Transmission Line Faults

c A.Stolp 12/9/08,
rev 12/26/22, 12/10/23

A fault is an undesired current flow (short circuit) or interruption of flow (open).
These faults may also occur in substations or distribution systems, not just transmission lines.

Importance of Fault Analysis

The power company must to be able to predict the effects of faults without using experimentation (intentional faults are usually frowned upon). Reasons:

1. Setup of fault detection and protection devices.
2. Design adequate robustness in the system and devices to avoid permanent damage when faults occur.
3. Faults instigate transient instabilities in the system -- a whole other important area of study.

Transient or Permanent Faults and Reclosures

Transient Faults

Most faults are short-duration short circuits cause by tree branches, birds or mylar balloons. All of these would be blown to smithereens by the large short-circuit current and not cause permanent damage, at least to the power system.

Sometimes a short-duration short circuit or a lightning strike can establish an arc from line to line or from a line to the tower (flashover). An arc through the air ionizes the air and makes it a conductor. Once the air is ionized, the arc typically continues until the power is shut off for a short time. Relays at both ends of a transmission lines should detect this short and cut the power using large circuit breakers. It is common for these relays to automatically reclose the breakers after a short time (1/3 second or so) to see if the problem has cleared. You have almost certainly experienced these very short power outages that make the lights flicker and alarms and microwaves go "beep".

Permanent Faults

If a reclosure relay makes one or more attempts to re-energize the line and the fault persists, then it opens the circuit breakers "permanently", or rather, until the fault has been found and repaired by a repair crew. Then, after safety protocols have been met, an operator will reclose the circuit breaker(s).

Local distribution lines are usually protected by fuses rather than expensive reclosure devices, so even transient faults in local systems can result in longer-term outages.

Types of Transient or Permanent Faults

Short circuits (Listed from most common to least common)

1. Single line to ground (70 - 80% of all faults) SLG
2. Line to line (ground not involved) LL
3. Double line to ground DLG
4. Balanced three lines to ground (Symmetric 3-line fault) 3P

Shorts may have zero impedance (called "bolted" or "solid") or non-zero fault impedance, $\mathbf{Z_f}$

Open circuits Open circuits can involve 1, 2, or all 3 lines

Any fault except the balanced 3-phase short or 3 open lines is called "unsymmetrical" and will result in unbalanced currents and voltages. Therefore we need a way to deal with unbalanced systems. Symmetrical faults can be analyzed in the same way as unsymmetrical faults, they're just simpler. So we'll jump straight to unbalanced systems.

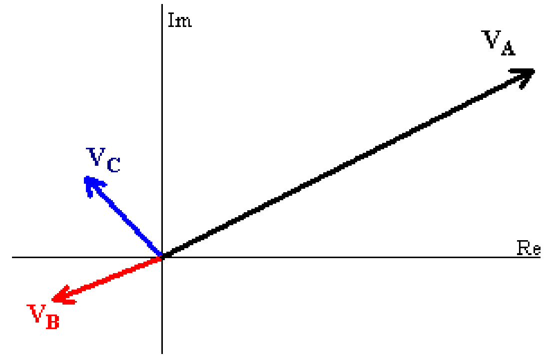
A generator, transformer or transmission line tripping off line (breakers disconnecting an item from the grid) could be analyzed as a symmetrical 3-phase fault.

Some terms

Balanced: Analysis of one phase is enough. Voltages and currents on the other two phases are the same magnitudes, just $\pm 120^o$ different in phase angle.

Unbalanced: Voltages and currents on the three phases are all different magnitudes and phase angles. Connections between the phases, like neutral and Δ-connected devices make standard circuit analysis very difficult. Impedances that change under unbalanced conditions make standard circuit analysis impossible. The trick here is to represent the unbalanced system with three symmetrical systems using the principal of superposition. Then you can add the results found for the three symmetrical components to solve the unbalanced system.

Symmetrical component: Except for known phase-angle differences (usually $\pm 120^\circ$), the same thing is happening on all three phases in any symmetrical component. For each component, analysis of one phase is enough. The components that we'll use are known as the three "sequences".



Unbalanced Systems

Up to now we've only dealt with balanced systems. We could separate out phase A, determine the currents and voltages for phase A, and phases B and C would be the same, $\pm 120^\circ$. Easy-peasy, but it only works for symmetric systems.

The voltages shown here are very unbalanced. $\mathbf{V_A}$, $\mathbf{V_B}$ and $\mathbf{V_C}$ are all different magnitudes and their phase angles don't have any discernible relationship to one another. So, it's also unsymmetric, meaning, you can't just analyze phase A to understand the whole system.

If only we could represent these unbalanced voltages with symmetrical components, we could go back to analyzing just phase A. That is exactly what we'll do.

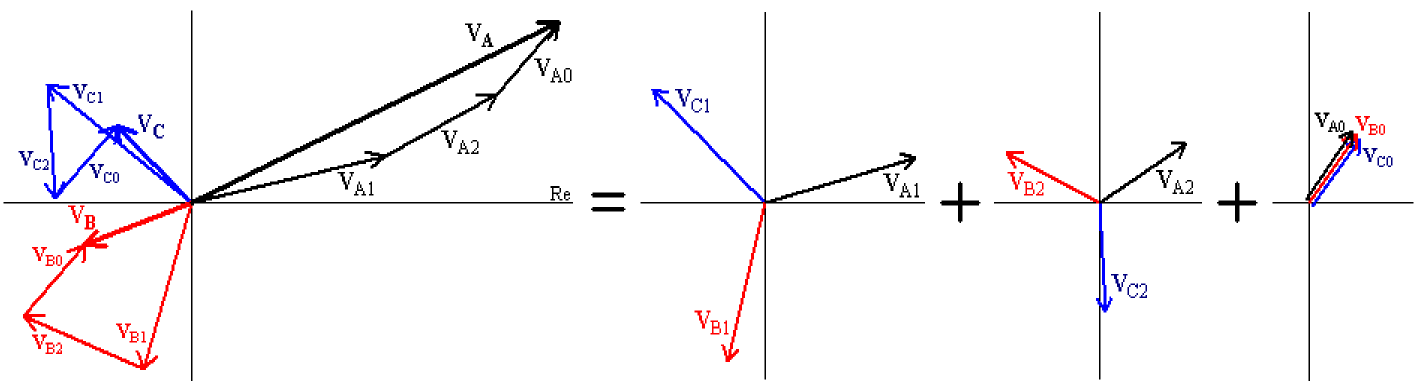
Any unsymmetrical set of three-phase voltages or currents can be broken down into three symmetrical sets of three-phase components. The voltages and currents of those three symmetrical components can then be found separately and finally added together like superposition.

However, it's a weird form of superposition. Superposition is usually used in circuits with multiple sources. This is different. Here there's still only one source per generator for phase A, but, the circuits are different for the different symmetrical components. Some knowledge about the voltage and current relationships at the fault will tell us how to add those circuits up. If voltages need to be added and the currents are equal, the circuits are placed in series. If currents need to be added and the voltages are equal, the circuits are placed in parallel. Or some mix of these.

Yeah, it's weird, but it's also not that terribly important that you fully understand how to assemble the individual circuits. It's already been done for all of the major fault types on generalized circuits. Mostly, you'll have to fit your case to the generalized case and find your specific impedances. (Plenty hard enough.)

Expression of Unbalanced 3-phase voltages and currents

Any three voltages can be expressed as a sum of a positive sequence, a negative sequence, a zero sequence.



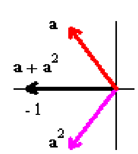
Any three voltages — Can be represented by: — Positive sequence (normal sequence) (a-b-c) — Negative sequence (reverse sequence) (a-c-b) — Zero sequence

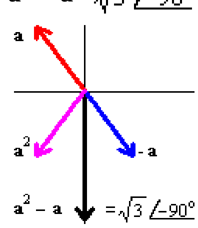
Same sequences for currents

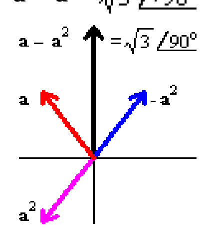
Positive sequence	Negative sequence	Zero sequence
$\mathbf{V}_{B1} = \mathbf{V}_{A1}1\angle{-120^\circ} = \mathbf{V}_{A1}1\angle{240^\circ}$	$\mathbf{V}_{B2} = \mathbf{V}_{A2}1\angle{120^\circ}$	$\mathbf{V}_{B0} = \mathbf{V}_{A0}$
$\mathbf{V}_{C1} = \mathbf{V}_{A1}1\angle{120^\circ}$	$\mathbf{V}_{C2} = \mathbf{V}_{A2}1\angle{-120^\circ} = \mathbf{V}_{A2}1\angle{240^\circ}$	$\mathbf{V}_{C0} = \mathbf{V}_{A0}$

Define: $\mathbf{a} = 1\angle{120^\circ}$ so $\mathbf{a}^2 = 1\angle{240^\circ}$ and $\mathbf{a}^3 = 1$

$\mathbf{a} + \mathbf{a}^2 = -1$ $\qquad$ $\mathbf{a}^2 - \mathbf{a} = \sqrt{3}\angle{-90^\circ}$ $\qquad$ $\mathbf{a} - \mathbf{a}^2 = \sqrt{3}\angle{+90^\circ}$







Which means that: $\mathbf{V}_{B1} = \mathbf{a}^2 \cdot \mathbf{V}_{A1}$ and $\mathbf{V}_{B2} = \mathbf{a} \cdot \mathbf{V}_{A2}$

$\mathbf{V}_{C1} = \mathbf{a} \cdot \mathbf{V}_{A1}$ $\qquad$ $\mathbf{V}_{C2} = \mathbf{a}^2 \cdot \mathbf{V}_{A2}$

We want to find the magnitudes (& angles) of our three sequences such that:

$$\mathbf{V_A} = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0}$$

$$\mathbf{V_B} = \mathbf{V}_{B1} + \mathbf{V}_{B2} + \mathbf{V}_{B0} = \mathbf{a}^2 \cdot \mathbf{V}_{A1} + \mathbf{a} \cdot \mathbf{V}_{A2} + \mathbf{V}_{A0}$$

$$\mathbf{V_C} = \mathbf{V}_{C1} + \mathbf{V}_{C2} + \mathbf{V}_{C0} = \mathbf{a} \cdot \mathbf{V}_{A1} + \mathbf{a}^2 \cdot \mathbf{V}_{A2} + \mathbf{V}_{A0}$$

$$\begin{bmatrix} \mathbf{V_A} \\ \mathbf{V_B} \\ \mathbf{V_C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V}_{A0} \\ \mathbf{V}_{A1} \\ \mathbf{V}_{A2} \end{bmatrix}$$

<u>Please note a source of confusion</u> The order of the sequences is 0,1,2 in the matrices, but 1,2,0 in the drawings & some equations.

Invert matrix to find components:

$$\begin{bmatrix} \mathbf{V}_{A0} \\ \mathbf{V}_{A1} \\ \mathbf{V}_{A2} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V_A} \\ \mathbf{V_B} \\ \mathbf{V_C} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 120^o & 240^o \\ 1 & 240^o & 120^o \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V_A} \\ \mathbf{V_B} \\ \mathbf{V_C} \end{bmatrix}$$

Input any three complex voltages

To get the magnitudes and angles of the three sequences which will add up to get those three voltages.

