## ECE 3600 Final, given: Fall 2019

Write Legibly! If I can't read what you've written or you answer is ambiguous, l'll assume you don't know.
(70 pts) Questions This part of the exam is Closed book, Closed notes, No Calculator.
Wrong answers may cost up to 3 times what a right answers is worth. Blanks cost the same as the right answer was worth.

1. Consider the single-phase system below. There are two sources, labeled S1 and S2 and two loads, labeled L1 and L2. All the variables shown or referred to in the questions are scalar or magnitudes of complex numbers. The same should be true of your answers.


$$
\mathrm{P}_{\mathrm{S} 1}+\mathrm{P}_{\mathrm{S} 2}=\mathrm{P}_{\mathrm{Rline}}+\mathrm{P}_{\mathrm{L} 1}+\mathrm{P}_{\mathrm{L} 2} \quad \mathrm{OR} \quad \mathrm{P}_{\mathrm{S} 1}+\mathrm{P}_{\mathrm{S} 2}=\mathrm{I}_{\text {line }} \cdot \mathrm{R}_{\text {line }}+\mathrm{P}_{\mathrm{L} 1}+\mathrm{P}_{\mathrm{L} 2} \quad \begin{gathered}
\text { (Only one answer is } \\
\text { necessary })
\end{gathered}
$$

Note: You are not being asked for FORMULAs. You are being asked to express basic concepts in a mathematical way.
a) Is there a simple relationship between all the reactive powers above?
NO YES

If yes, express that relationship in a mathematical way.
b) Is there a simple relationship between all the apparent powers above? NO YES

If yes, express that relationship in a mathematical way.
c) Is there a simple relationship between all the power factors above? NO YES

If yes, express that relationship in a mathematical way.
d) Express $\mathrm{I}_{\text {line }}$ in terms of source real and/or reactive powers and $\mathrm{V}_{\mathrm{S}}$. (Please remember that these variables are all scalar or magnitudes.)
e) Express $\mathrm{I}_{\text {line }}$ in terms of load real and/or reactive powers and $\mathrm{V}_{\mathrm{L}}$.
f) Express $I_{L 1}$ in terms of load real and/or reactive powers and $V_{L}$.
g) Express the efficiency in terms of real and/or reactive powers.
2. Consider the balanced three-phase load shown. Except for the Z's, all the variables shown or referred to in the questions are scalar or magnitudes of complex numbers. The same should be true of your answers. Where possible, express answers mathematically.
a) If this is a balanced load, what can be said about the $\mathbf{Z}$ 's?
b) What is the value $V_{4}$ ? (may be expressed in terms of $\mathrm{V}_{1}$ )
c) What is the value $\mathrm{I}_{4}$ ? (may be expressed in terms of $\mathrm{I}_{2}$ )
 O YES If yes, which one?
e) Is one of the currents shown also known as the line current? NO YES

If yes, which one?
f) Is this load connected Y or $\Delta$ ?
g) Could we find an equivalent load connected the other way ( $\Delta$ if now Y , or Y if now $\Delta$ )? NO YES If yes what $\mathbf{Z}$ values should be used? Finish one of these two expressions:

$$
\mathbf{Z}_{\Delta}=\quad \text { OR } \quad \mathbf{Z}_{\mathbf{Y}}=
$$

3. A source and load are connected to a model of a non-ideal transformer as shown. All the variables shown or referred to in the questions are scalar or magnitudes of complex numbers. The same should be true of your answers. Where possible, express answers mathematically.

b) What is the relationship between $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ ?
c) What is the relationship between $\mathrm{I}_{1}$ and $\mathrm{I}_{2}$ ?
d) Can the dotted box and the load be replaced with a simpler equivalent, $\mathrm{R}_{\mathrm{eq}}+\mathrm{j} \cdot \mathrm{X}_{\mathrm{eq}}$ ? NO YES

If yes, express: $\quad \mathrm{R}_{\mathrm{eq}}=\quad \mathrm{X}_{\mathrm{eq}}=$
these may be used in expressions below
e) Express $I_{1}$ in terms of impedances and $V_{p}$. (Please remember that these variables are all scalar or magnitudes.)
f) Express the real power provided by the source in terms of impedances, $\mathrm{V}_{\mathrm{p}}$ and $\mathrm{I}_{1}$.
g) Express the reactive power provided by the source in terms of impedances, $\mathrm{V}_{\mathrm{p}}$ and $\mathrm{I}_{1}$.
h) Express $\mathrm{I}_{\mathrm{p}}$ in whatever terms you can find above. ("above" means in the figure and/or answers of this problem.)
i) Express the efficiency in whatever terms you can find above.
4. The following questions pertain to a 3-phase synchronous machine.
a) Label all the phasors and angles shown.
b) Is this phasor diagram for a motor or a generator?

c) Judging by the phasor diagram, is the machine making + or - reactive power?
d) Label the diagram with the voltage and current labels used in a). Also label anything else of importance.

e) Express the relationship between the 3 voltage phasors above (they can be complex numbers).
5. The following questions pertain to a 3-phase induction motor. A model of one phase is shown. Bold variables are complex, all others are scalar.

a) What is the variable "s" called?
b) A partial schematic is shown at right.

