omega$ $\arg \left(\mathbf{Z}_{\mathbf{Y t o t}}\right)=9.516 \cdot \mathrm{deg}$

$$
\left|\mathbf{I}_{\mathbf{L}}\right|=224.082 \cdot \mathrm{~A}
$$

$$
\arg \left(\mathbf{I}_{\mathbf{L}}\right)=-9.516 \cdot \operatorname{deg}
$$

$\mathbf{V}_{\text {LNload }}=251.311-32.025 \mathrm{j} \cdot \mathrm{V}$
$\mathbf{V}_{\text {Lload }}=435.283-55.47 \mathrm{j} \cdot \mathrm{V}$
b) What is the voltage drop on the transmission lines?
$\mathbf{V}_{\text {linedrop }}:=\mathbf{I}_{\mathbf{L}} \cdot \mathbf{Z}_{\text {line }} \quad \mathbf{V}_{\text {linedrop }}=25.817+32.025 \mathrm{j} \cdot \mathrm{V}$
$\left|\mathbf{V}_{\text {linedrop }}\right|=41.136 \cdot \mathrm{~V}$
$\arg \left(\mathbf{V}_{\text {linedrop }}\right)=51.126 \cdot \mathrm{deg}$
Check: $\quad \mathbf{V}_{\mathbf{Y}}-\mathbf{V}_{\mathbf{L N l o a d}}=25.817+32.025 \mathrm{j} \cdot \mathrm{V}$
c) Find the real and reactive powers supplied to each load.

$$
\begin{array}{llll}
\mathrm{I}_{\phi 1}:=\frac{\left|\mathbf{V}_{\mathbf{L N l o a d}}\right|}{\left|\mathbf{Z}_{\boldsymbol{\phi} 1}\right|} & \mathrm{I}_{\phi 1}=101.337 \cdot \mathrm{~A} & \mathrm{I}_{\mathrm{L} 2}:=\frac{\left|\mathbf{V}_{\mathbf{L N l o a d}}\right|}{\left|\mathbf{Z}_{\mathbf{Y} \boldsymbol{2} 2}\right|} & \mathrm{I}_{\mathrm{L} 2}=152.006 \cdot \mathrm{~A} \\
\mathrm{P}_{3 \phi 1}:=3 \cdot \mathrm{I}_{\phi 1} \cdot{ }^{2} \cdot \operatorname{Re}\left(\mathbf{Z}_{\boldsymbol{\phi} 1}\right) & \mathrm{P}_{3 \phi 1}=61.615 \cdot \mathrm{~kW} & \mathrm{P}_{3 \phi 2}:=3 \cdot \mathrm{I}_{\mathrm{L} 2} \cdot{ }^{2} \cdot \operatorname{Re}\left(\mathbf{Z}_{\mathbf{Y} \boldsymbol{\phi} 2}\right) & \mathrm{P}_{3 \phi 2}=108.562 \cdot \mathrm{~kW} \\
\mathrm{Q}_{3 \phi 1}:=3 \cdot \mathrm{I}_{\phi 1}{ }^{2} \cdot \operatorname{Im}\left(\mathbf{Z}_{\boldsymbol{\phi} 1}\right) & \mathrm{Q}_{3 \phi 1}=46.212 \cdot \mathrm{kVAR} & \mathrm{Q}_{3 \phi 2}:=3 \cdot \mathrm{I}_{\mathrm{L} 2}{ }^{2} \cdot \operatorname{Im}\left(\mathbf{Z}_{\mathbf{Y} \boldsymbol{\phi} \mathbf{2}}\right) & \mathrm{Q}_{3 \phi 2}=-39.513 \cdot \mathrm{kVAR}
\end{array}
$$

d) Find the real and reactive power losses in the transmission line.

$$
\begin{array}{ll}
\mathrm{P}_{3 \phi \mathrm{~L}}:=3 \cdot\left(\left|\mathbf{I}_{\mathbf{L}}\right|\right)^{2} \cdot \operatorname{Re}\left(\mathbf{Z}_{\text {line }}\right) & \mathrm{P}_{3 \phi \mathrm{~L}}=13.557 \cdot \mathrm{~kW} \\
\mathrm{Q}_{3 \phi \mathrm{~L}}:=3 \cdot\left(\left|\mathbf{I}_{\mathbf{L}}\right|\right)^{2} \cdot \operatorname{Im}\left(\mathbf{Z}_{\text {line }}\right) & \mathrm{Q}_{3 \phi \mathrm{~L}}=24.102 \cdot \mathrm{kVAR}
\end{array}
$$

e) Find the real power, reactive power, and power factor supplied by the generator.