Same goes for currents.

$$\begin{bmatrix} \mathbf{I_A} \\ \mathbf{I_B} \\ \mathbf{I_C} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I}_{A0} \\ \mathbf{I}_{A1} \\ \mathbf{I}_{A2} \end{bmatrix} \quad \& \quad \begin{bmatrix} \mathbf{I}_{A0} \\ \mathbf{I}_{A1} \\ \mathbf{I}_{A2} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{I_A} \\ \mathbf{I_B} \\ \mathbf{I_C} \end{bmatrix}$$

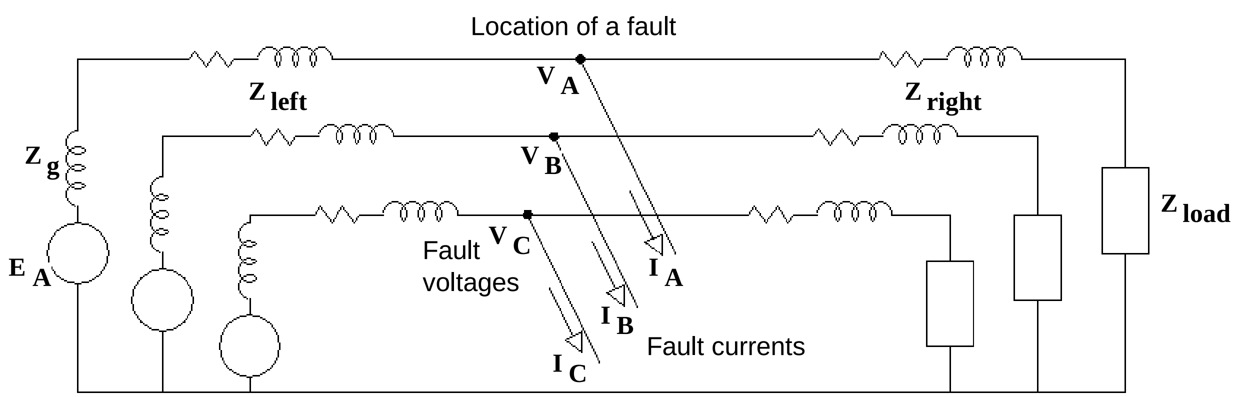
Now we can use these to analyze faults

<u>Some Assumptions about Faults</u>

1. The system was balanced before the fault, so negative- and zero-sequence voltages and currents were all zero.
2. There is only one fault location at a time. This is the location where the negative- and zero-sequence voltages and currents interact.
3. Generators (and other large 50+ hp rotating machines) should be represented by their subtransient synchronous reactances ($\mathbf{Z''_g}$ or $\mathbf{X''_g}$) and a generated emf that would have resulted in the pre-fault voltages and currents with that special reactance. It's actually even more complicated than that. There's even a DC current component. Refer you to chapter 12 of the Chapman text if you want to know more. The added complexities are beyond the scope of this class.

<u>Our Generalized Circuit</u>

A basic 3-phase system separated on a per-phase basis. We'll apply a fault at the center of our system. The way it's drawn here is good for short-circuit faults. We'll draw it a little differently later for open-circuit faults.



Now, a natural question at this point should be: "Why not just do conventional superposition using the circuit above and the three sources?" The problem is that the circuit above is based on assumptions of a steady-state, balanced system and are no longer valid when we suddenly unbalance the system with an unsymmetric fault.

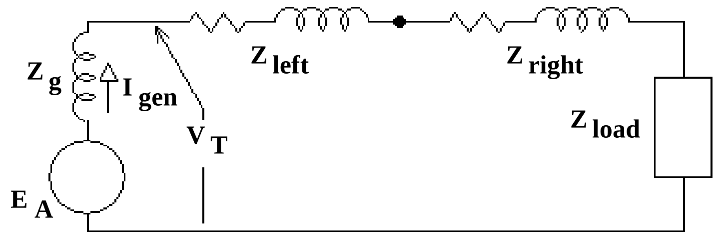
Some problematic assumptions in the circuit above:

1. Δ-connected transformers and loads can be replaced with Y-connected equivalents -- no longer true!
2. Y-connected transformers and loads don't need neutral connections because the neutral current is zero -- no longer true!
3. Neutral connections that do exist carry little or no current, so impedances in the neutral path can be neglected -- no longer true!
4. Impedances are the same in transient and/or unbalanced conditions as they are for steady-state, balanced conditions -- no longer true, especially for generators and motors. In fact, even the EMF source in generators ($\mathbf{E_A}$) need to be adjusted for transient conditions.

So, first let's do some pre-fault analysis while the circuit above is still valid. Essentially we want to replace the synchronous impedance of the generator with it's subtransient equivalent and adjust the generator EMF ($\mathbf{E_A}$) to still get the same terminal voltage ($\mathbf{V_\phi}$ or $\mathbf{V_T}$). (See ch.12 of your Chapman textbook for an explanation the subtransient.) The new $\mathbf{E_A}$ will be called $\mathbf{E''_A}$.

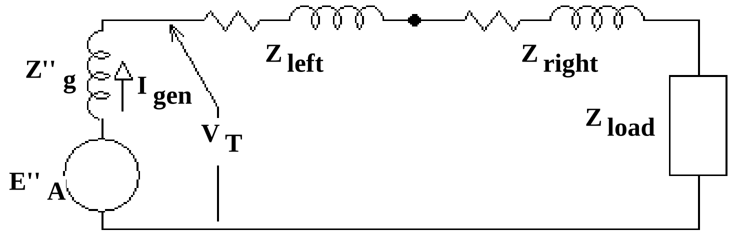
Pre-Fault Setup

Draw the per-phase drawing of phase A, pre-fault.



Find the terminal voltage of the generator ($\mathbf{V_T}$) and the generator current ($\mathbf{I_{gen}}$).

Now replace $\mathbf{Z_g}$ (or $\mathbf{X_g}$) with the subtransient impedance of the generator ($\mathbf{Z''_g}$ or $\mathbf{X''_g}$)



Find the value of $\mathbf{E''_A}$ that will result in the same $\mathbf{V_T}$ and $\mathbf{I_{gen}}$, usually like this:

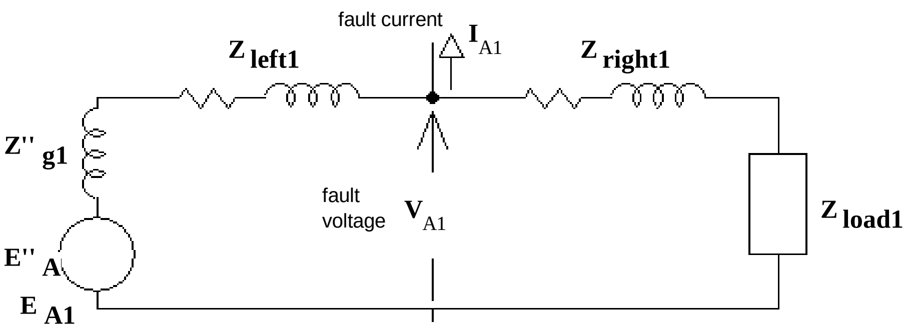
$$E''_A = V_T - I_{gen} \cdot j \cdot X''_g$$

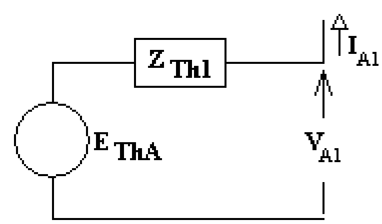
OR $$E''_A = V_T - I_{gen} \cdot Z''_g$$

Phase A of the Generalized Circuit Represented as 3 Circuits for the 3 Sequences

Sequence-components per-phase drawings of phase A

Thévenin equivalents:
helpful for the short-circuit faults



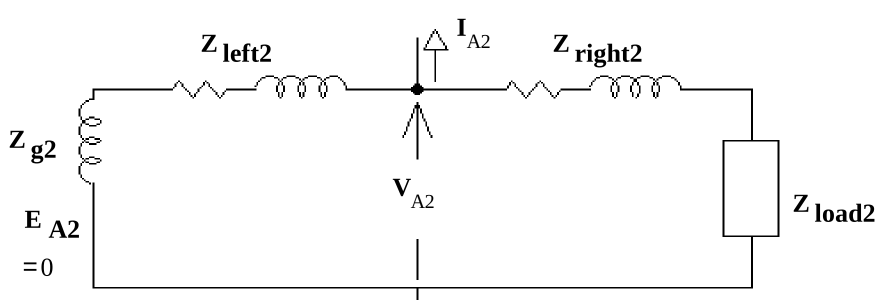


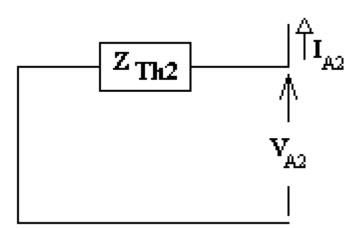
$$E_{ThA} = E_{A1} \cdot \frac{Z_{right1} + Z_{load1}}{Z''_{g1} + Z_{left1} + Z_{right1} + Z_{load1}}$$

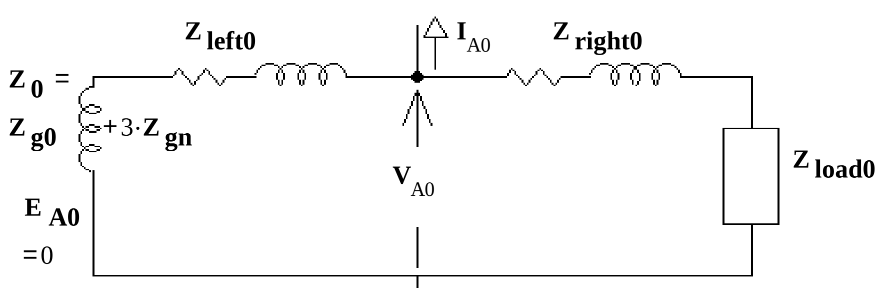
$$Z_{Th1} = (Z''_{g1} + Z_{left1}) \,||\, (Z_{right1} + Z_{load1})$$

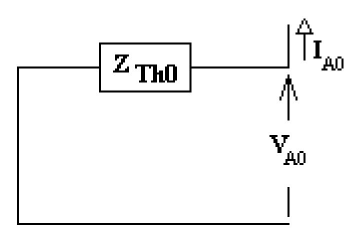
$$Z_{Th2} = (Z_{g2} + Z_{left2}) \,||\, (Z_{right2} + Z_{load2})$$

$$Z_{Th0} = (Z_{g0} + Z_{left0}) \,||\, (Z_{right0} + Z_{load0})$$





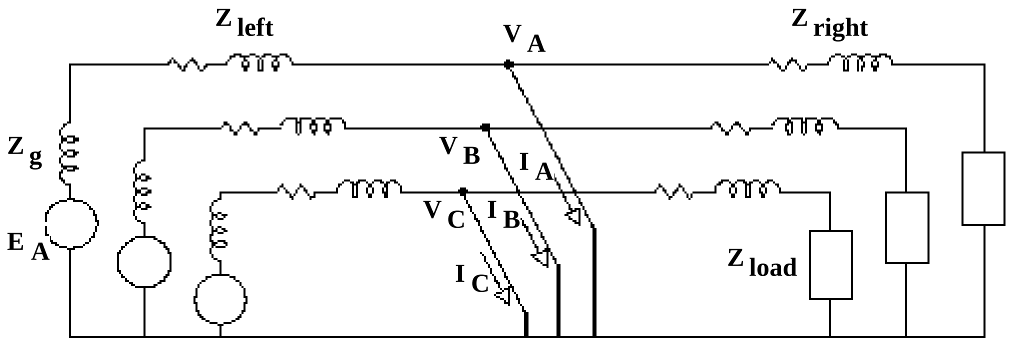




You will notice that all the impedances in the circuits are different for the different sequences. Except for the generator and motor impedances, the values are almost always the same for the positive- and negative- sequences. The zero-sequence impedances are often quite different. This is covered well in section 13.3 (p.599) in your textbook and later in these notes.

