ECE 3600 Phasor Examples

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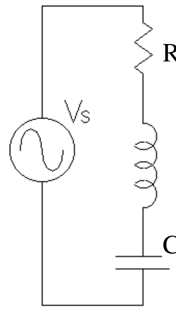
rev

Ex 1. Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2\cdot kHz$

$\mathbf{V}(j\omega)$

$6\cdot V\cdot e^{j0}$

$f := 2\cdot kHz$



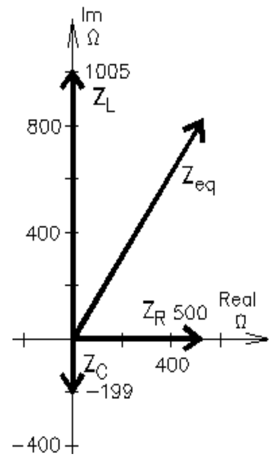
$R := 500\cdot\Omega$ $\quad\omega := 2\cdot\pi\cdot f$ $\quad\omega = 1.257\cdot 10^{4}\cdot\frac{rad}{sec}$

$L := 80\cdot mH$ $\quad\mathbf{Z_L} := j\cdot\omega\cdot L$ $\quad\mathbf{Z_L} = 1.005j\cdot k\Omega$

$C := 0.4\cdot\mu F$ $\quad\mathbf{Z_C} := \frac{1}{j\cdot\omega\cdot C}$ $\quad\mathbf{Z_C} = -0.199j\cdot k\Omega$

$\mathbf{Z_{eq}} := R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}$ $\quad\mathbf{Z_{eq}} = 500 + 806.366j\ \cdot\Omega$

$\sqrt{500^2 + 806^2} = 948.491$ $\quad\operatorname{atan}\left(\frac{806}{500}\right) = 58.187\cdot deg$ $\quad\mathbf{Z_{eq}} = 948.5\Omega\ \underline{/\ 58.2^o}$



find the current: $\mathbf{I} := \frac{6\cdot V\cdot e^{j\cdot 0}}{\mathbf{Z_{eq}}}$ magnitude: $\frac{6\cdot V}{948.5\cdot\Omega} = 6.326\cdot mA$ angle: $0\cdot deg - 58.2\cdot deg = -58.2\cdot deg$

$\mathbf{I} = 6.326mA\ \underline{/-58.2}^o$

	<u>find the magnitude</u>	<u>find the angle</u>	
$\mathbf{V_R} := \mathbf{I}\cdot R$	$6.326\cdot mA\cdot 500\cdot\Omega = 3.163\cdot V$	$-58.2\cdot deg + 0\cdot deg = -58.2\cdot deg$	$\mathbf{V_R} = 3.163V\ \underline{/-58.2}^o$
$\mathbf{V_L} := \mathbf{I}\cdot\mathbf{Z_L}$	$6.326\cdot mA\cdot 1005\cdot\Omega = 6.358\cdot V$	$-58.2\cdot deg + 90\cdot deg = 31.8\cdot deg$	$\mathbf{V_L} = 6.358V\ \underline{/\ 31.8}^o$
$\mathbf{V_C} := \mathbf{I}\cdot\mathbf{Z_C}$	$6.326\cdot mA\cdot(-199)\cdot\Omega = -1.259\cdot V$	$-58.2\cdot deg + (90)\cdot deg = 31.8\cdot deg$	$\mathbf{V_C} = -1.259V\ \underline{/\ 31.8}^o$
OR:	$6.326\cdot mA\cdot(199)\cdot\Omega = 1.259\cdot V$	$-58.2\cdot deg + (-90)\cdot deg = -148.2\cdot deg$	$\mathbf{V_C} = 1.259V\ \underline{/-148.2}^o$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\frac{1}{j\cdot\omega\cdot C}}{R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) + j\cdot\omega\cdot L\cdot(j\cdot\omega\cdot C) + 1}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) - \omega^2\cdot L\cdot C + 1}\cdot 6\cdot V$$

$$= \frac{1}{\left(1 - \omega^2\cdot L\cdot C\right) + j\cdot\omega\cdot R\cdot C}\cdot 6\cdot V \qquad \left(1 - \omega^2\cdot L\cdot C\right) = -4.053 \qquad j\cdot\omega\cdot R\cdot C = 2.513j$$

$$= \frac{6\cdot V}{-4.053 + 2.513\cdot j}\cdot\frac{(-4.053 - 2.513\cdot j)}{(-4.053 - 2.513\cdot j)} = \frac{6\cdot V\cdot(-4.053 - 2.513\cdot j)}{(-4.053)^2 + 2.513^2}$$