Find (write) an expression for the combined impedance, $\mathbf{Z}$.

$$
\begin{aligned}
& \mathbf{Z}= \\
& \text { Impedance of } \\
& \text { these } 3 \text { parts }
\end{aligned}
$$


c) Write an expression for $\mathbf{E}_{\mathbf{1}}$ in terms of $\mathrm{V}_{\phi}$ and the impedances given or found above.
d) Write an expression for $\left|\mathbf{I}_{\mathbf{2}}\right|$ in terms of $\mathbf{E}_{\mathbf{1}}$ and the impedances given or found above.
e) Express the stator-copper-loss of this $3 \phi$ motor in terms given or found above.
f) Express the rotor-copper-loss in terms given or found above.
g) Express the air-gap power in terms given or found above.
h) Express the power converted to mechanical power in terms given or found above.
$\qquad$

Problems Closed Book, Closed notes except for those given in class for Exam 1, 2, and final, Calculators OK, Show all work to receive credit.Circle answers, show units, and round off reasonably

1. ( 32 pts ) A 345 kV transmission line has the following length and line parameters. $\mathrm{S}:=$ siemens
len $:=150 \cdot \mathrm{~km}$
$\mathrm{r}:=0.08 \cdot \frac{\Omega}{\mathrm{~km}}$
$\mathrm{x}:=0.76 \cdot \frac{\Omega}{\mathrm{~km}}$
$\mathrm{g}:=0 \cdot \frac{\mathrm{~S}}{\mathrm{~km}}$
$\mathrm{y}:=5 \cdot 10^{-6} \cdot \frac{\mathrm{~S}}{\mathrm{~km}}$
a) Choose the most appropriate model for this transmission line and draw it, including the impedance and/or admittance value(s). Add a $3 \phi$ load at the receiving end of the transmission line.

The line voltage at the source is 345 kV . The line current from the source ( $\mathbf{I}_{\mathbf{S}}$ ) is 250 A and it leads the line-to-neutral voltage by $15^{\circ}$.
b) Find the line current in your model, $\mathbf{I}_{\text {Line }}\left(\right.$ not $\left.\mathbf{I}_{\mathbf{S}}\right)$ in a complex-number form. $\quad \mathbf{I}_{\text {Line }}=$ ?
c) Find the load phase voltage, $\mathbf{V}_{\mathbf{R}}$, magnitude and phase. $\quad \mathbf{V}_{\mathbf{R}}=$ ?
d) What is the line voltage at the load (magnitude)?
e) What is the "power angle" ( $\delta$ )?

1, continued f) Find the impedance of one phase of the load, assuming Y-connected.
g) Find the power consumed by the entire load.
h) Find the power factor of the load.
2. (36 pts) A 1.5-hp, separately excited dc motor runs at $60 \%$ overall efficiency (includes power needed for the field) when operated at full load. Both the armature and field are hooked to a single 200 V source. The rotational losses are proportional to the motor speed. Other important information is given below.

$$
\eta:=60 \cdot \% \quad \mathrm{~V}_{\mathrm{T}}:=200 \cdot \mathrm{~V} \quad \mathrm{n}_{\mathrm{fL}}:=900 \cdot \mathrm{rpm} \quad \mathrm{R}_{\mathrm{A}}:=3 \cdot \Omega \quad \mathrm{R}_{\mathrm{F}}:=250 \cdot \Omega \quad 1 \cdot \mathrm{hp}=745.7 \cdot \mathrm{~W}
$$

a) Find the power converted from electrical to mechanical, $\mathrm{P}_{\text {conv }}=$ ?
b) Find the rotational losses, $\quad \mathrm{P}_{\text {rot }}=$ ?

2, continued c) Find the no-load armature current. Show the algebra needed to find $\mathrm{I}_{\mathrm{A}}$ from the basic equations.
The rotational losses are proportional to the motor speed.
Hint 1: This also means that the rotational losses are proportional to $\mathrm{E}_{\mathrm{A}}$, like this:
Hint 2: This turns out to be amazingly easy to calculate, no quadratic required.
$P_{\text {rot2 }}=P_{\text {rot1 }} \cdot \frac{E_{A 2}}{E_{A 1}}$
d) The full-load speed is given above as $n_{\mathrm{fL}}$. Find the no-load shaft speed.
e) The mechanical load on the shaft is reduced so $\quad \mathrm{P}_{\text {out }}:=1 \cdot \mathrm{hp} \quad$ Find the new shaft speed.

