Ex. 1 A 3-hp dc motor has the following nameplate information: 150 V, 1400 rpm, 18 A, $R_A = 0.8 \ \Omega$, and $R_F = 300 \ \Omega$. The field is shunt connected and the 18 A includes the field current. Assume rotational losses are constant.

a) Find the efficiency of the motor at nameplate operation. (Include the field in your calculations)

\[
\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = 82.86\% 
\]

\[
P_{\text{in}} = V_T I_{FL} 
\]

\[
P_{\text{out}} = 2.237 \ \text{kW} 
\]

\[
V_T = 150 \ \text{V} 
\]

\[
I_{FL} = 18 \ \text{A} 
\]

\[
I_F = 0.5 \ \text{A} 
\]

\[
I_{AFL} = 17.5 \ \text{A} 
\]

\[
E_A = 136 \ \text{V} 
\]

\[
P_{\text{conv}} = 2.38 \ \text{kW} 
\]

\[
P_{\text{rot}} = 142.9 \ \text{W} 
\]

\[
P_{\text{rot}} = 0.192 \ \text{hp} 
\]

b) Find the rotational losses at nameplate operation.

\[
P_{\text{rot}} = 142.9 \ \text{W} 
\]

\[
P_{\text{rot}} = 0.192 \ \text{hp} 
\]

c) Find the required current for a developed power of 1.5 hp with $V_T = 150 \ \text{V}$.

\[
P_{\text{conv}} = 1.119 \ \text{kW} 
\]

\[
P_{\text{conv}} = 2.38 \ \text{kW} 
\]

\[
P_{\text{rot}} = 142.9 \ \text{W} 
\]

\[
P_{\text{rot}} = 0.192 \ \text{hp} 
\]

d) Find the output power if the developed power is 1.5 hp with $V_T = 150 \ \text{V}$.

\[
P_{\text{out}} = 975.7 \ \text{W} 
\]

\[
P_{\text{out}} = 1.308 \ \text{hp} 
\]

e) Find the shaft speed if the developed power is 1.5 hp with $V_T = 150 \ \text{V}$.

\[
\omega = 155 \ \text{rad} / \text{sec} 
\]

\[
\omega = 1480 \ \text{rpm} 
\]

\[
n = 1480 \ \text{rpm} 
\]

\[
\omega = 155 \ \text{rad} / \text{sec} 
\]

g) Find the load torque if the developed power is still 1.5 hp with $V_T = 150 \ \text{V}$.

\[
\tau = 2.518 \ \text{N-m} 
\]

\[
\tau = 2.518 \ \text{N-m} 
\]
Ex. 2 A dc motor has the following nameplate information: 5-hp, 1200 rpm, Armature: 200 V, 22 A, 1 Ω; Field: 200 V, 1 A

a) Find the rotational losses at nameplate operation.

\[ V_T := 200-V \quad n_{FL} := 1200\text{-rpm} \]

\[ I_{FL} := 22-A \quad R_A := 1-\Omega \]

\[ P_{outFL} := 5\text{-hp} \times \frac{745.7\text{-W}}{\text{hp}} \]

\[ P_{outFL} = 3.728\text{ kW} \]

\[ E_{AFL} := V_T - I_{FL}\cdot R_A \]

\[ E_{AFL} = 178\text{ V} \]

\[ P_{convFL} := E_{AFL}\cdot I_{FL} \]

\[ P_{convFL} = 3.916\text{ kW} \]

\[ P_{rotFL} := P_{convFL} - P_{outFL} \]

\[ P_{rotFL} = 187.5\text{ W} \quad P_{rotFL} = 0.251\text{ hp} \]

b) The load torque is constant (not dependent on speed) and the rotational losses are proportional to speed, find the shaft speed at half the rated armature voltage and full rated field voltage.

\[ P = \tau \cdot \omega_m \quad \text{so, if a power is proportional to speed, then the torque is constant.} \]

\[ \text{OR, conversely, if the torque is constant, the power is proportional to speed.} \]

In this case, ALL the power converted is proportional to speed and ALL the the induced torque is constant.

\[ P_{conv} = \frac{n_{new}}{n_{FL}} \cdot P_{convFL} \]

One way: \[ \tau_{ind} = K\phi I_A \quad \text{so, if} \ \tau_{ind} \ \text{and the field current are constant, then} \ I_A \ \text{is constant.} \]

\[ I_A := I_{FL} \]

\[ V_T := \frac{200\text{-V}}{2} \quad E_A := V_T - I_A\cdot R_A \quad E_A = 78\text{ V} \quad n_{new} = \frac{E_A}{E_{AFL}} \cdot n_{FL} = 525.8\text{ rpm} \]

Another solution: recognize that: \[ E_A = \frac{n_{new}}{n_{FL}} \cdot E_{AFL} \quad \text{because the field is constant} \]

\[ V_T = E_A + I_A\cdot R_A = E_A + \frac{P_{convFL}}{E_A}\cdot R_A = E_A + \frac{n_{new}\cdot P_{convFL}}{n_{FL}\cdot E_{AFL}}\cdot R_A = E_A + \frac{P_{convFL}}{E_{AFL}}\cdot R_A \]

\[ E_A := \frac{200\text{-V}}{2} - \frac{P_{convFL}}{E_{AFL}}\cdot R_A \]

\[ E_A = 78\text{ V} \quad n_{new} = \frac{E_A}{E_{AFL}} \cdot n_{FL} = 525.8\text{ rpm} \]

c) The torque is constant (like part b)) , find the shaft speed at half the rated voltage for both the armature and field. You may assume that the flux is proportional to the field current.