$6\cdot V\cdot(-4.053 - 2.513\cdot j) = -24.318 - 15.078j\ \cdot V$

$(-4.053)^2 + 2.513^2 = 22.742$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078\cdot j}{22.742}\right)\cdot V = -1.069 - 0.663j\ \cdot V$$

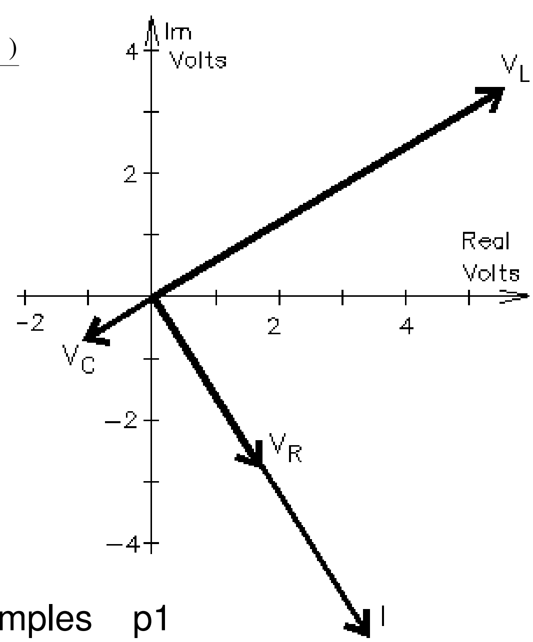
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

angle: $\operatorname{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81\cdot deg$

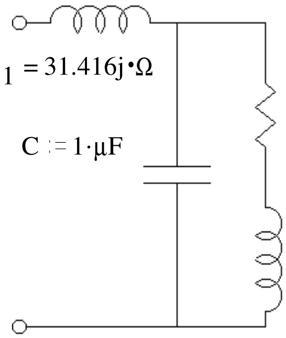
but this is actually in the third quadrant,
so modify your calculator's results:

$31.81\cdot deg - 180\cdot deg = -148.19\cdot deg$

$= 1.258V\ \underline{/-148.2}^o$



Ex 2. a) Find $\mathbf{Z_{eq}}$. $f := 2.5\cdot kHz$ $\quad \omega := 2\cdot\pi\cdot f$ $\quad \omega = 1.571\cdot 10^{4}\ \cdot\frac{rad}{sec}$



$L_1 := 2\cdot mH$ $\quad j\cdot\omega\cdot L_1 = 31.416j\cdot\Omega$ $\quad C := 1\cdot\mu F$ $\quad R := 200\cdot\Omega$ $\quad L_2 := 8\cdot mH$ $\quad j\cdot\omega\cdot L_2 = 125.664j\cdot\Omega$

Left branch

$$\mathbf{Z_l} := \frac{1}{j\cdot\omega\cdot C} \qquad \mathbf{Z_l} = -63.662j\cdot\Omega$$

Right branch

$$\mathbf{Z_r} := j\cdot\omega\cdot L_2 + R \qquad \mathbf{Z_r} = 200 + 125.664j\ \cdot\Omega$$

$$\mathbf{Z_{eq}} := j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + \frac{1}{\frac{1}{j\cdot\omega\cdot C}}} = j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C} = j\cdot\omega\cdot L_1 + \frac{R + j\cdot\omega\cdot L_2}{1 + j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right)}$$

$$= j\cdot\omega\cdot L_1 + \frac{R + j\cdot\omega\cdot L_2}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}$$

Sometimes it's worth simplifying a little before puttiing in numbers.

$$\mathbf{Z_{eq}} = 31.416\cdot j\ \cdot\Omega + \frac{(200 + 125.664\cdot j)\cdot\Omega}{-0.974 + 3.142\cdot j}\cdot\left(\frac{-0.974 - 3.142\cdot j}{-0.974 - 3.142\cdot j}\right) = 31.416\cdot j\ \cdot\Omega + \frac{(200 + 125.664\cdot j)\cdot(-0.974 - 3.142\cdot j\ )}{0.974^2 + 3.142^2}$$

$$= 31.416\cdot j\ \cdot\Omega + \frac{((200\cdot(-0.974) - 125.664\cdot(-3.142)) + (125.664\cdot(-0.974) - 200\cdot 3.142)\cdot j)\cdot\Omega}{0.974^2 + 3.142^2}$$