Symmetrical 3-phase to ground fault

(2 - 3% of faults)



Note:

$\mathbf{a}^3 = 1$

$\mathbf{a}^4 = \mathbf{a}$

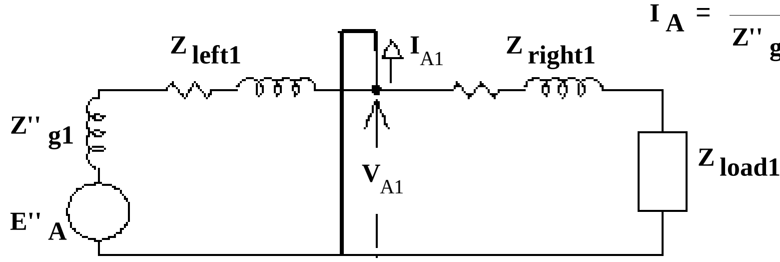
$$\begin{bmatrix} \mathbf{I}_{A0} \\ \mathbf{I}_{A1} \\ \mathbf{I}_{A2} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{I}_A \\ 0 \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} \mathbf{I}_A + \mathbf{I}_A\cdot 240^o + \mathbf{I}_A\cdot 120^o \\ \mathbf{I}_A + \mathbf{I}_A\cdot 360^o + \mathbf{I}_A\cdot 360^o \\ \mathbf{I}_A\cdot 120^o + \mathbf{I}_A\cdot 360^o + \mathbf{I}_A\cdot 240^o \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\cdot\begin{bmatrix} \mathbf{I}_A \\ \mathbf{a}^2\cdot\mathbf{I}_A \\ \mathbf{a}\cdot\mathbf{I}_A \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & 120^o & 240^o \\ 1 & 240^o & 120^o \end{bmatrix}\cdot\begin{bmatrix} \mathbf{I}_A \\ \mathbf{I}_A\cdot 240^o \\ \mathbf{I}_A\cdot 120^o \end{bmatrix}$$

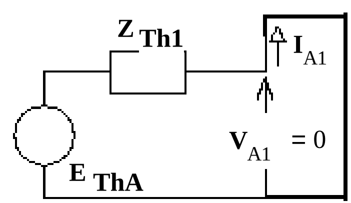
$$\begin{bmatrix} \mathbf{V}_{A0} \\ \mathbf{V}_{A1} \\ \mathbf{V}_{A2} \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\cdot\begin{bmatrix} \mathbf{V}_A \\ \mathbf{V}_B \\ \mathbf{V}_C \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & 120^o & 240^o \\ 1 & 240^o & 120^o \end{bmatrix}\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

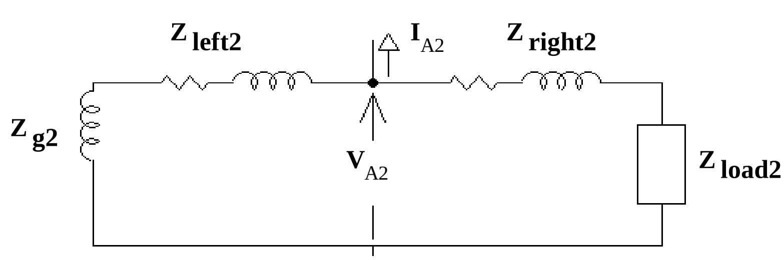
Sequence-components per-phase drawings of phase A

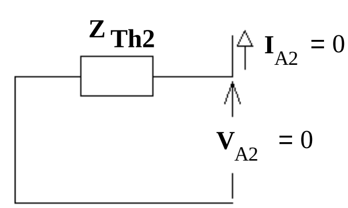
Thévenin equivalents:

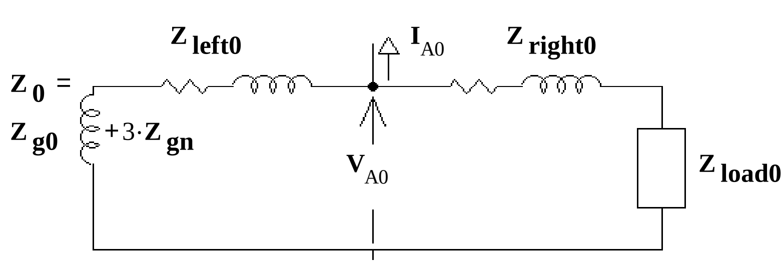
$$\mathbf{I}_A = \frac{\mathbf{E''}_A}{\mathbf{Z''}_g + \mathbf{Z}_{left1}} \qquad \text{OR} \qquad \mathbf{I}_A = \frac{\mathbf{E}_{ThA}}{\mathbf{Z}_{Th1}}$$

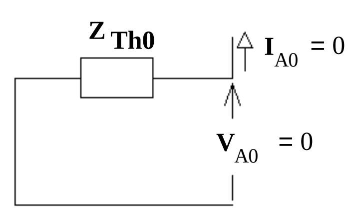








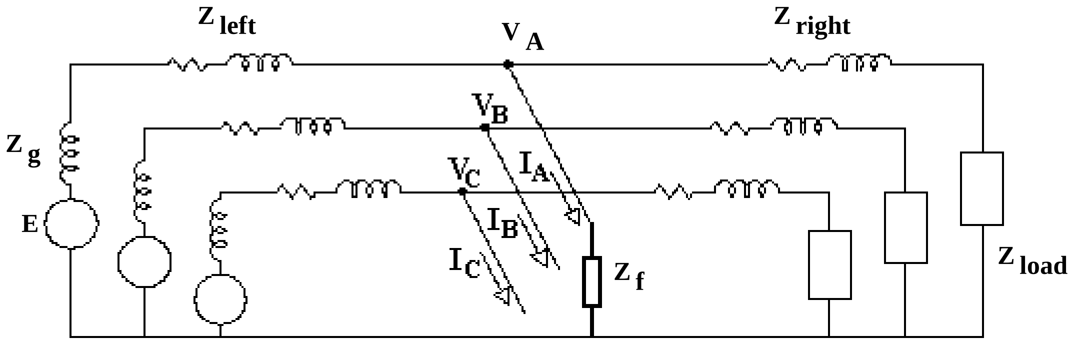




Notice that the negative- and zero-sequences are NOT involved.
Exactly what you should expect with a balanced, 3-phase fault.

Single line to ground fault

(70 - 80% of faults)



$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \begin{bmatrix} \frac{I_A}{3} \\ \frac{I_A}{3} \\ \frac{I_A}{3} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} I_A \\ 0 \\ 0 \end{bmatrix}$$

From circuit above: $I_A = \dfrac{V_A}{Z_f}$

from basic relations (p.3) $V_A = V_{A0} + V_{A1} + V_{A2}$

$$I_A = \frac{V_{A0} + V_{A1} + V_{A2}}{Z_f}$$

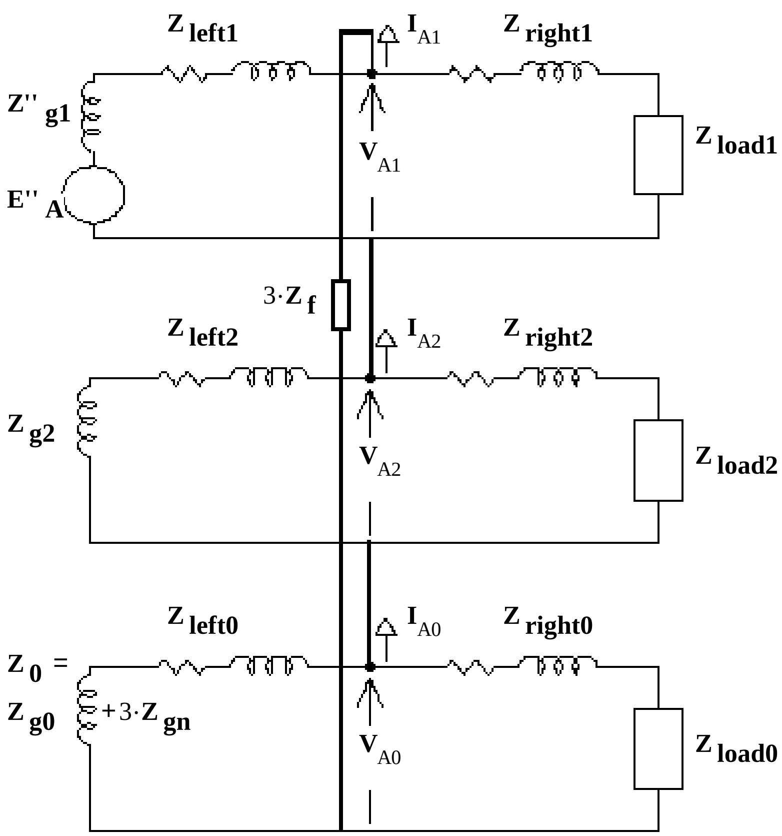
$I_{A0} = I_{A1} = I_{A2}$ implies a series connection

also: $I_{A1} = \dfrac{I_A}{3} = \dfrac{V_A}{3 \cdot Z_f}$

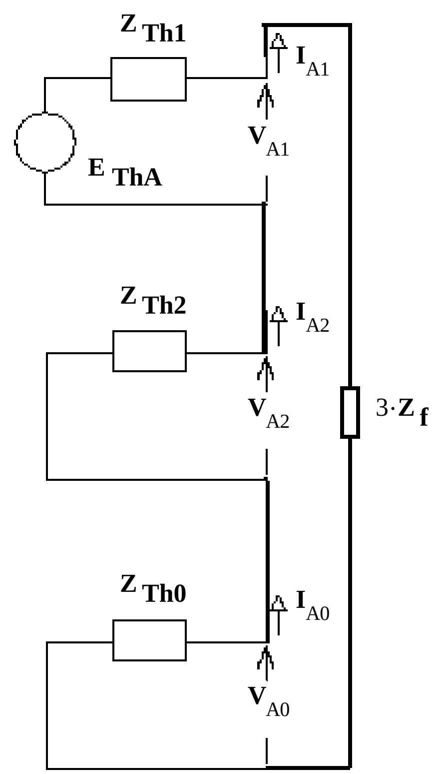
Voltage matrix eq. isn't helpful

$I_{A1} = \dfrac{V_{A0} + V_{A1} + V_{A2}}{3 \cdot Z_f}$ gives location and multiplier for Z_f