$$
\begin{array}{ll}
\mathrm{P}_{3 \phi \text { gen }}:=\mathrm{P}_{3 \phi \mathrm{~L}}+\mathrm{P}_{3 \phi 1}+\mathrm{P}_{3 \phi 2} & \mathrm{P}_{3 \phi \text { gen }}=183.734 \cdot \mathrm{~kW} \\
\mathrm{Q}_{3 \phi \mathrm{gen}}:=\mathrm{Q}_{3 \phi \mathrm{~L}}+\mathrm{Q}_{3 \phi 1}+\mathrm{Q}_{3 \phi 2} & \mathrm{Q}_{3 \phi \text { gen }}=30.801 \cdot \mathrm{kVAR}
\end{array} \quad \mathrm{pf}=\frac{\mathrm{P}_{3 \phi \text { gen }}}{3 \cdot\left|\mathbf{V}_{\mathbf{Y}} \cdot\right| \cdot\left|\mathbf{I}_{\mathbf{L}}\right|}=0.986
$$

f) What is the efficiency of this system?

$$
\eta=\frac{\mathrm{P}_{3 \phi 1}+\mathrm{P}_{3 \phi 2}}{\mathrm{P}_{3 \phi \mathrm{gen}}}=92.621 \cdot \%
$$

The next example uses a "one-line diagram" to show how a generator is connected to 3 loads. In these diagrams, one line represents all 3 phases and neutral. Because the individual lines are not shown, there may be notes or symbols to indicate $Y$ or $\Delta$ connections. All powers given will be 3-phase values, all voltages will be line voltages (that is line-to-line) and all currents will line currents. The term "bus" refers common connection area.
Ex. 6 The one-line diagram below shows a single, Y-connected generator and 3 loads. Assume all lines are lossless.


Find:
a) The phase voltage and currents in Load 1.

$$
\begin{array}{lrl}
\mathrm{V}_{\mathrm{LL}}:=480 \cdot \mathrm{~V} & \mathrm{~V}_{\mathrm{LN}}:=\frac{\mathrm{V}_{\mathrm{LL}}}{\sqrt{3}} & \mathrm{~V}_{\mathrm{LN}}=277.128 \cdot \mathrm{~V}=\mathrm{V}_{\mathrm{L} 1 \phi} \\
\mathrm{pf}_{\mathrm{L} 1}:=0.9 & \mathrm{~S}_{\mathrm{L} 1.1 \phi}:=\frac{100 \cdot \mathrm{~kW}}{3 \cdot \mathrm{pf}_{\mathrm{L} 1}} & \mathrm{I}_{1}:=\frac{\mathrm{S}_{\mathrm{L} 1.1 \phi}}{\mathrm{~V}_{\mathrm{LN}}}
\end{array} \quad \mathrm{I}_{1}=133.646 \cdot \mathrm{~A}=\mathrm{I}_{\mathrm{L} 1 \phi}
$$

b) The phase voltage and currents in Load 2.

$$
\begin{aligned}
\mathrm{V}_{\mathrm{LL}}:=480 \cdot \mathrm{~V}=\mathrm{V}_{\mathrm{L} 2 \phi} & \mathrm{pf}_{\mathrm{L} 2}:=0.8 \quad \mathrm{~S}_{\mathrm{L} 2.1 \phi}:=\frac{80 \cdot \mathrm{kVA}}{3} \\
\mathrm{I}_{2}:=\frac{\mathrm{S}_{\mathrm{L} 2.1 \phi}}{\mathrm{~V}_{\mathrm{LN}}} & \mathrm{I}_{2}=96.225 \cdot \mathrm{~A}=\sqrt{3} \cdot \mathrm{I}_{\mathrm{L} 2 \phi} \quad \mathrm{I}_{\mathrm{L} 2 \phi}=\frac{\mathrm{I}_{2}}{\sqrt{3}}=55.556 \cdot \mathrm{~A}
\end{aligned}
$$

c) The real, reactive and apparent power supplied by the generator with the switch to load 3 open.

