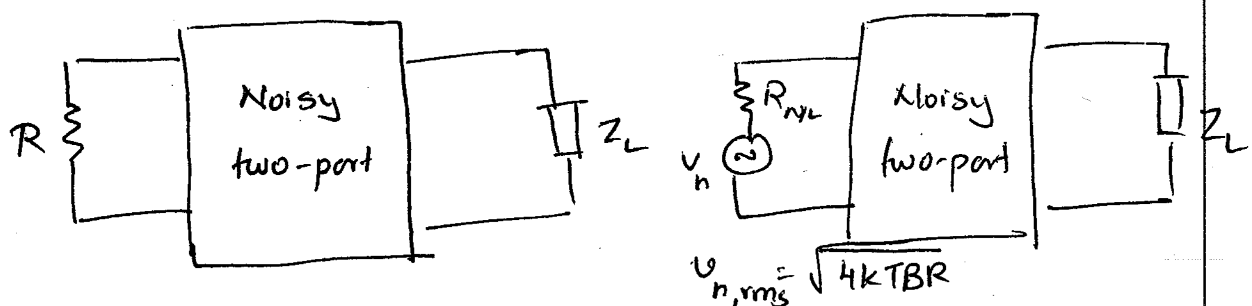
Noise in Two-Port Networks

→ In a microwave amplifier, even when there is no input signal, a small output voltage can be measured.

→ This small output power is known as the amplifier noise power.

The model of a noisy two-port microwave amplifier is shown in figure below



- Noise i/p power can be modeled by a source resistor that produces thermal or Johnson noise. This noise is produced by the random fluctuations of the electrons due to thermal agitation. The rms value of the noise voltage ($v_{n,rms}$) produced by the noisy resistor R over a frequency range $f_H - f_L$

$$v_{n,rms} = \sqrt{4kTBR}$$

$k \rightarrow$ Boltzmann's constant ($k = 1.374 \times 10^{-23}$ J/°K)

$T \rightarrow$ resistor noise temperature

B ⇒ Bandwidth $(f_H - f_L)$

The equation shows that the thermal noise power depends on the bandwidth and not on a given center frequency. Such a distribution of noise is called white noise.

The available noise power from R is

$$P_N = \frac{U_{n,rms}^2}{4 \cdot R} = KTB$$

The noise figure (F) describes quantitatively the performance of a noisy microwave amplifier.

- Noise figure of a microwave amplifier is defined as the ratio of the total available noise power at the output ~~due to ther~~ of the amplifier to the available noise power at the output due to thermal noise from the input termination R, where R is at the standard temperature $T = T_0 = 290°K$.

The noise figure can be expressed in the form

$$F = \frac{P_{N_0}}{P_{N_i} G_A}$$

P_{N_0} → total available noise power at the output of the amplifier

$P_{N_i} = kT_0B$ ⇒ available power due to R at $T = T_0 = 290°K$ in a bandwidth B, & G_A is the available power Gain

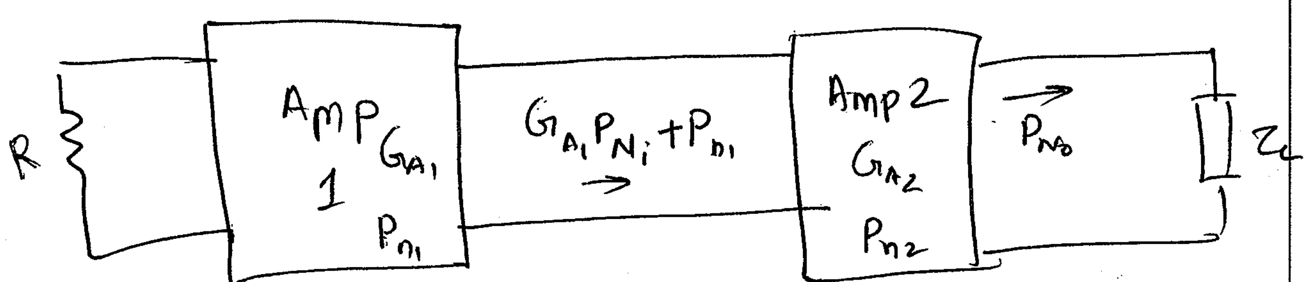
$$G_A = \frac{P_{S_o}}{P_{S_i}}$$

$P_{S_o} \rightarrow$ available signal power at the output
$P_{S_i} \rightarrow$ available signal power at the i/p.

$$F = \frac{P_{S_i}/P_{N_i}}{P_{S_o}/P_{N_o}}$$

F can also be defined as the ratio of the available signal-to-noise power ratio at the input to the available signal-to-noise power ratio at the output.
A minimum noise figure is obtained by properly selecting the source reflection coefficient of the amplifier.

A model for calculating the noise figure of a two stage amplifier is shown in figure below.



The total available noise power at the o/p is given by

$$P_{N_o} = G_{A_2}(G_{A_1}P_{N_i} + P_{n_1}) + P_{n_2}$$

$$F = \frac{P_{N_o}}{P_{N_i} G_{A_1} G_{A_2}} = 1 + \frac{P_{n_1}}{P_{N_i} G_{A_1}} + \frac{P_{n_2}}{P_{N_i} G_{A_1} G_{A_2}}$$

or

$$F = F_1 + \frac{F_2 - 1}{G_{A_1}} \qquad \text{— (A)}$$

where

$$F_1 = 1 + \frac{P_{n_1}}{P_{N_i} G_{A_1}}$$

&

$$F_2 = 1 + \frac{P_{n_2}}{P_{N_i} G_{A_2}}$$

F_1 & F_2 are recognized as the individual noise figures of the first and second stages.