$$= 31.416\cdot j\ \cdot\Omega + \frac{(200.036288 - 750.796736\cdot j)\cdot\Omega}{10.82084} = 31.416\cdot j\ \cdot\Omega + 18.486\cdot\Omega - 69.384\cdot j\cdot\Omega = 18.486 - 37.968j\ \cdot\Omega$$

$$\sqrt{18.49^2 + 37.97^2} = 42.233 \qquad \operatorname{atan}\left(\frac{-37.97}{18.49}\right) = -64.036\cdot deg \qquad \mathbf{Z_{eq}} = 42.24\Omega\ \underline{/-64.04^o}$$

b) $\mathbf{V_{in}} := 12\cdot V\cdot e^{j\cdot 20\cdot deg}$ Find $\mathbf{I_{L1}}$, $\mathbf{V_C}$ $\quad \mathbf{I_{L1}} := \frac{\mathbf{V_{in}}}{\mathbf{Z_{eq}}}$ $\quad \frac{12\cdot V}{42.24\cdot\Omega} = 284.091\cdot mA$ $\quad 20\cdot deg - (-64.04)\cdot deg = 84.04\cdot deg$

$$\mathbf{I_{L1}} = 284mA\ \underline{/\ 84.04^o} = 284\cdot mA\cdot e^{j\cdot 84.04\cdot deg} \qquad \mathbf{I_{L1}} = 29.485 + 282.569j\ \cdot mA$$

$$\mathbf{V_C} := \mathbf{I_{L1}}\cdot(18.486 - 69.384\cdot j\ )\cdot\Omega \qquad 284\cdot mA\cdot\sqrt{18.486^2 + 69.384^2}\cdot\Omega = 20.392\cdot V$$

$$84.04\cdot deg + \operatorname{atan}\left(\frac{-69.384}{18.486}\right) = 8.959\cdot deg \qquad \mathbf{V_C} = 20.4V\ \underline{/\ 8.96^o}$$

To find V_C directly:

You could then use another voltage divider to find $\mathbf{V_R}$ or $\mathbf{V_{L2}}$.

$$\mathbf{V_C} := \frac{\frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}{j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}\cdot\mathbf{V_{in}} = \frac{1}{j\cdot\omega\cdot L_1\cdot\left(\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C\right) + 1}\cdot\mathbf{V_{in}} = \frac{1}{\frac{j\cdot\omega\cdot L_1}{R + j\cdot\omega\cdot L_2} - \omega^2\cdot L_1\cdot C + 1}$$

$$= \frac{1}{\frac{j\cdot\omega\cdot L_1}{R + j\cdot\omega\cdot L_2} - \omega^2\cdot L_1\cdot C + 1}\cdot\mathbf{V_{in}} = \frac{1}{\frac{j\cdot\omega\cdot L_1\cdot\left(R - j\cdot\omega\cdot L_2\right)}{R^2 + \left(\omega\cdot L_2\right)^2} - \omega^2\cdot L_1\cdot C + 1}\cdot\mathbf{V_{in}}$$

$$= \frac{1}{\left[\frac{\omega^2\cdot L_1\cdot L_2}{R^2 + \left(\omega\cdot L_2\right)^2} - \omega^2\cdot L_1\cdot C + 1\right] + j\cdot\frac{\omega\cdot L_1\cdot R}{R^2 + \left(\omega\cdot L_2\right)^2}}\cdot\mathbf{V_{in}} = \frac{12\cdot V\cdot e^{j\cdot 20\,deg}}{0.58816\cdot e^{j\cdot 11.039\,deg}} = \frac{12\cdot V}{0.58816}\ \underline{/\ 20 - 11.039^o}$$

$$= 20.4V\ \underline{/\ 8.96^o} \qquad \text{Same}$$

$$\mathbf{V_C} = 20.153 + 3.178j\ \cdot V$$

Ex 2. Continued Find $\mathbf{I_{L2}}$.

$$\mathbf{Z_r} := R + j\cdot\omega\cdot L_2$$

$$\mathbf{Z_r} = 200 + 125.664j \ \cdot\Omega \qquad \sqrt{200^2 + 125.664^2} = 236.202 \qquad \operatorname{atan}\left(\frac{125.664}{200}\right) = 32.142 \cdot deg$$

$$\mathbf{I_{L2}} = \frac{20.4\cdot V\cdot e^{j\cdot 8.96\cdot deg}}{236.202\cdot\Omega\cdot e^{j\cdot 32.142\cdot deg}} = \frac{20.4\cdot V}{236.202\cdot\Omega} \ \underline{/ 8.96 - 32.142^o} \quad = 86.4mA \ \underline{/ -23.18}$$