One way: \[ \tau_{ind} = K\phi I_A \quad \text{so, if} \ \tau_{ind} \ \text{is constant and the field current (and flux) is halved,} \]

\[ \text{then} \ I_A \ \text{is constant at twice the value it used to be.} \]

\[ I_A := 2\cdot I_{FL} \]

\[ V_T := \frac{200\text{-V}}{2} \quad E_A := V_T - I_A\cdot R_A \quad E_A = 56\cdot V \quad n_{new} = \frac{E_A}{E_{AFL}} \cdot n_{FL} = 377.5\text{ rpm} \]

Another solution: recognize that: \[ E_A = \frac{1}{2} \cdot \frac{E_{AFL}}{n_{FL}} \cdot n_{new} \]

\[ V_T = E_A + I_A\cdot R_A = E_A + \frac{P_{convFL}}{E_A}\cdot R_A = E_A + \frac{n_{new}\cdot P_{convFL}}{2\cdot n_{FL}\cdot E_{AFL}}\cdot R_A = E_A + \frac{P_{convFL}}{E_{AFL}}\cdot R_A \]

\[ E_A := \frac{200\text{-V}}{2} - 2\cdot \frac{P_{convFL}}{E_{AFL}}\cdot R_A \quad E_A = 56\cdot V \quad n_{new} = \frac{E_A}{E_{AFL}} \cdot n_{FL} = 377.5\text{ rpm} \]
d) If the load torque and rotational loss torque are both proportional to speed (most like real life), find the shaft speed when the motor armature is hooked to half the rated voltage. The field is at rated voltage.

\[ \tau_{\text{ind}} = \frac{n_{\text{new}}}{n_{FL}} \tau_{\text{indFL}} \]

\[ P = \tau \omega_m \]

which leads to:

\[ P_{\text{conv}} = \left( \frac{n_{\text{new}}}{n_{FL}} \right)^2 P_{\text{convFL}} \]

One way: \[ \tau_{\text{ind}} = K \phi I_A \]

so, if \( \tau_{\text{ind}} \) is proportional to speed and the field current is constant,

then \( I_A \) is also proportional to speed.

\[ I_A \approx \frac{n_{\text{new}}}{n_{FL}} I_A \]

\[ V_T = E_A + I_A R_A = E_A + \frac{n_{\text{new}}}{n_{FL}} I_{FL} R_A = E_A + \frac{n_{\text{new}}}{n_{FL}} I_{FL}^2 R_A \]

\[ = \frac{n_{\text{new}}}{n_{FL}} (E_{AFL} + \frac{n_{\text{new}}}{n_{FL}} I_{FL} R_A) \]

\[ = \frac{n_{\text{new}}}{n_{FL}} (200 \cdot V) \]

\[ n_{\text{new}} = \frac{100 \cdot V}{200 \cdot V} n_{FL} = 600 \text{rpm} \]

Another solution: \[ E_A = \frac{n_{\text{new}}}{n_{FL}} E_{AFL} \]

because the field is constant

\[ V_T = E_A + I_A R_A = E_A + \frac{P_{\text{conv}}}{E_A} R_A = E_A + \frac{n_{\text{new}}}{n_{FL}} E_{AFL} R_A \]

\[ = \frac{n_{\text{new}}}{n_{FL}} (E_{AFL} + \frac{n_{\text{new}}}{n_{FL}} I_{FL} R_A) \]

\[ = \frac{n_{\text{new}}}{n_{FL}} (E_{AFL} + \frac{P_{\text{conv}}}{E_A} R_A) \]

\[ \text{same as above} \]

\[ \tau_{\text{load}} = \frac{n_{\text{new}}}{n_{FL}} \tau_{\text{loadFL}} \]

\[ \tau_{\text{loss}} = \frac{n_{\text{new}}}{n_{FL}} \tau_{\text{lossFL}} = \frac{P_{\text{rotFL}}}{\omega_{mFL}} \]

\[ \tau_{\text{ind}} = \frac{n_{\text{new}}}{n_{FL}} \tau_{\text{loadFL}} + \frac{n_{\text{new}}}{n_{FL}} \tau_{\text{lossFL}} = K \phi I_A \]

\[ I_A = \frac{n_{\text{new}}}{n_{FL}} \tau_{\text{loadFL}} + \frac{n_{\text{new}}}{n_{FL}} \tau_{\text{lossFL}} = \frac{n_{\text{new}}}{n_{FL}} I_{\text{loadFL}} + I_{\text{AlossFL}} \]

the current can be thought of as composed of two parts

\[ I_{\text{AloadFL}} := \frac{P_{\text{outFL}}}{E_{AFL}} \]

\[ I_{\text{AloadFL}} = 20.947 \cdot A \]

\[ I_{\text{AlossFL}} := \frac{P_{\text{rotFL}}}{E_{AFL}} \]

\[ I_{\text{AlossFL}} = 1.053 \cdot A \]

\[ V_T = E_A + I_A R_A = \frac{n_{\text{new}}}{n_{FL}} E_{AFL} + \left( \frac{n_{\text{new}}}{n_{FL}} I_{\text{AloadFL}} + I_{\text{AlossFL}} \right) R_A \]

\[ = \frac{n_{\text{new}}}{n_{FL}} E_{AFL} + \frac{n_{\text{new}}}{n_{FL}} I_{\text{AloadFL}} R_A + I_{\text{AlossFL}} R_A \]

\[ V_T - I_{\text{AlossFL}} R_A = \frac{n_{\text{new}}}{n_{FL}} (E_{AFL} + I_{\text{AloadFL}} R_A) \]

\[ n_{\text{new}} = \frac{V_T - I_{\text{AlossFL}} R_A}{E_{AFL} + I_{\text{AloadFL}} R_A} n_{FL} = 596.8 \text{rpm} \]