Review of Phasors

ECE 3600

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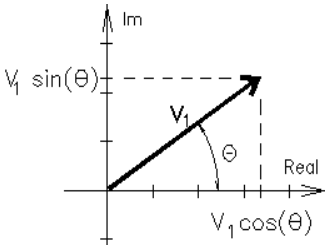
9/3/08
rev,

For steady-state sinusoidal response ONLY

Phasors



Time domain

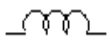
$v(t) = V_1 \cdot \cos(377 \cdot t + \theta)$

Phasor, frequency domain

$\mathbf{V_1} = V_1 \cdot e^{j \cdot \theta} = V_1 \angle \theta = V_1 \cdot \cos(\theta) + j \cdot V_1 \cdot \sin(\theta)$

Impedances,

Inductor



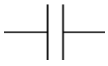
$v_L = L \cdot \frac{d}{dt} i_L = L \cdot \frac{d}{dt} I_p \cdot e^{j \cdot (\omega \cdot t + \theta)} = j \cdot \omega \cdot L \cdot \left[I_p \cdot e^{j \cdot (\omega \cdot t + \theta)} \right]$

$\mathbf{V_L}(\omega) = j \cdot \omega \cdot L \cdot \mathbf{I}(\omega)$

AC impedance

$\mathbf{Z_L} = j \cdot \omega \cdot L$

Capacitor



$i_C = C \cdot \frac{d}{dt} v_C = C \cdot \frac{d}{dt} V_p \cdot e^{j \cdot (\omega \cdot t + \theta)} = j \cdot \omega \cdot C \cdot \left[V_p \cdot e^{j \cdot (\omega \cdot t + \theta)} \right]$

$\mathbf{I_C}(\omega) = j \cdot \omega \cdot C \cdot \mathbf{V}(\omega)$ $\quad \mathbf{V_C}(\omega) = \frac{1}{j \cdot \omega \cdot C} \cdot \mathbf{I}(\omega)$

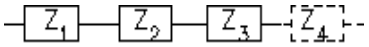
$\mathbf{Z_C} = \frac{1}{j \cdot \omega \cdot C} = \frac{-j}{\omega \cdot C}$

Resistor

$v_R = i_R \cdot R$ $\quad \mathbf{V_R}(\omega) = R \cdot \mathbf{I}(\omega)$ $\quad \mathbf{Z_R} = R$

You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

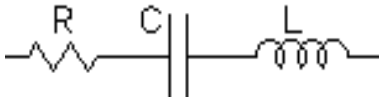
series:



$\mathbf{Z_{eq}} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots$

$f := 60 \cdot Hz$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad \omega = 377 \cdot \frac{rad}{sec}$

Example:



$R := 20 \cdot \Omega$ $\quad L := 80 \cdot mH$ $\quad C := 60 \cdot \mu F$

$j \cdot \omega \cdot L = 30.159j \cdot \Omega$

$\frac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega$

$\mathbf{Z_{eq}} := R + \frac{1}{j \cdot \omega \cdot C} + j \cdot \omega \cdot L = 20 \cdot \Omega - 44.21 \cdot j \cdot \Omega + 30.16 \cdot j \cdot \Omega = 20 - 14.05j \cdot \Omega$

$\sqrt{(20 \cdot \Omega)^2 + (14.05 \cdot \Omega)^2} = 24.44 \cdot \Omega$ $\quad \operatorname{atan}\left(\frac{-14.05 \cdot \Omega}{20 \cdot \Omega} \right) = -35.09 \cdot deg$

$\mathbf{Z_{eq}} = 24.44\Omega \angle -35.1^o$

If: $\mathbf{V} := 120 \cdot V \cdot e^{j \cdot 0 \cdot deg}$ $\quad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z_{eq}}} = \frac{120 \cdot V}{24.44 \cdot \Omega} = 4.91 \cdot A$ $\quad \angle \; 0 - -35.1 = 35.1 \; deg$

$4.91 \cdot \cos(35.1 \cdot deg) = 4.017$ $\quad 4.91 \cdot \sin(35.1 \cdot deg) = 2.823$ $\quad \mathbf{I} = 4.017 + 2.822j \cdot A$

slight roundoff error

Voltage divider:

$$V_{Zn} = V_{total} \cdot \frac{Z_n}{Z_1 + Z_2 + Z_3 + \ldots}$$

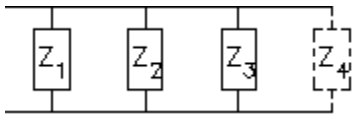
Eg: $V_C := V \cdot \frac{\frac{1}{j \cdot \omega \cdot C}}{Z_{eq}} = 120 \cdot V \cdot e^{j \cdot 0 \cdot deg} \cdot \frac{44.21 \cdot e^{-j \cdot 90 \cdot deg} \cdot \Omega}{24.44 \cdot e^{-j \cdot 35.1 \cdot deg} \cdot \Omega}$