Sequence-components per-phase drawings of phase A

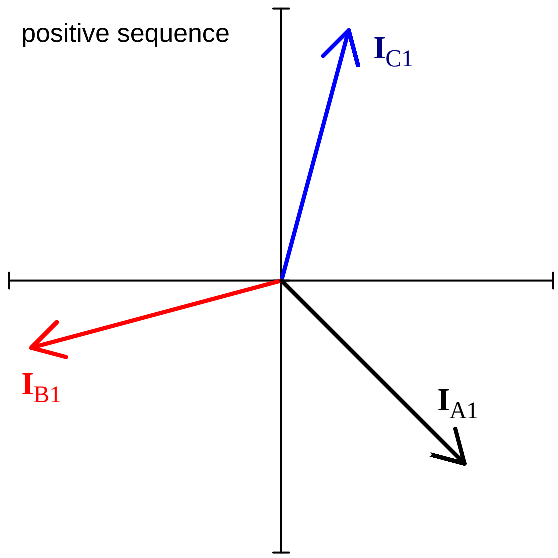


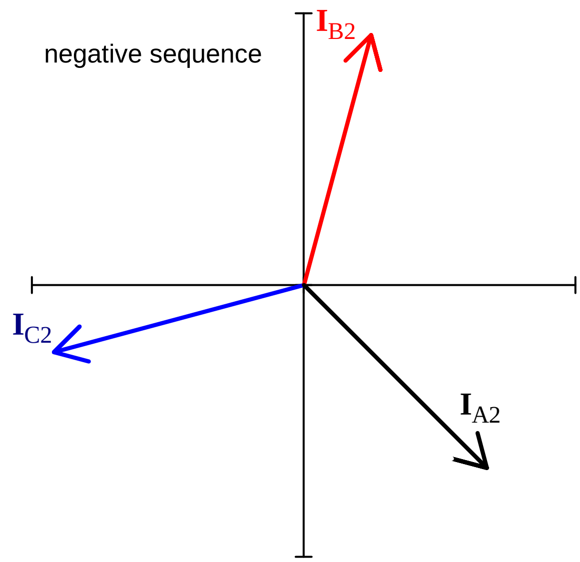
<u>Thévenin equivalents:</u>

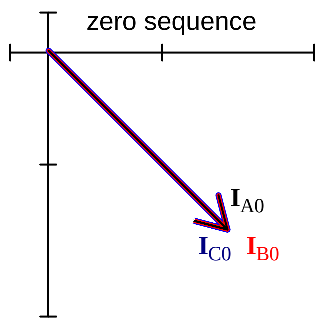


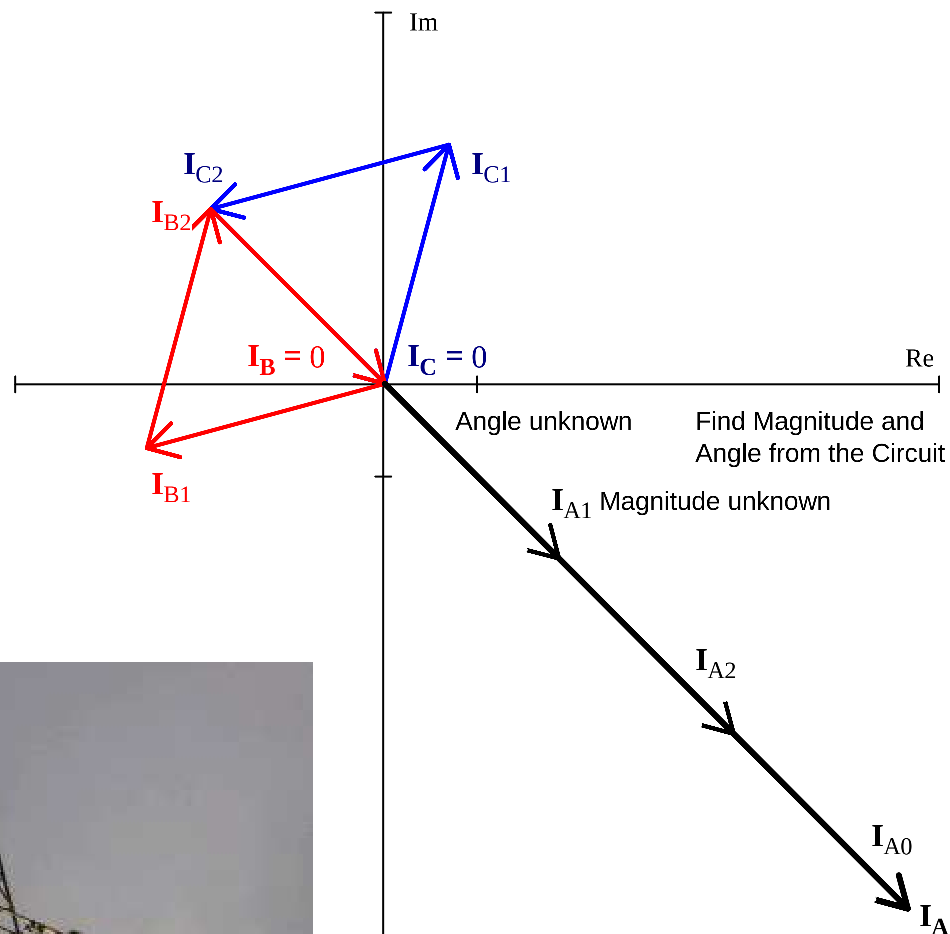
Find $I_{A1} = \dfrac{E_{ThA}}{Z_{Th1} + Z_{Th0} + Z_{Th2} + 3 \cdot Z_f}$

Find $\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} -I_{A1} \cdot Z_{Th0} \\ E_{ThA} - I_{A1} \cdot Z_{Th1} \\ -I_{A1} \cdot Z_{Th2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix}$


positive sequence
I_{C1}
I_{B1}
I_{A1}

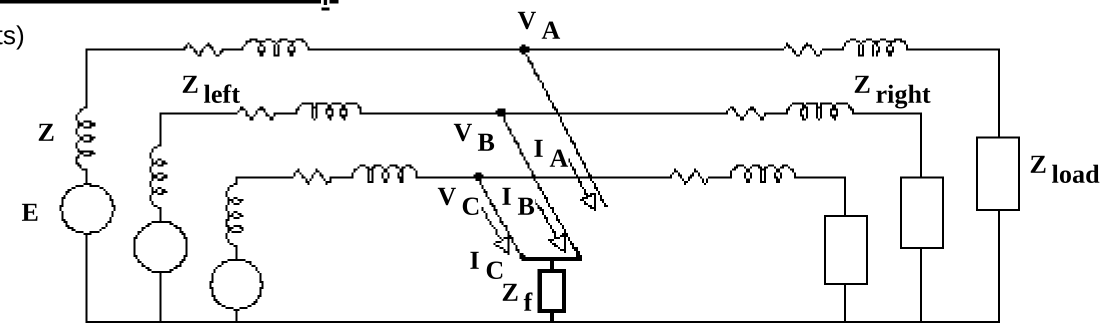

negative sequence
I_{B2}
I_{C2}
I_{A2}


zero sequence
I_{A0}
I_{C0} I_{B0}


Im
I_{C2}
I_{C1}
I_{B2}
$I_B = 0$
$I_C = 0$
Re
Angle unknown
Find Magnitude and
Angle from the Circuit
I_{B1}
I_{A1} Magnitude unknown
I_{A2}
I_{A0}
I_A

Double line to ground fault with Z_f

(5 - 10% of faults)



From circuit above: $I_A = I_{A0} + I_{A1} + I_{A2} = 0$ $\qquad V_B = V_C = (I_B + I_C)\cdot Z_f$

implies that all flow into same node $\qquad (I_B + I_C) = \dfrac{V_B}{Z_f}$

$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} V_A + 2\cdot V_B \\ V_A - V_B \\ V_A - V_B \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} V_A + 2\cdot V_B \\ V_A + (a + a^2)\cdot V_B \\ V_A + (a^2 + a)\cdot V_B \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} V_A + V_B + V_B \\ V_A + a\cdot V_B + a^2\cdot V_B \\ V_A + a^2\cdot V_B + a\cdot V_B \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\cdot\begin{bmatrix} V_A \\ V_B \\ V_B \end{bmatrix} \quad V_C = V_B$$

rows 2 & 3: $V_{A1} = V_{A2}$ connected $\qquad a + a^2 = -1$

$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 0 + I_B + I_C \\ 0 + a\cdot I_B + a^2\cdot I_C \\ 0 + a^2\cdot I_B + a\cdot I_C \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 0 + I_B + I_C \\ 0 + a\cdot I_B + a^2\cdot I_C \\ 0 + a^2\cdot I_B + a\cdot I_C \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\cdot\begin{bmatrix} 0 \\ I_B \\ I_C \end{bmatrix}$$

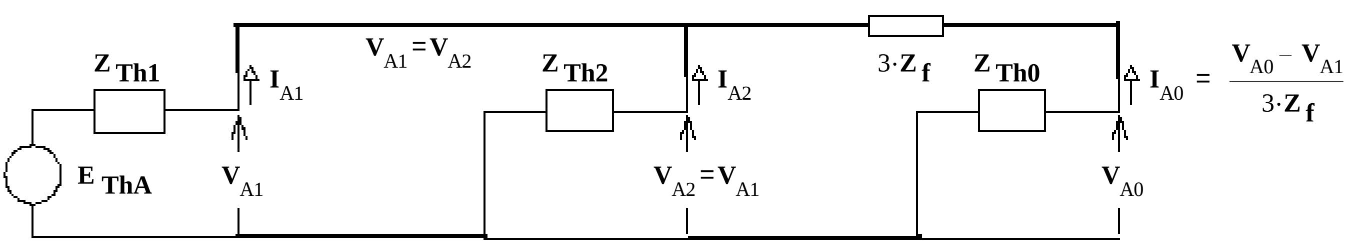
1st row: $3\cdot I_{A0} = I_B + I_C = \dfrac{V_B}{Z_f}$ $\qquad V_B = V_{B0} + V_{B1} + V_{B2} = V_{A0} + a^2\cdot V_{A1} + a\cdot V_{A2}$ from basic relations (p.3)

$$a^2 + a = -1$$

$$3\cdot I_{A0} = \frac{V_{A0} + a^2\cdot V_{A1} + a\cdot V_{A2}}{Z_f} \qquad 3\cdot I_{A0} = \frac{V_{A0} + a^2\cdot V_{A1} + a\cdot V_{A1}}{Z_f} = \frac{V_{A0} + (a^2 + a)\cdot V_{A1}}{Z_f} = \frac{V_{A0} - V_{A1}}{Z_f}$$

$I_{A0} = \dfrac{V_{A0} - V_{A1}}{3\cdot Z_f}$ $\qquad 3\cdot Z_f$ is connected between V_{A0} and V_{A1}

Sequence-components per-phase drawings of phase A Thévenin equivalents:



$I_{A0} + I_{A1} + I_{A2} = 0$ all flow into same node

$$I_{A1} = \frac{E_{ThA}}{Z_{Th1} + \dfrac{1}{\dfrac{1}{Z_{Th2}} + \dfrac{1}{3\cdot Z_f + Z_{Th0}}}} \qquad V_{A1} = I_{A1}\cdot\left(\frac{1}{\dfrac{1}{Z_{Th2}} + \dfrac{1}{3\cdot Z_f + Z_{Th0}}}\right) \qquad I_{A2} = -\frac{V_{A1}}{Z_{Th2}}$$

$$I_{A0} = -\frac{V_{A1}}{3\cdot Z_f + Z_{Th0}}$$

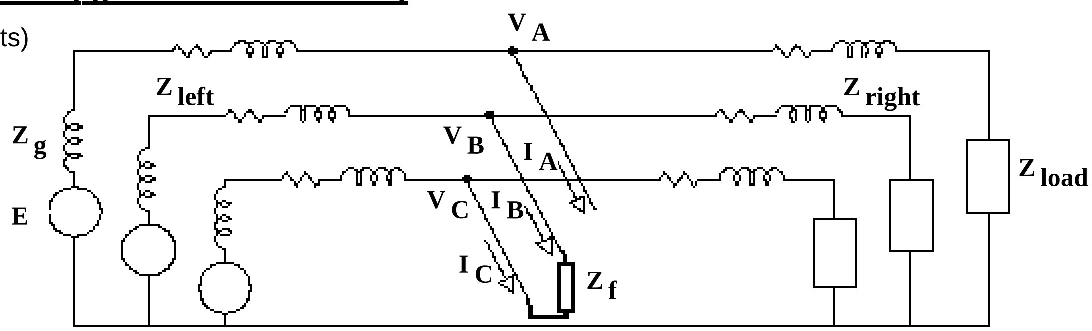
Find: $I_B = I_{A0} + a^2\cdot I_{A1} + a\cdot I_{A2}$

$I_C = I_{A0} + a\cdot I_{A1} + a\cdot I_{A2}$ $\qquad V_B = V_C = (I_B + I_C)\cdot Z_f$

$V_A = V_{A0} + V_{A1} + V_{A2} = 2\cdot V_{A1} + I_{A1}\cdot Z_{Th0}$

Line-to-Line fault (ground not involved)

(15 - 20% of faults)



$$\begin{bmatrix} I_{A0}\\ I_{A1}\\ I_{A2}\end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix}0\\ j\cdot\sqrt{3}\cdot I_B\\ -j\cdot\sqrt{3}\cdot I_B\end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix}0\\ \left(a-a^2\right)\cdot I_B\\ \left(a^2-a\right)\cdot I_B\end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix}0+I_B-I_B\\ 0+a\cdot I_B-a^2\cdot I_B\\ 0+a^2\cdot I_B-a\cdot I_B\end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix}1&1&1\\1&a&a^2\\1&a^2&a\end{bmatrix}\cdot\begin{bmatrix}0\\ I_B\\ -I_B\end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix}1&1&1\\1&a&a^2\\1&a^2&a\end{bmatrix}\cdot\begin{bmatrix}I_A\\ I_B\\ I_C\end{bmatrix}$$

$I_{A0} = 0$ & $I_{A1} = -I_{A2}$ connected so that both flow into same node

not connected to anything

$$I_B = I_{A0} + a^2\cdot I_{A1} + a\cdot I_{A2} = 0 + a^2\cdot I_{A1} + a\cdot I_{A2} = \left(a^2-a\right)\cdot I_{A1} = -I_{A1}\cdot j\cdot\sqrt{3}$$
$$= \sqrt{3}\ \underline{/-90^o}\ I_{A1}$$