Show the algebra needed to find $\mathrm{E}_{\mathrm{A}}$ from the basic equations. (Yeah, algebra is a prerequisite for Engineering.) Hints: This is NOT trivial to calculate, algebra and quadratic required.

Remember $\mathrm{P}_{\text {conv }}:=\mathrm{P}_{\text {out }}+\mathrm{P}_{\text {rot }} \quad$ and, $\mathrm{P}_{\text {rot }}$ is dependent on speed as described above.
3. (22 pts) A $1 / 2-\mathrm{hp}, 120-\mathrm{V}, 60-\mathrm{Hz}$, single-phase, capacitor-run, induction motor has two windings set $90^{\circ}$ apart in the motor housing. The windings are NOT the same. At Startup, winding 1 draws 6 A at $30^{\circ} \mathrm{lag}$. Winding 2 in series with an $80-\mu \mathrm{F}$ capacitor draws 4.5 A at $35^{\circ}$ lead.

$$
\mathrm{V}_{\mathrm{T}}:=120 \cdot \mathrm{~V} \quad \mathbf{I}_{\mathbf{1}}:=6 \cdot \mathrm{~A} \cdot \mathrm{e}^{-\mathrm{j} \cdot 30 \cdot \mathrm{deg}} \quad \mathbf{I}_{\mathbf{2}}:=4.5 \cdot \mathrm{~A} \cdot \mathrm{e}^{\mathrm{j} \cdot 35 \cdot \mathrm{deg}} \quad \mathrm{C}:=80 \cdot \mu \mathrm{~F} \text { in series with win }
$$

a) Find the impedance of winding 1 and winding 2 without the capacitor. Find both in rectangular form.
b) If the capacitor were disconnected from winding 2 and placed in series with winding 1 instead, find the new phase angle difference and the new current magnitudes. Did anything improve?
c) There will be one other major change in the motor startup with this new configuration. We didn't directly discuss this in class, but you can figure it out if you understand how the startup works. What will be different?

## Answers Questions

1. a) $\mathrm{Q}_{\mathrm{S} 1}+\mathrm{Q}_{\mathrm{S} 2}=\mathrm{I}_{\text {line }}{ }^{2} \cdot \mathrm{X}_{\text {line }}+\mathrm{Q}_{\mathrm{L} 1}+\mathrm{Q}_{\mathrm{L} 2}$
e) $I_{\text {line }}=\frac{\sqrt{\left(P_{\mathrm{L} 1}+P_{L 2}\right)^{2}+\left(Q_{L 1}+Q_{L 2}\right)^{2}}}{V_{\mathrm{L}}}$
2. a) $\mathbf{Z}_{\mathbf{1}}=\mathbf{Z}_{\mathbf{2}}=\mathbf{Z}_{\mathbf{3}}$
b) $\sqrt{3} \cdot V_{1}$
c) 0
d) $\mathrm{V}_{4}$
b) NO
c) NO
d) $\mathrm{I}_{\text {line }}=\frac{\sqrt{\left(\mathrm{P}_{\mathrm{S} 1}+\mathrm{P}_{\mathrm{S} 2}\right)^{2}+\left(\mathrm{Q}_{\mathrm{S} 1}+\mathrm{Q}_{\mathrm{S} 2}\right)^{2}}}{\mathrm{~V}_{\mathrm{S}}}$ g) $\mathbf{Z}_{\Delta}=3 \cdot \mathbf{Z}_{\mathbf{Y}}$ OR $3 \cdot \mathbf{Z}_{\mathbf{1}}$ either answer $\mathbf{Z}_{\mathbf{Y}}=\frac{\mathbf{Z}_{\boldsymbol{\Delta}}}{3}$
e) $I_{2}$ f) $Y$
b) $\frac{V_{1}}{V_{2}}=\frac{N_{1}}{N_{2}}=\frac{I_{2}}{I_{1}} \quad$ d) $R_{e q}=R_{L} \cdot\left(\frac{N_{1}}{N_{2}}\right)^{2} \quad X_{e q}=X_{L} \cdot\left(\frac{N_{1}}{N_{2}}\right)^{2}$
3. a) Ideal transformer
f) $I_{L 1}=\frac{\sqrt{\left(P_{L 1}\right)^{2}+\left(Q_{L 1}\right)^{2}}}{V_{L}}$
g) $\eta=\frac{P_{L 1}+P_{L 2}}{O_{\mathrm{P}}{ }^{P_{S}+P_{S}}{ }^{2}}$
$\eta=\frac{P_{L 1}+P_{L 2}}{P_{L 1}+P_{L 2}+P_{\text {Rline }}}$
e) $I_{1}=\frac{}{\sqrt{[ }}$