Or, directly by Current divider:

$$\mathbf{I_{L2}} := \frac{\dfrac{1}{R + j\cdot\omega\cdot L_2}}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{1}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I_{L1}} = \frac{\mathbf{I_{L1}}}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}$$

denominator: $\sqrt{\left(1 - \omega^2\cdot C\cdot L_2\right)^2 + (\omega\cdot C\cdot R)^2} = 3.289 \qquad \operatorname{atan}\left(\frac{\omega\cdot C\cdot R}{1 - \omega^2\cdot C\cdot L_2}\right) + 180\cdot deg = 107.224 \cdot deg$

$$\mathbf{I_{L2}} = \frac{284\cdot mA\cdot e^{j\cdot 84.04\cdot deg}}{3.289\cdot e^{j\cdot 107.224\cdot deg}} = \frac{284\cdot mA}{3.289} \ \underline{/ 84.04 - 107.224^o} \quad = 86.4mA \ \underline{/ -23.18}$$

$$\mathbf{I_{L2}} = 79.404 - 34.001j \ \cdot mA$$

How about $\mathbf{I_C}$?

$$\mathbf{I_C} := \frac{\mathbf{V_C}}{\left(\dfrac{1}{j\cdot\omega\cdot C}\right)} = \mathbf{V_C}\cdot j\cdot\omega\cdot C \quad = 20.4V \ \underline{/ 8.96^o} \ 0.015708 \ \underline{/ 90^o} \ \frac{1}{\Omega} \quad = 320mA \ \underline{/ 98.96^o}$$

Or, directly by Current divider:

$$\mathbf{I_C} := \frac{j\cdot\omega\cdot C}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right)}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I_{L1}} = \frac{-\omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}\cdot\mathbf{I_{L1}}$$

numerator: $\sqrt{\left(\omega^2\cdot C\cdot L_2\right)^2 + (\omega\cdot C\cdot R)^2} = 3.71 \qquad \operatorname{atan}\left(\frac{\omega\cdot C\cdot R}{-\omega^2\cdot C\cdot L_2}\right) + 180\cdot deg = 122.142 \cdot deg$

Second quadrant

denominator is the same as above.

$$\mathbf{I_C} = \frac{3.71\cdot e^{j\cdot 122.14\cdot deg}}{3.289\cdot e^{j\cdot 107.224\cdot deg}}\cdot 284\cdot mA\cdot e^{j\cdot 84.04\cdot deg} = \frac{3.71}{3.289}\cdot 284\cdot mA \ \underline{/ 122.14 - 107.224 + 84.04^o} \quad = 320mA \ \underline{/ 98.96^o}$$

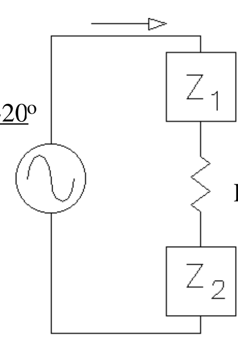
This current is greater than the input current. What's going on?

The angle between $\mathbf{I_C}$ & $\mathbf{I_{L2}}$ is big enough that they somewhat cancel each other out.

Check Kirchoff's Current Law: $\mathbf{I_C} + \mathbf{I_{L2}} = 29.485 + 282.569j \ \cdot mA \quad = \quad \mathbf{I_{L1}} = 29.485 + 282.569j \ \cdot mA$

Ex 3. a) Find $\mathbf{Z_2}$.

$$i(t) = 25\cdot mA\cdot\cos\left(377\cdot\frac{rad}{sec}\cdot t + 10\cdot deg\right)$$



$\mathbf{V_{in}} = 10V \ \underline{/ -20^o}$

$\mathbf{Z_1} := (120 - 60\cdot j)\cdot\Omega$

$R := 50\cdot\Omega$

$\mathbf{Z_2} = ?$

$$\mathbf{I} := 25\cdot mA\cdot e^{j\cdot 10\cdot deg}$$

$$\mathbf{V_{in}} := 10\cdot V\cdot e^{-j\cdot 20\cdot deg}$$

$$\mathbf{Z_T} := \frac{\mathbf{V_{in}}}{\mathbf{I}} = \frac{10\cdot V}{25\cdot mA} \ \underline{/ -20 - 10^o} \ = 400\Omega \ \underline{/ -30^o}$$