$V_{A0} = 0$ from circuit below

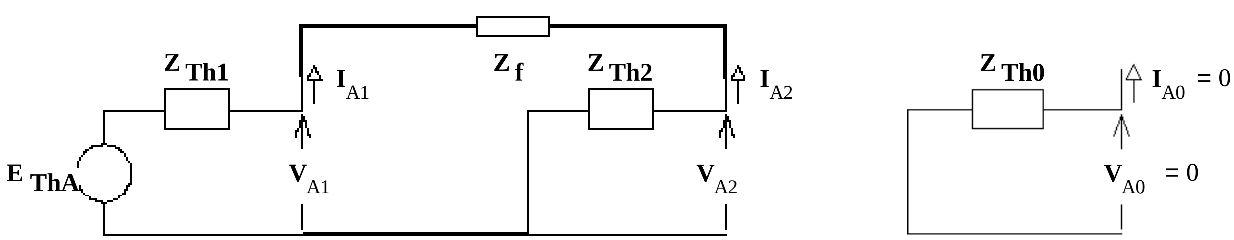
from basic relations (p.3)

$$V_B = V_{A0} + a^2\cdot V_{A1} + a\cdot V_{A2} = 0 + a^2\cdot V_{A1} + a\cdot V_{A2}$$
$$V_C = V_{A0} + a\cdot V_{A1} + a^2\cdot V_{A2} = 0 + a\cdot V_{A1} + a^2\cdot V_{A2}$$

from circuit above: $V_B - V_C = I_B\cdot Z_f$

$$\left(a^2\cdot V_{A1} + a\cdot V_{A2}\right) - \left(a\cdot V_{A1} + a^2\cdot V_{A2}\right) = I_B\cdot Z_f$$
$$\left(a^2-a\right)\cdot V_{A1} - \left(a^2-a\right)\cdot V_{A2} = \left(a^2-a\right)\cdot I_{A1}\cdot Z_f$$
$$V_{A1} - V_{A2} = I_{A1}\cdot Z_f$$ gives location of Z_f

Sequence-components per-phase drawings of phase A Thévenin equivalents:



$$I_{A1} = \frac{E_{ThA}}{Z_{Th1} + Z_f + Z_{Th2}}$$

$$\begin{bmatrix}I_A\\ I_B\\ I_C\end{bmatrix} = \begin{bmatrix}1&1&1\\1&a^2&a\\1&a&a^2\end{bmatrix}\cdot\begin{bmatrix}I_{A0}\\ I_{A1}\\ I_{A2}\end{bmatrix}$$

see page 2

$$\begin{bmatrix}V_A\\ V_B\\ V_C\end{bmatrix} = \begin{bmatrix}1&1&1\\1&a^2&a\\1&a&a^2\end{bmatrix}\cdot\begin{bmatrix}V_{A0}\\ V_{A1}\\ V_{A2}\end{bmatrix}$$

$$I_B = \overset{0}{I_{A0}} + a^2\cdot I_{A1} + a\cdot I_{A2} = a^2\cdot I_{A1} - a\cdot I_{A1} = \left(a^2-a\right)\cdot I_{A1} = -I_{A1}\cdot j\cdot\sqrt{3} = I_{A1}\cdot\sqrt{3}\,\underline{/-90^o}$$

$$I_C = -I_B = I_{A1}\cdot\sqrt{3}\,\underline{/+90^o} = I_{A1}\cdot j\cdot\sqrt{3} = I_{A1}\cdot\sqrt{3}\,\underline{/\ 90^o}$$

$$V_A = 0 + V_{A1} + V_{A2} = I_{A1}\cdot\left(Z_f + 2\cdot Z_{Th2}\right)$$

$a^2 = 1\underline{/-120^o}$ $\left(a^2 + a\right) = -1$

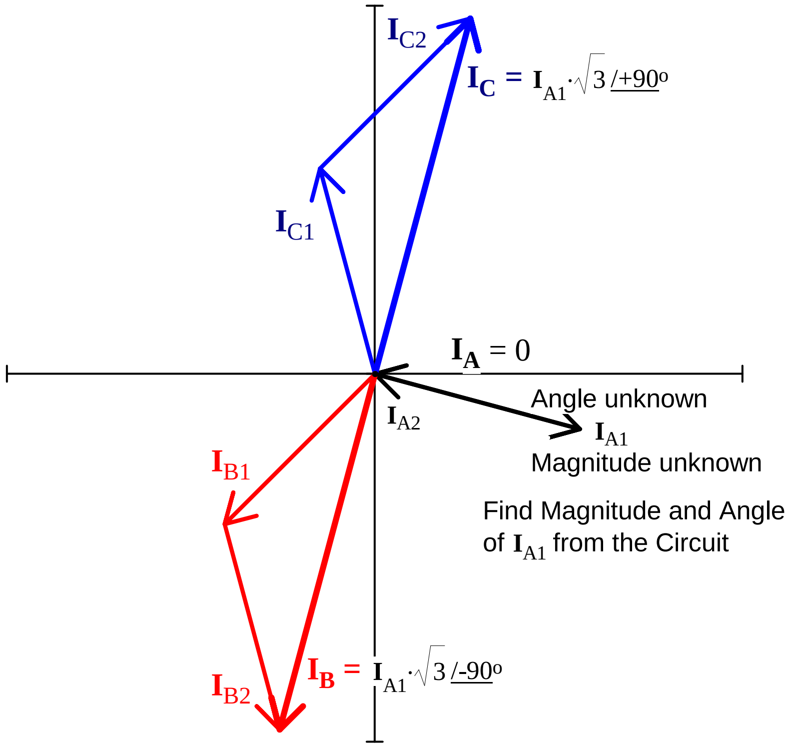
$$V_B = V_{A0} + a^2\cdot V_{A1} + a\cdot V_{A2} = a^2\cdot I_{A1}\cdot\left(Z_f + Z_{Th2}\right) + a\cdot I_{A1}\cdot Z_{Th2} = a^2\cdot I_{A1}\cdot Z_f + \left(a^2+a\right)\cdot I_{A1}\cdot Z_{Th2} = 1\underline{/-120^o}\ I_{A1}Z_f - I_{A1}Z_{Th2}$$

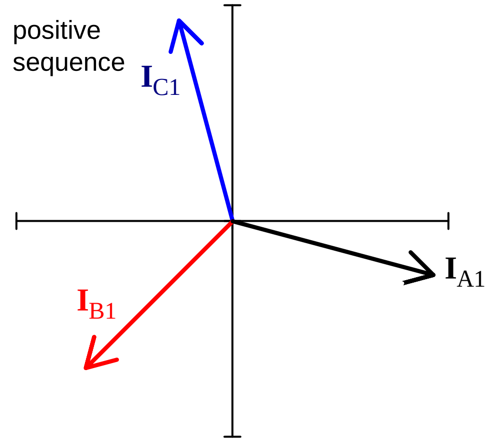
from basic relations (p.3)

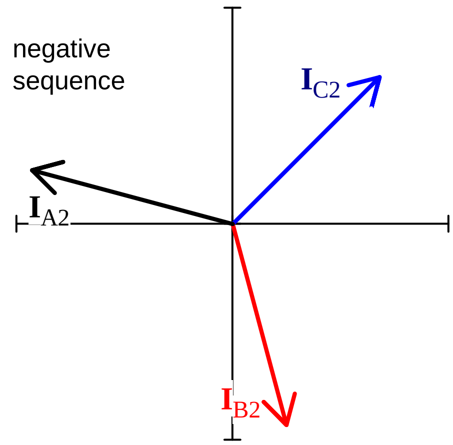
$$V_C = a\cdot I_{A1}\cdot Z_f + \left(a+a^2\right)\cdot I_{A1}\cdot Z_{Th2} = 1\underline{/120^o}\ I_{A1}Z_f - I_{A1}Z_{Th2}$$

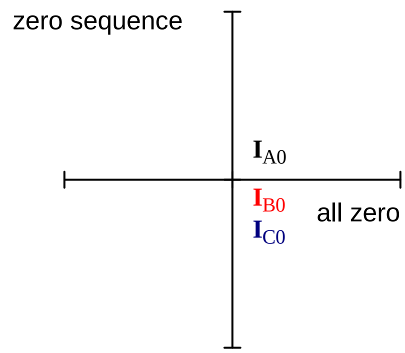
Transmission Line Faults p10

Currents in a Line-to-Line (LL) fault (ground not involved)

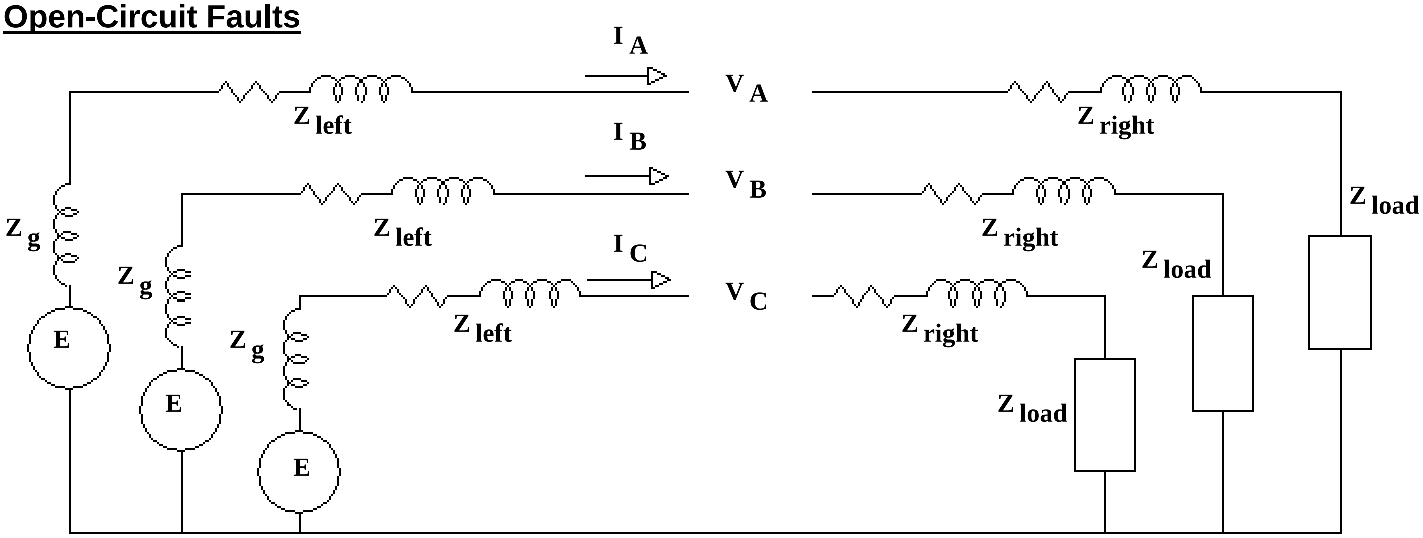






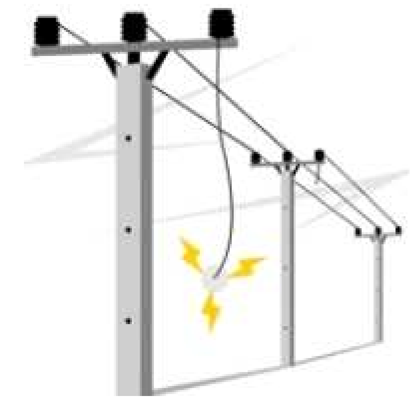


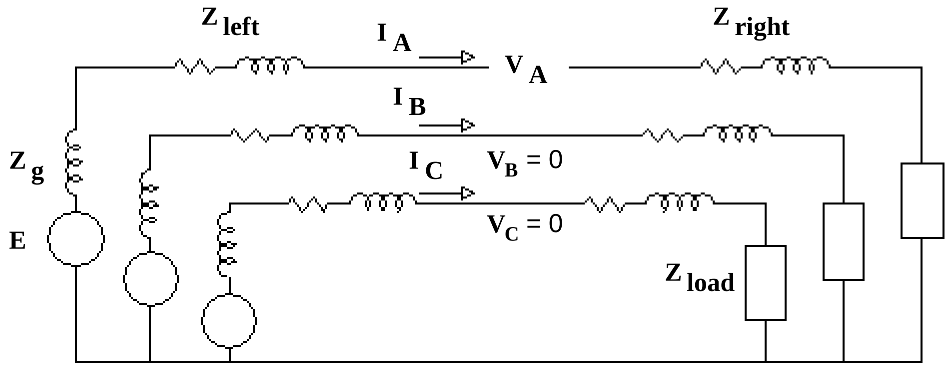
Open-Circuit Faults



Now the voltages are measured across the fault point and the "fault" currents are the line currents.