From (A) we see that noise figure of the second stage is reduced by G_{A_1}. Therefore noise contribution from second stage is small if G_{A_1} is large & can be significant if the gain G_{A_1} is low.
It is not always important to minimize the first stage noise if the gain reduction is too large.
- We can also select a higher gain, even if F_1 is higher than the minimum noise figure of the first stage.
such that a low value of F is obtained. In a design a tradeoff between gain & noise figure is generally made

A specific calculation can be made for two cascaded amplifiers to determine which one must be used first in order to achieve the lowest noise figure.

Consider two amplifiers with noise figures F_1 and F_2 and gains G_{A_1} and G_{A_2}

If amplifier 1 is connected before amplifier 2, the total noise figure, denoted by F_{12}, is

$$F_{12} = F_1 + \frac{F_2 - 1}{G_{A_1}}$$

- if amplifier 2 is connected at the i/p, the total noise figure denoted by F_{21} is

$$F_{21} = F_2 + \frac{F_1 - 1}{G_{A_2}}$$

The configuration with amplifier 1 connected at i/p produces a lower total ~~low~~ noise figure when $F_{12} < F_{21}$

$$F_1 + \frac{F_2 - 1}{G_{A_1}} < F_2 + \frac{F_1 - 1}{G_{A_2}}$$

$$F_1 - 1 + \frac{F_2 - 1}{G_{A_1}} < F_2 - 1 + \frac{F_1 - 1}{G_{A_2}}$$

or $$\frac{F_1 - 1}{1 - \frac{1}{G_{A_1}}} < \frac{F_2 - 1}{1 - \frac{1}{G_{A_2}}}$$

which can be written as

$$M_1 < M_2$$

where

$$M = \frac{F-1}{1-\frac{1}{G_A}}$$

$M \rightarrow$ noise measure

When two amplifiers are cascaded, the lower total noise figure is achieved when the amplifier with the lowest value of M is connected at the input

For the case of chain of n amplifiers the total noise figure is given by

$$F = F_1 + \frac{F_2-1}{G_{A_1}} + \frac{F_3-1}{G_{A_1}G_{A_2}} + \frac{F_4-1}{G_{A_1}G_{A_2}G_{A_3}} + \cdots \qquad (B)$$

If the transistors are identical with $F_1 = F_2 = \cdots = F_n$ and $G_{A_1} = G_{A_2} = \cdots = G_{A_n}$ the eqn (B) reduces to

$$F = 1 + \frac{F_1-1}{1-\frac{1}{G_{A_1}}} = 1 + M_1$$

Constant noise figure Circles.

Noise figure of a two port amplifier is given by

$$F = F_{min} + \frac{r_n}{g_s} |y_s - y_{opt}|^2 \quad -(1)$$

$r_n \rightarrow$ equivalent normalized noise resistance of the two-port (ie $r_n = R_n/z_o$)

$y_s = g_s + jb_s \rightarrow$ normalized source admittance

$y_{opt} = g_{opt} + jb_{opt} \rightarrow$ normalized source admittance that results in minimum noise figure, F_{min}

y_s and y_{opt} can be represented in terms of reflection coefficients Γ_s and Γ_{opt}

$$y_s = \frac{1-\Gamma_s}{1+\Gamma_s} \quad -(2) \qquad y_{opt} = \frac{1-\Gamma_{opt}}{1+\Gamma_{opt}} \quad -(3)$$

Substituting (2) and (3) in (1) we get

$$F = F_{min} + \frac{4 r_n \, |\Gamma_s - \Gamma_{opt}|^2}{(1-|\Gamma_s|^2)\, |1+\Gamma_{opt}|^2} \quad -(4)$$

This equation depends on F_{min}, r_n and Γ_{opt}.

These are the noise parameters and are provided by designers.

The r_n can be obtained as

$$r_n = \left(F_{\Gamma_s=0} - F_{min}\right) \frac{|1+\Gamma_{opt}|^2}{4|\Gamma_{opt}|^2}$$

F_{min} is a function of device operating current and frequency and there is one value of Γ_{opt} associated with each F_{min}.

For a given noise figure $F = F_i$ we can design Γ_s as follows

$$\frac{|\Gamma_s - \Gamma_{opt}|^2}{1-|\Gamma_s|^2} = \frac{F_i - F_{min}}{4r_n} |1+\Gamma_{opt}|^2 \quad — (5)$$

we observe that for a given noise figure right had side of eqn (5) is constant

Hence, defining noise figure N_i as

$$N_i = \frac{F_i - F_{min}}{4r_n} |1+\Gamma_{opt}|^2$$

$$\frac{|\Gamma_s - \Gamma_{opt}|^2}{1-|\Gamma_s|^2} = N_i$$

which when furthur solved becomes

$$|\Gamma_s|^2 - \frac{2}{1+N_i} \operatorname{Re}(\Gamma_s \Gamma_{opt}^*) + \frac{|\Gamma_{opt}|^2}{1+N_i} = \frac{N_i}{1+N_i}$$

This equation is that of a ○ in the Γ_s plane.

This can be expressed as:

$$\left|\Gamma_s - \frac{\Gamma_{opt}}{1+N_i}\right|^2 = \frac{N_i^2 + N_i(1-|\Gamma_{opt}|^2)}{(1+N_i)^2}$$

center

$$C_{F_i} = \frac{\Gamma_{opt}}{1+N_i}$$

radius

$$r_{F_i} = \frac{1}{1+N_i}\sqrt{N_i^2 + N_i(1-|\Gamma_{opt}|^2)}$$

N_i is calculated for various F_i and constant noise ○s are drawn.

$F_i = F_{min}$; $N_i = 0$, $C_{F_{min}} = \Gamma_{opt}$ & $r_{F_{min}} = 0$

ie centerd F_{min} ○ is located at Γ_{opt} with radius 0.