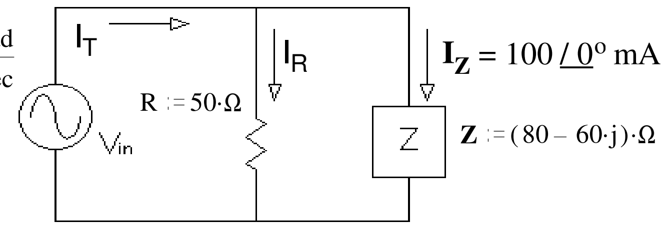
$$\mathbf{Z_T} = 346.41 - 200j \ \cdot\Omega$$

$$\mathbf{Z_2} := \mathbf{Z_T} - R - \mathbf{Z_1} = (346.41 - 200\cdot j)\cdot\Omega - 50\cdot\Omega - (120 - 60\cdot j)\cdot\Omega = 176.41 - 140j \ \cdot\Omega$$

b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^o > -20^o$.

ii) The source voltage leads the source current

Ex 4. a) Find $\mathbf{V_{in}}$ in polar form.



$\mathbf{I_Z} := 100\cdot mA$ $\mathbf{Z} := (80 - 60\cdot j)\cdot\Omega$ $\omega := 1000\cdot\frac{rad}{sec}$

$\mathbf{V_{in}} := \mathbf{I_Z}\cdot\mathbf{Z}$ $\mathbf{V_{in}} = 8 - 6j\ \cdot V$

$\sqrt{8^2 + 6^2} = 10$ $\operatorname{atan}\left(\frac{-6}{8}\right) = -36.87\ \cdot deg$

$\mathbf{V_{in}} = 10V\ \underline{/-36.9^o}$

b) Find $\mathbf{I_T}$ in polar form. $\mathbf{I_R} := \frac{\mathbf{V_{in}}}{R} = \frac{10\cdot V}{50\cdot\Omega}\ \underline{/-36.9^o} = \frac{10\cdot V}{50\cdot\Omega}\cdot\cos(-36.9\cdot deg) + j\cdot\frac{10\cdot V}{50\cdot\Omega}\cdot\sin(-36.9\cdot deg) = 160 - 120i\ \ \cdot mA$

$\mathbf{I_T} := \mathbf{I_R} + \mathbf{I_Z} = (160 - 120\cdot j\ )\cdot mA + 100\cdot mA = 260 - 120j\ \ \cdot mA$

$\sqrt{260^2 + 120^2} = 286.356$ $\operatorname{atan}\left(\frac{-120}{260}\right) = -24.78\ \cdot deg$ $\mathbf{I_T} = 286mA\ \underline{/-24.8^o}$

c) Circle 1: i) The source current leads the source voltage   ii) The source voltage leads the source current

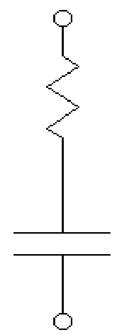
answer i), $-24.8^o > -36.9^o$

Ex 5. a) The impedance **Z** (above) is made of two components in series. What are they and what are their values?

$\mathbf{Z} = 80 - 60j\ \ \cdot\Omega$

Must have a resistor because there is a real part.

$R := \operatorname{Re}(\mathbf{Z})$ $R = 80\cdot\Omega$



Must have a capacitor because the imaginary part is negative.

$\operatorname{Im}(\mathbf{Z}) = -60\cdot\Omega\ = \frac{-1}{\omega\cdot C}$ $C := \frac{-1}{\omega\cdot\operatorname{Im}(\mathbf{Z})}$ $C = 16.667\cdot\mu F$

b) The impedance **Z** is made of two components in <u>parallel</u>. What are they and what are their values?

$\mathbf{Z} = 80 - 60j\ \ \cdot\Omega$

Must have a resistor because there is a real part.

Must have a capacitor because the imaginary part is negative.

$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C}$ $\frac{1}{\mathbf{Z}} = \frac{1}{(80 - 60\cdot j\ )\cdot\Omega}\cdot\left(\frac{80 + 60\cdot j}{80 + 60\cdot j}\right) = \frac{80 + 60\cdot j}{80^2 + 60^2} = \frac{80 + 60\cdot j}{10,000}\cdot\frac{1}{\Omega}$

$\frac{1}{\mathbf{Z}} = 8\cdot 10^{-3} + 6\cdot 10^{-3} j\ \ \ \cdot\Omega^{-1} = \frac{1}{R} + j\cdot\omega\cdot C$

$\frac{1}{R} = .008\cdot\frac{1}{\Omega}$ $R := \frac{1}{.008\cdot\frac{1}{\Omega}}$ $R = 125\cdot\Omega$

$\omega\cdot C = .006\cdot\frac{1}{\Omega}$ $C := \frac{.006\cdot\frac{1}{\Omega}}{\omega}$ $C = 6\cdot\mu F$