$120 \cdot V \cdot \frac{44.21 \cdot \Omega}{24.44 \cdot \Omega} = 217.07 \cdot V \qquad \angle\ 0 + -90 - -35.1 = -54.9 \ \ deg$

$V_C = 217.1V \angle -54.9^o \qquad V_C = 124.771 - 177.604j \ \cdot V$

$217.1 \cdot \cos(-54.9 \cdot deg) = 124.8 \qquad 217.1 \cdot \sin(-54.9 \cdot deg) = -177.6$

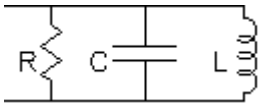
parallel:



$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$$

Example:

$f := 60 \cdot Hz \qquad \omega := 2 \cdot \pi \cdot f \qquad \omega = 377 \cdot \frac{rad}{sec}$



$L := 80 \cdot mH \qquad j \cdot \omega \cdot L = 30.159j \cdot \Omega$

$R := 20 \cdot \Omega \qquad C := 60 \cdot \mu F \qquad \frac{1}{\omega \cdot L} = 3.316 \cdot 10^{-2} \cdot \frac{1}{\Omega}$

$\frac{1}{j \cdot \omega \cdot C} = -44.21j \cdot \Omega \qquad \omega \cdot C = 2.262 \cdot 10^{-2} \cdot \frac{1}{\Omega}$

$$Z_{eq} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{j \cdot \omega \cdot L}} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C - \frac{j}{\omega \cdot L}} = \frac{1}{\frac{1}{20 \cdot \Omega} + 2.262 \cdot 10^{-2} \cdot j \cdot \frac{1}{\Omega} - 3.316 \cdot 10^{-2} \cdot j \cdot \frac{1}{\Omega}} = \frac{1}{\left(5 \cdot 10^{-2} - 1.054 \cdot 10^{-2} \cdot j\right) \cdot \frac{1}{\Omega}}$$

$$= \frac{1}{\left(5 \cdot 10^{-2} - 1.054 \cdot 10^{-2} \cdot j\right) \cdot \frac{1}{\Omega}} \cdot \frac{5 \cdot 10^{-2} + 1.054 \cdot 10^{-2} \cdot j}{\left(5 \cdot 10^{-2} + 1.054 \cdot 10^{-2} \cdot j\right)} = 19.149 + 4.037j \ \cdot \Omega$$

OR, If you want a polar result, it's actually easier to change the denominator to polar and then do polar division.

$$\sqrt{\left(5 \cdot 10^{-2} \cdot \frac{1}{\Omega}\right)^2 + \left(1.054 \cdot 10^{-2} \cdot \frac{1}{\Omega}\right)^2} = 5.11 \cdot 10^{-2} \cdot \frac{1}{\Omega} \qquad \operatorname{atan}\left(\frac{-1.054 \cdot 10^{-2} \cdot \Omega}{5 \cdot 10^{-2} \cdot \Omega}\right) = -11.9 \cdot deg$$

$\frac{1}{5.11 \cdot 10^{-2} \cdot \frac{1}{\Omega}} = 19.569 \cdot \Omega \qquad \angle\ 0 - -11.9 = 11.9 \ \ deg \qquad Z_{eq} = 19.57\Omega \angle 11.9^o$

If: $V := 120 \cdot V \cdot e^{j \cdot 0 \cdot deg} \qquad I := \frac{V}{Z_{eq}} = \frac{120 \cdot V}{19.57 \cdot \Omega} = 6.132 \cdot A \qquad \angle\ 0 - 11.9 = -11.9 \ \ deg$

$6.132 \cdot \cos(-11.9 \cdot deg) = 6 \qquad 6.132 \cdot \sin(-11.9 \cdot deg) = -1.264 \qquad I = 6 - 1.265j \ \cdot A$

slight roundoff error

Current divider:

$$I_{Zn} = I_{total} \cdot \frac{\frac{1}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$$

Eg: $I_L := I \cdot \frac{\frac{1}{j \cdot \omega \cdot L}}{\frac{1}{R} + j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L}} = I \cdot \frac{\left(\frac{1}{j \cdot \omega \cdot L}\right)}{\left(\frac{1}{Z_{eq}}\right)} = I \cdot \frac{Z_{eq}}{j \cdot \omega \cdot L}$

$= 6.132 \cdot A \cdot e^{j \cdot -11.9 \cdot deg} \cdot \frac{19.57 \cdot e^{j \cdot 11.9 \cdot deg} \cdot \Omega}{30.159 \cdot e^{j \cdot 90 \cdot deg} \cdot \Omega}$

$I_L = 6.132 \cdot A \cdot \frac{19.57 \cdot \Omega}{30.159 \cdot \Omega} = 3.979 \cdot A \qquad \angle\ -11.9 + 11.9 - 90 = -90 \ \ deg \qquad I_L = -3.979 \cdot 10^3 j \ \cdot mA$

Duh... $\frac{V}{j \cdot \omega \cdot L} = -3.979 \cdot 10^3 j \ \cdot mA$