Single line Open





$I_A = I_{A0} + I_{A1} + I_{A2} = 0$ implies that all flow into same node

NOT HELPFUL

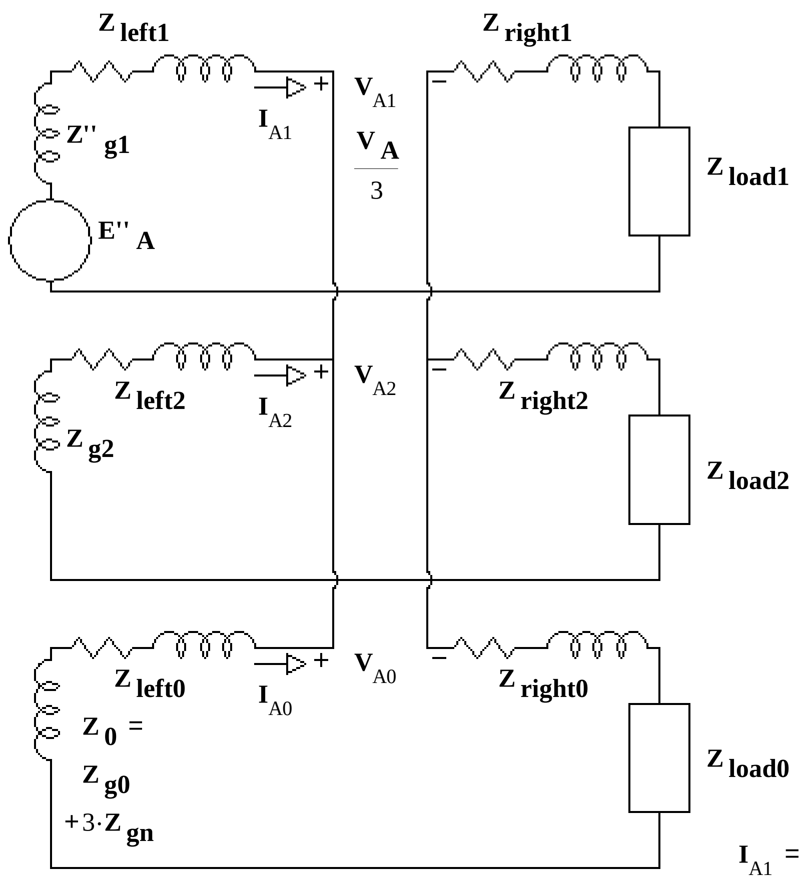
$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} V_A \\ V_A \\ V_A \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} V_A \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} I_B + I_C \\ a \cdot I_B + a^2 \cdot I_C \\ a^2 \cdot I_B + a \cdot I_C \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} 0 \\ I_B \\ I_C \end{bmatrix}$$

$V_{A0} = V_{A1} = V_{A2}$ implies a parallel connection

$V_A = 3 \cdot V_{A1}$

Sequence-components per-phase drawings of phase A



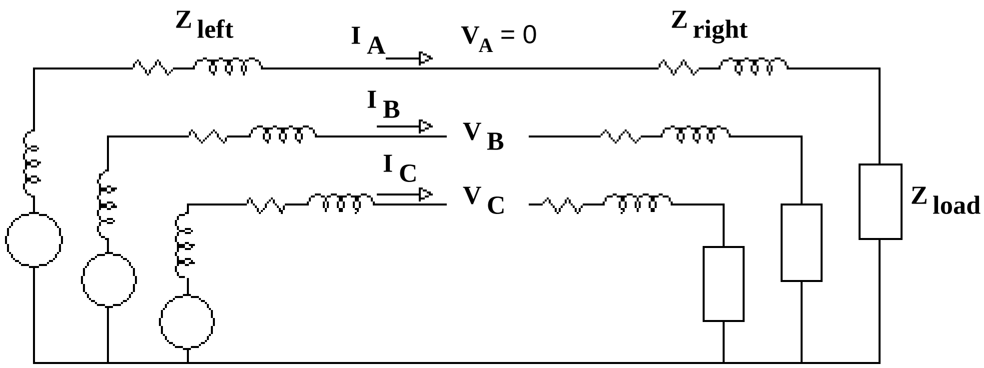
$$Z_{tot2} = Z_{left2} + Z''_{g2} + Z_{load2} + Z_{right2}$$

$$Z_{tot0} = Z_{left0} + Z''_{g0} + Z_{load0} + Z_{right0}$$

$$I_{A1} = \frac{E''_A}{Z''_{g1} + Z_{left1} + \dfrac{1}{\dfrac{1}{Z_{tot2}} + \dfrac{1}{Z_{tot0}}} + Z_{right1} + Z_{load1}}$$

$$V_A = 3 \cdot V_{A1} = 3 \cdot I_{A1} \cdot \left(\frac{1}{\dfrac{1}{Z_{tot2}} + \dfrac{1}{Z_{tot0}}} \right)$$

Double line B & C Open



NOT HELPFUL

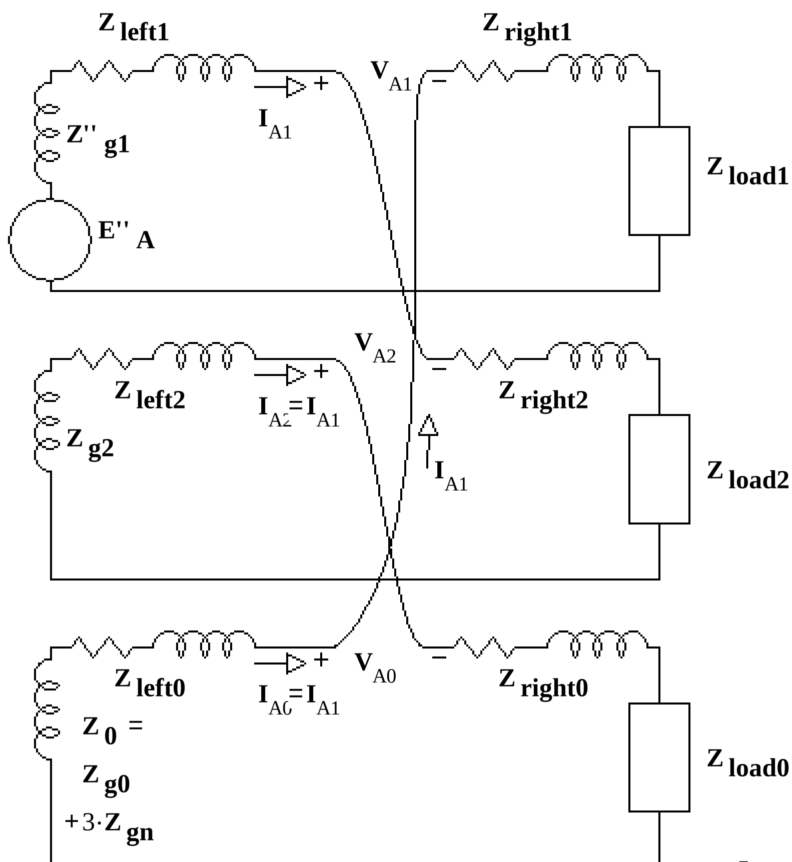
$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} V_B + V_C \\ a\cdot V_B + a^2\cdot V_C \\ a^2\cdot V_B + a\cdot V_C \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\cdot\begin{bmatrix} 0 \\ V_B \\ V_C \end{bmatrix}$$

$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} I_A \\ I_A \\ I_A \end{bmatrix} = \frac{1}{3}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\cdot\begin{bmatrix} I_A \\ 0 \\ 0 \end{bmatrix}$$

$I_{A0} + I_{A1} + I_{A2}$ implies a series connection

from basic relations (p.3) $V_A = V_{A0} + V_{A1} + V_{A2} = 0$ also indicates series connection

Sequence-components per-phase drawings of phase A



$$Z_{tot2} = Z_{left2} + Z''_{g2} + Z_{load2} + Z_{right2}$$

$$Z_{tot0} = Z_{left0} + Z''_{g0} + Z_{load0} + Z_{right0}$$

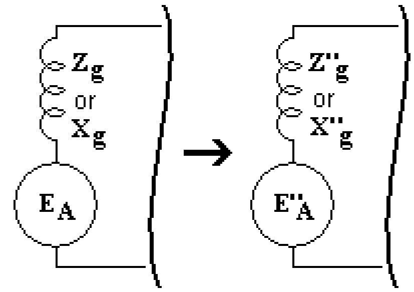
$$I_{A1} = \frac{E''_A}{Z''_{g1} + Z_{left1} + Z_{tot2} + Z_{tot0} + Z_{right1} + Z_{load1}}$$

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\cdot\begin{bmatrix} -\left(I_{A1}\cdot Z_{tot0}\right) \\ I_{A1}\cdot\left(Z_{tot2} + Z_{tot0}\right) \\ -\left(I_{A1}\cdot Z_{tot2}\right) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\cdot\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix}$$

Sequence Impedances (and generated voltages)

Positive-Sequence Impedances (and generated voltages)

Most are exactly the same as what we've been using all along for our balanced analysis except for generators and large motors.



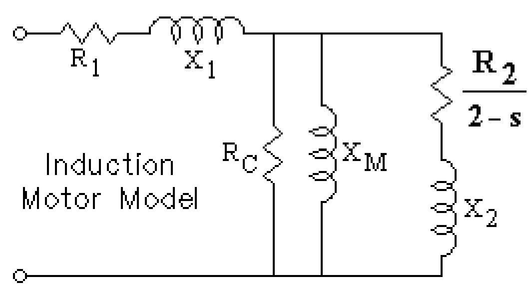
1. Generator reactances should be replaced by their subtransient synchronous reactances ($\mathbf{Z''_g}$ or $\mathbf{X''_g}$). Subtransient reactances are 10% to 25% of the normal synchronous reactance-- so quite a bit less. You will not be expected to find these values in this class.
2. Generator voltages ($\mathbf{E''}$) are found from prefault currents and subtransient synchronous reactances. See the "Pre-Fault Setup" section earlier in these notes.

It's actually even more complicated than that. There's even a DC current component. Refer to chapter 12 of the Chapman text if you want to know more. The added complexities are beyond the scope of this class.

Negative-Sequence Impedances

Most are the same as the positive sequence except for generators and large motors, esp. induction motors.

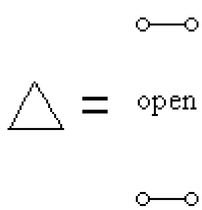
1. There is no negative-sequence generator voltage ($\mathbf{E} = 0$).
2. Negative-sequence generator reactances ($\mathbf{Z_{g2}}$ or $\mathbf{X_{g2}}$) are similar in value to the subtransient synchronous reactances.
3. Induction motors are usually characterized as an impedance which is dependent on the slip. The greater the slip, the smaller the impedance. This is the reason motors have such large startup currents. Now consider that the negative sequence is actually trying to turn the motor in the opposite direction, meaning the slip much greater than 1, resulting in a negative-sequence impedance much, much less than for the positive sequence.



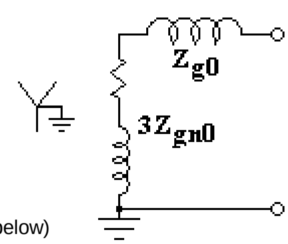
Zero-Sequence Impedances

Most are different.