$$
\frac{v_{p}}{\sqrt{\left[R_{s}+R_{L} \cdot\left(\frac{N_{1}}{N_{2}}\right)^{2}\right]^{2}+\left[X_{s}+X_{L}\left(\frac{N_{1}}{N_{2}}\right)^{2}\right]^{2}}}=\frac{v_{p}}{\text { OR }}
$$

f) $P_{S}=\frac{V_{p}{ }^{2}}{R_{m}}+I_{1}{ }^{2} \cdot\left(R_{s}+R_{e q}\right) \quad$ g) $Q_{S}=\frac{V_{p}{ }^{2}}{X_{m}}+I_{1}{ }^{2} \cdot\left(X_{s}+X_{e q}\right)$
h) $I_{p}=\frac{\sqrt{P_{S}{ }^{2}+Q_{S}{ }^{2}}}{V_{p}} \quad$ i) $\eta=\frac{I_{1}{ }^{2} \cdot R_{e q}}{P_{S}}$
4. a)


OR
e) $\mathbf{E}_{\mathbf{A}}=\mathbf{V}_{\boldsymbol{\phi}}{ }^{+} \mathbf{V}_{\mathbf{X s}}=\mathbf{V}_{\boldsymbol{\phi}}{ }^{+} \mathbf{I}_{\mathbf{A}} \cdot \mathrm{j} \cdot \mathbf{X}_{\mathbf{s}}$
c) - d)

5. a) The slip
b) $\mathbf{Z}=\frac{1}{\frac{1}{j \cdot X_{m}}+\frac{1}{\left(\frac{R_{2}}{s}+j \cdot X_{2}\right)}}$
c) $\mathbf{E}_{\mathbf{1}}=V_{\phi} \cdot \frac{\mathbf{Z}}{R_{1}+\mathrm{j} \cdot \mathrm{X}_{1}+\mathbf{Z}}$
d) $\left|\mathbf{I}_{\mathbf{2}}\right|=\frac{\left|\mathbf{E}_{\mathbf{1}}\right|}{\sqrt{\left(\frac{\mathrm{R}_{2}}{\mathrm{~s}}\right)^{2}+\mathrm{X}_{2}{ }^{2}}}$
e) $\mathrm{P}_{\mathrm{SCL}}=3 \cdot\left[\left(\left|\mathbf{I}_{\mathbf{1}}\right|\right)^{2} \cdot \mathrm{R}_{1}\right]$
f) $\mathrm{P}_{\mathrm{RCL}}=3 \cdot\left[\left(\left|\mathbf{I}_{\mathbf{2}}\right|\right)^{2} \cdot \mathrm{R}_{2}\right]$
g) $\mathrm{P}_{\mathrm{AG}}=3 \cdot\left[\left(\left|\mathbf{I}_{\mathbf{2}}\right|\right)^{2} \cdot \frac{\mathrm{R}_{2}}{\mathrm{~s}}\right]$
h) $\mathrm{P}_{\text {conv }}=(1-\mathrm{s}) \cdot \mathrm{P}_{\mathrm{AG}}=\mathrm{P}_{\mathrm{AG}^{-}} \mathrm{P}_{\mathrm{RCL}}$

## Problems

1. a)

b) $241.7 \cdot \mathrm{~A} /-2.369^{\circ}$
c) $197.1 \cdot \mathrm{kV} \quad \underline{-} 7.995^{\circ}$
d) $341.3 \cdot \mathrm{kV}$
e) 7.9950
f) $(785.1+163.9 \cdot \mathrm{j} \Omega \Omega$ $802.0 \cdot \Omega \quad 11.79^{\circ}$
g) $142.2 \cdot \mathrm{MW}$
h) 0.979
2. a) $1.486 \cdot \mathrm{~kW}$
b) $367 \cdot 9 \cdot \mathrm{~W}$
c) $2.109 \cdot \mathrm{~A}$
d) $999.3 \cdot \mathrm{rpm}$ OR $104.6 \cdot \frac{\mathrm{rad}}{\mathrm{sec}}$
e) $935.6 \cdot \mathrm{rpm}$ OR $97.98 \cdot \frac{\mathrm{rad}}{\mathrm{sec}}$
3. a) $17.32+10 \cdot \mathrm{j} \Omega$
$21.84+17.86 \cdot \mathrm{j} \Omega$
b) Angle difference is much closer to the ideal of $90^{\circ}$.
c) It will start spinning in the opposite direction
Currents are both less and closer in value