$$
\begin{aligned}
& \mathrm{P}_{1}:=100 \cdot \mathrm{~kW} \quad \mathrm{P}_{2}:=80 \cdot \mathrm{kVA} \cdot \mathrm{pf} \mathrm{~L} 2^{\mathrm{P}_{2}=64 \cdot \mathrm{~kW}} \quad \mathrm{P}_{\mathrm{G}}:=\mathrm{P}_{1}+\mathrm{P}_{2} \quad \mathrm{P}_{\mathrm{G}}=164 \cdot \mathrm{~kW} \\
& \mathrm{Q}_{1}:=\sqrt{\left(\frac{100 \cdot \mathrm{~kW}}{\mathrm{pf})^{2}-(100 \cdot \mathrm{~kW})^{2}} \quad \mathrm{Q}_{1}=48.432 \cdot \mathrm{kVAR}\right.} \quad \mathrm{Q}_{2}:=\sqrt{(80 \cdot \mathrm{kVA})^{2}-(64 \cdot \mathrm{~kW})^{2}} \quad \mathrm{Q}_{1}=48.432 \cdot \mathrm{kVAR} \\
& \mathrm{Q}_{\mathrm{G}}:=\mathrm{Q}_{1}+\mathrm{Q}_{2} \quad \mathrm{Q}_{\mathrm{G}}=96.432 \cdot \mathrm{kVAR} \\
& \mathrm{~S}_{\mathrm{G}}:=\sqrt{\mathrm{P}_{\mathrm{G}}{ }^{2}+\mathrm{Q}_{\mathrm{G}}{ }^{2}} \quad \mathrm{~S}_{\mathrm{G}}=190.25 \cdot \mathrm{kVAR} \\
& \text { d) The total line current from the generator, } \mathrm{I}_{\mathrm{G}} \text {, with the switch to load 3 open. } \quad \mathrm{I}_{\mathrm{G}}=\frac{\left(\frac{\mathrm{S}_{\mathrm{G}}}{3}\right)}{\mathrm{V}_{\mathrm{LN}}}=228.836 \cdot \mathrm{~A}
\end{aligned}
$$

e) The real, reactive and apparent power supplied by the generator with the switch to load 3 closed.
$\mathrm{pf}_{\mathrm{L} 3}:=0.65$
$\mathrm{S}_{\mathrm{L} 3.1 \phi}:=\frac{80 \cdot \mathrm{~kW}}{3 \cdot \mathrm{pf}}$
$\mathrm{Q}_{3}:=-\sqrt{\left(\frac{80 \cdot \mathrm{~kW}}{\mathrm{pf} \mathrm{L} 3}\right)^{2}-(80 \cdot \mathrm{~kW})^{2}}$
$\mathrm{Q}_{3}=-93.53 \cdot \mathrm{kVAR}$
$P_{G}:=P_{1}+P_{2}+80 \cdot \mathrm{~kW} \quad P_{G}=244 \cdot \mathrm{~kW}$
$\mathrm{Q}_{\mathrm{G}}:=\mathrm{Q}_{1}+\mathrm{Q}_{2}+\mathrm{Q}_{3}$
$\mathrm{Q}_{\mathrm{G}}=2.902 \cdot \mathrm{kVAR}$
$S_{G}:={ }^{P} G^{2}+Q_{G}{ }^{2}$
$\mathrm{S}_{\mathrm{G}}=244.017 \cdot \mathrm{kVAR}$
f) How does the total line apparent power from the generator, $\mathrm{S}_{\mathrm{G}}$, compare to the sum of the three individual apparent powers, $\mathrm{S}_{1}+\mathrm{S}_{2}+\mathrm{S}_{3}$ ? If they aren't equal, why not? (Switch closed)

$$
3 \cdot \mathrm{~S}_{\mathrm{L} 1.1 \phi}+80 \cdot \mathrm{kVAR}+3 \cdot \mathrm{~S}_{\mathrm{L} 3.1 \phi}=314.188 \cdot \mathrm{kVAR} \not \not \neq \mathrm{S}_{\mathrm{G}}=244.017 \cdot \mathrm{kVAR}
$$

## Can't Add Magnitudes

g) The total line current from the generator, $\mathrm{I}_{\mathrm{G}}$, with the switch to load 3 closed.

$$
\mathrm{I}_{\mathrm{G}}:=\frac{\left(\frac{\mathrm{S}_{\mathrm{G}}}{3}\right)}{\mathrm{V}_{\mathrm{LN}}} \quad \mathrm{I}_{\mathrm{G}}=293.507 \cdot \mathrm{~A}
$$

h) How does the total line current from the generator, $\mathrm{I}_{\mathrm{G}}$, compare to the sum of the three individual currents, $\mathrm{I}_{1}+\mathrm{I}_{2}+\mathrm{I}_{3}$ ? If they aren't equal, why not? (Switch closed)

$$
\begin{array}{rc}
\mathrm{I}_{3}:=\frac{\mathrm{S}_{\mathrm{L} 3.1 \phi}}{\mathrm{~V}_{\mathrm{LN}}} & \mathrm{I}_{3}=148.039 \cdot \mathrm{~A} \\
\mathrm{I}_{1}+\mathrm{I}_{2}+\mathrm{I}_{3}=377.909 \cdot \mathrm{~A} \not \not \neq \mathrm{I}_{\mathrm{G}}=293.507 \cdot \mathrm{~A}
\end{array}
$$