1. There is no zero-sequence generator voltage ($\mathbf{E} = 0$).
2. Generators
   A. Δ-connected = open. The zero-sequence currents through anything Δ-connected is 0 (all three phases are the same).

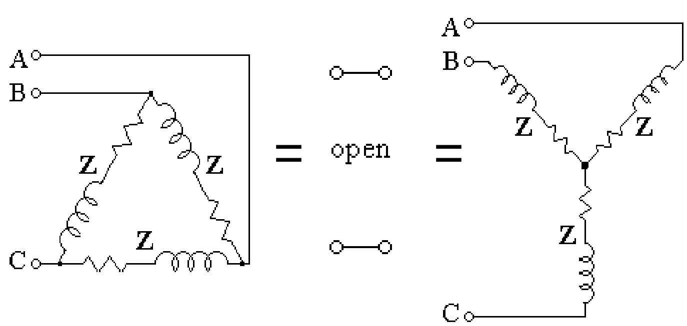


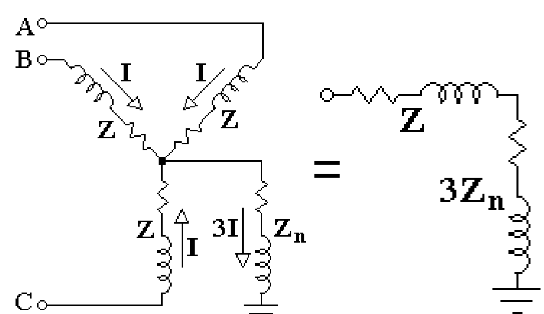
   B. Y-connected, the zero-sequence generator reactances ($\mathbf{Z_{g0}}$ or $\mathbf{Xg0}$, which are about a third of the subtransient synchronous reactances) + 3x the ground connection impedance.



(See load connection below)

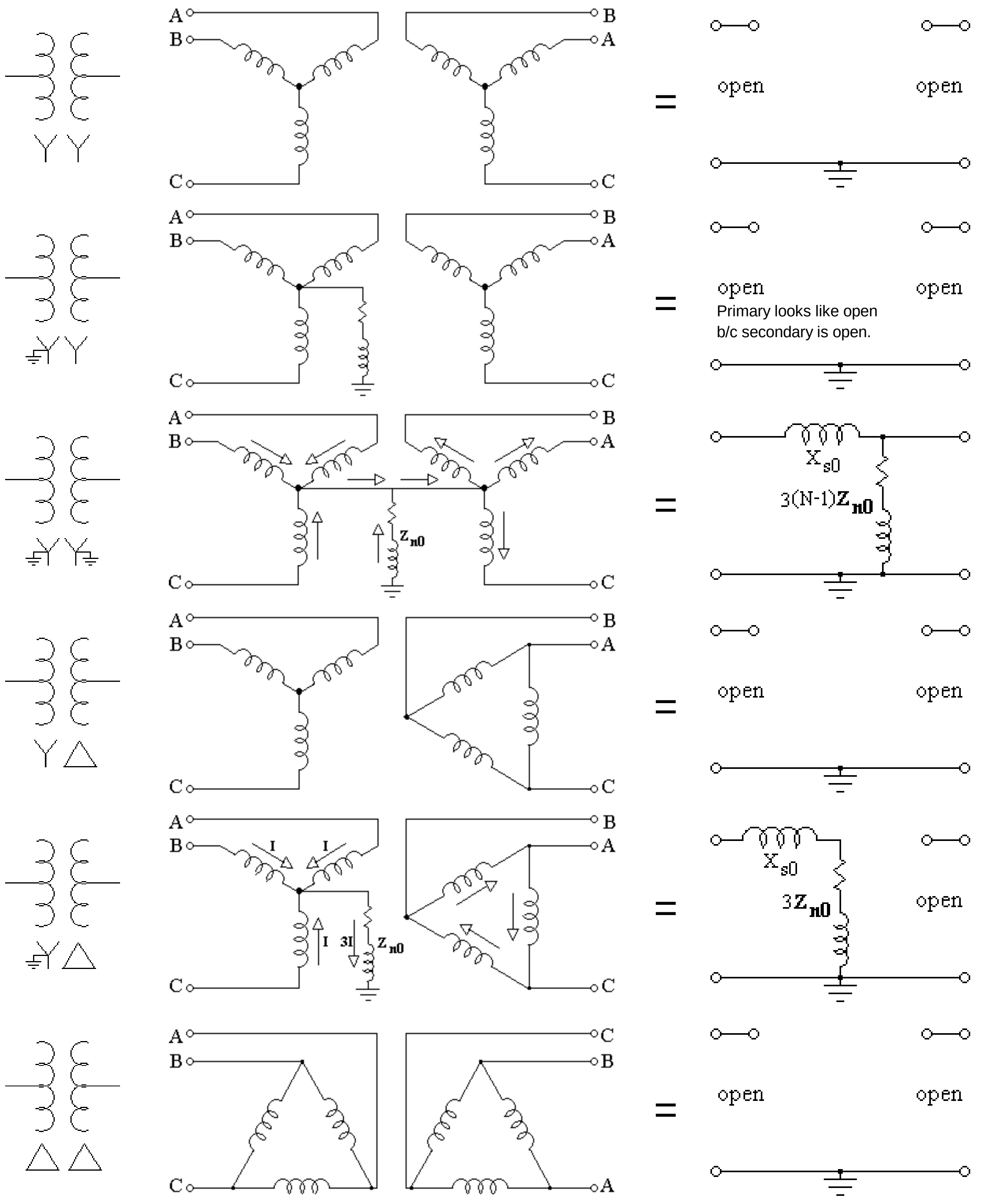
3. Loads
   A. Δ-connected = open. The zero-sequence voltages across a Δ-connected load is 0 (all three phases are the same).
   B. Y-connected
      a. No neutral connection (common for motors) = open. Same reason as a Δ-connected load
      b. With a neutral connection. Load impedance + 3x the ground connection impedance (all three currents flow through that one connection).





4. Transformers. Consider the side of a transformer facing the fault as the primary and the other the secondary.
   A. Δ-connected primary = open
   B. Y-connected primary
      a. No neutral connection = open
      b. With neutral connection, must look at secondary.
         i. Δ-connected secondary = short. Transformer looks like the leakage reactance in series with 3x the ground connection impedance.
         ii. Y-connected secondary without neutral = open, the whole transformer = open.
         iii. Y-connected secondary with neutral, the transformer = the leakage reactance in series with whatever is connected to the secondary.

One-line Symbol	Transformer connections [Currents (if shown) are for a fault to the left]	Fault is left of transformer	Fault is right of transformer



The Bottom Line

1. Pre-Fault Setup

If you don't have the values for E'' and Z''_g or X''_g for your generator(s), you will have to find them.

Generator voltages (E'') are found from prefault currents and subtransient synchronous reactances. See the "Pre-Fault Setup" section earlier in these notes.

2. Symmetrical Faults (easy & therefore not likely your step 2.)

Balanced 3-phase short or 3 open lines are "symmetrical" and will result in balanced currents and voltages. Symmetrical faults can be analyzed just like any other load, except for generators and other large 50+ hp rotating machines. They should be represented by their subtransient synchronous reactances (Z''_g or X''_g) and E'' .

Unsymmetrical Faults

Set up the circuit that needs to be solved. Despite all the time and energy spent to explain the "3-sequences" and how they are used to set up the basic circuits, you don't need to do that. (That was most of pages 2 - 10 of these notes.) There aren't that many types of faults, and that work has already been done, so:

2. Find the basic circuit that matches your fault on the page showing all the faults..

Now the devil is in the details. Those basic circuits are just representative. You have to figure out how your actual case fits.

3. If you don't know your sequence impedances, you'll need to find them. Zero-sequence impedances are the tricky ones. See the previous 2 pages.

4. Combine impedances where you can and fit values in your basic circuit.

5. Analyze the circuit to find I_{A1} and/or V_{A1}.

Sometimes It can be difficult to see series and parallel relationships. Thevenin equivalent circuits can be helpful.

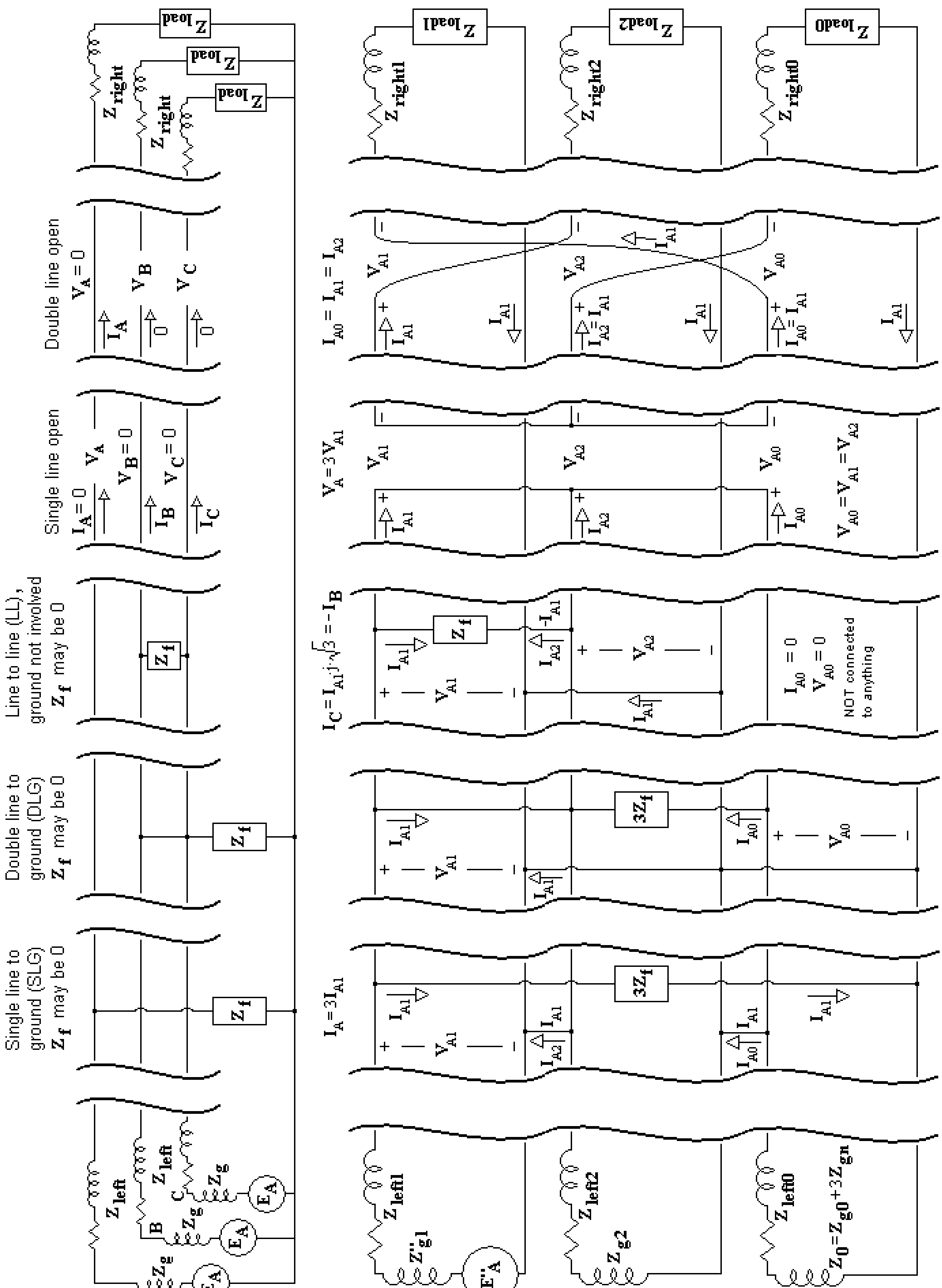
6. Use relationships and equations found in the notes above to find the actual fault currents and/or voltages.

Other Faults

A single line to ground (SLG) fault on line B is not really a different type of fault. Just do your analysis as though line A was shorted and then adjust all your answers by subtracting 120^o from their phase angles. Same for line C, only now you'll adjust your answers by adding 120^o to their phase angles. Same concept works for all the other faults if the lines involved are different than those shown above.

More complex faults are rare and many can be approximated by one of those we've studied here. Otherwise, they are beyond the scope of this class.

Faults are not limited to transmission lines. They can happen anywhere, including at generators, substations and transformers.

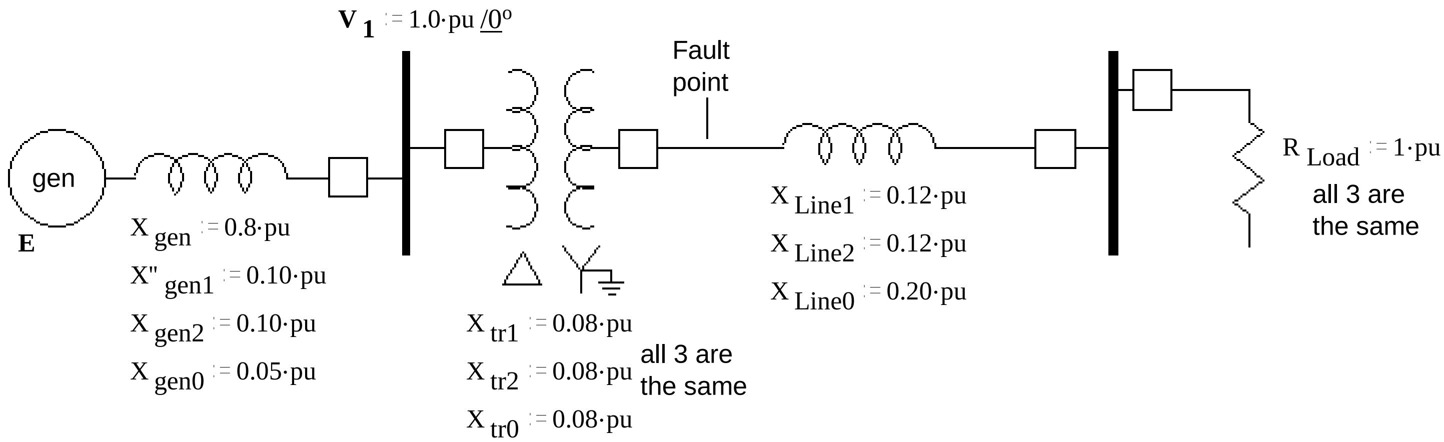


Phases A, B & C

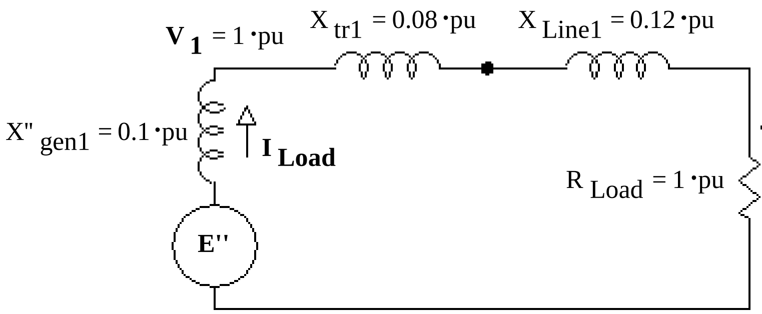
Positive sequence

Negative sequence

Zero sequence



Create a per-phase diagram of the pre-fault system, using all the positive sequence impedances. Only one weirdity, instead of using $\mathbf{X_{gen}}$ and $\mathbf{E_A}$ for the generator, use $\mathbf{X''}$ to find $\mathbf{E''}$. That's because we need to find $\mathbf{E''}$. $\mathbf{E''}$ is found such that $\mathbf{E''}$ in series with $\mathbf{X''}$ would produce the same current and terminal voltage at the generator as the original $\mathbf{E_A}$ and $\mathbf{X_{gen}}$ in pre-fault conditions.



$$\mathbf{I_{Load}} := \frac{\mathbf{V_1}}{R_{Load} + j\cdot(X_{tr1} + X_{Line1})}$$

$$|\mathbf{I_{Load}}| = 0.9806\cdot pu$$

$$\arg(\mathbf{I_{Load}}) = -11.31\cdot deg$$

Pre-fault, back-calculate $\mathbf{E''}$ as though $\mathbf{X''}$ were valid at that time:

$$\mathbf{E''} := \mathbf{V_1} + \mathbf{I_{Load}}\cdot X''_{gen1}\cdot j$$

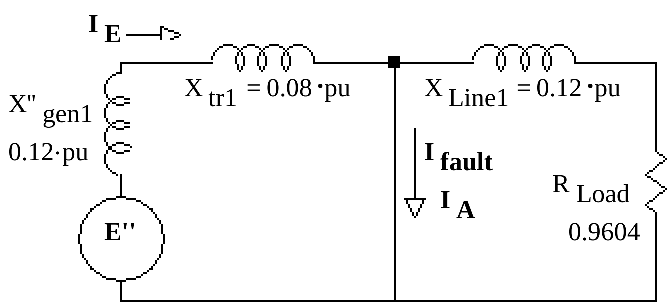
$$|\mathbf{E''}| = 1.024 \qquad \arg(\mathbf{E''}) = 5.389\cdot deg$$

See item 3, under "Some Assumptions about Faults", way back on page 2 of the notes.

Note: in this case we had the pre-fault generator terminal voltage ($\mathbf{V_1}$) and could find the pre-fault current ($\mathbf{I_{Load}}$). If we didn't have those numbers, we might need to find them using a pre-fault diagram with the original $\mathbf{E_A}$ and $\mathbf{X_{gen}}$.

a) A three-phase fault (all three lines grounded) occurs at the fault point. Find the fault current.

Since it's a symmetrical fault, we only need the positive sequence drawing.



$$\mathbf{I_{fault}} := \frac{\mathbf{E''}}{(X''_{gen1} + X_{tr1})\cdot j} \qquad |\mathbf{I_{fault}}| = 5.6875 \qquad \arg(\mathbf{I_{fault}}) = -84.611\cdot deg$$

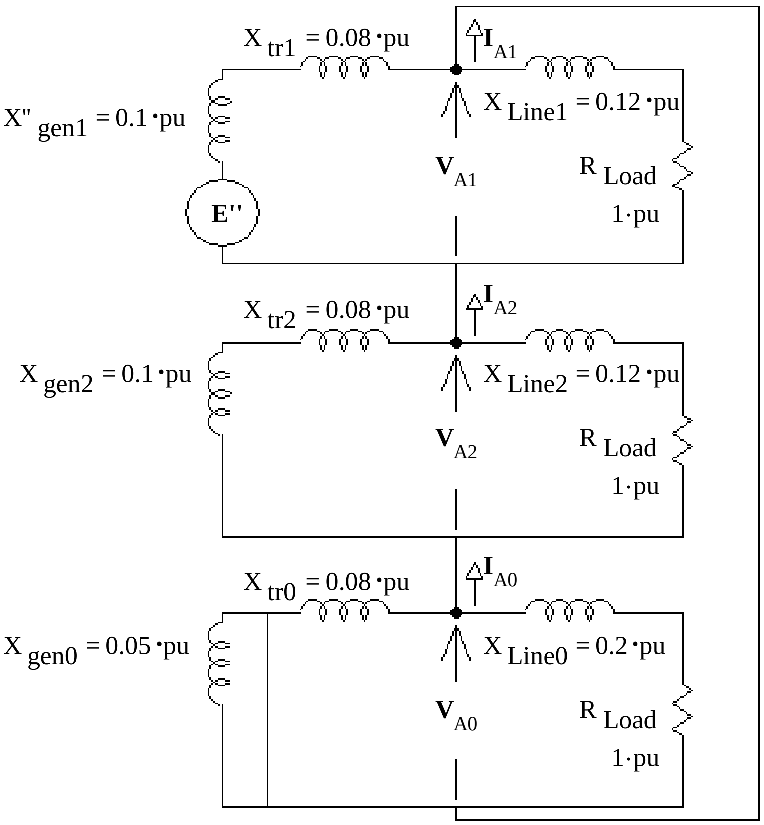
$$\mathbf{I_{fault}} = 5.69\ pu\ \underline{/-88.6^o}$$

Transmission Line Faults p18

Example   Continued

b) Instead of a three-phase fault, a single-line to ground (SLG) fault occurs at the fault point. Find the fault current.

We don't have to do anything with the matrices, since that's already been figured out for the SLG fault.



/\

Note the short here.
Caused by the secondary Δ– winding of the transformer. We're also assuming 0-impedance in the ground connection

define: $\mathbf{Z}_{A1} := X_{Line1} \cdot j + R_{Load}$

$\mathbf{Z}_{A1} = 1 + 0.12j \cdot pu$

$|\mathbf{Z}_{A1}| = 1.0072 \cdot pu \quad \arg(\mathbf{Z}_{A1}) = 6.843 \cdot deg$

define:

$$\mathbf{Z}_{A2} := \frac{1}{\dfrac{1}{(X_{gen2} + X_{tr2}) \cdot j} + \dfrac{1}{X_{Line2} \cdot j + R_{Load}}}$$

$\mathbf{Z}_{A2} = 29.725 + 171.083j \cdot 10^{-3} \cdot pu$

$|\mathbf{Z}_{A2}| = 0.1736 \cdot pu \qquad \arg(\mathbf{Z}_{A2}) = 80.144 \cdot deg$

define: $$\mathbf{Z}_{A0} := \frac{1}{\dfrac{1}{(X_{tr0}) \cdot j} + \dfrac{1}{X_{Line0} \cdot j + R_{Load}}}$$

$\mathbf{Z}_{A0} = 5.935 + 78.338j \cdot 10^{-3} \cdot pu$

$|\mathbf{Z}_{A0}| = 0.0786 \cdot pu \qquad \arg(\mathbf{Z}_{A0}) = 85.668 \cdot deg$

$$\mathbf{Z}_{tot} := (X''_{gen1} + X_{tr1}) \cdot j + \frac{1}{\left(\dfrac{1}{\mathbf{Z}_{A1}} + \dfrac{1}{\mathbf{Z}_{A2} + \mathbf{Z}_{A0}}\right)}$$

$\mathbf{Z}_{tot} = 82.424 + 395.564j \cdot 10^{-3} \cdot pu$

$|\mathbf{Z}_{tot}| = 0.4041 \cdot pu \qquad \arg(\mathbf{Z}_{tot}) = 78.23 \cdot deg$

$\mathbf{I}_E := \dfrac{\mathbf{E''}}{\mathbf{Z}_{tot}} \qquad |\mathbf{I}_E| = 2.5337 \cdot pu \qquad \arg(\mathbf{I}_E) = -72.84 \cdot deg$

$\mathbf{V}_{A1} := \mathbf{E''} - \mathbf{I}_E \cdot (X''_{gen1} + X_{tr1}) \cdot j \qquad \mathbf{V}_{A1} = 583.47 - 38.4j \cdot 10^{-3} \cdot pu$

$|\mathbf{V}_{A1}| = 0.5847 \cdot pu \qquad \arg(\mathbf{V}_{A1}) = -3.765 \cdot deg$

$\mathbf{I}_{A1} := \dfrac{\mathbf{V}_{A1}}{\mathbf{Z}_{A2} + \mathbf{Z}_{A0}} \qquad |\mathbf{I}_{A1}| = 2.3208 \cdot pu \qquad \arg(\mathbf{I}_{A1}) = -85.629 \cdot deg$

$\mathbf{I}_{A2} = \mathbf{I}_{A0} = \mathbf{I}_{A1} \qquad \mathbf{I}_A := 3 \cdot \mathbf{I}_{A1} \qquad |\mathbf{I}_A| = 6.9623 \cdot pu \qquad \arg(\mathbf{I}_A) = -85.629 \cdot deg$

= The fault current