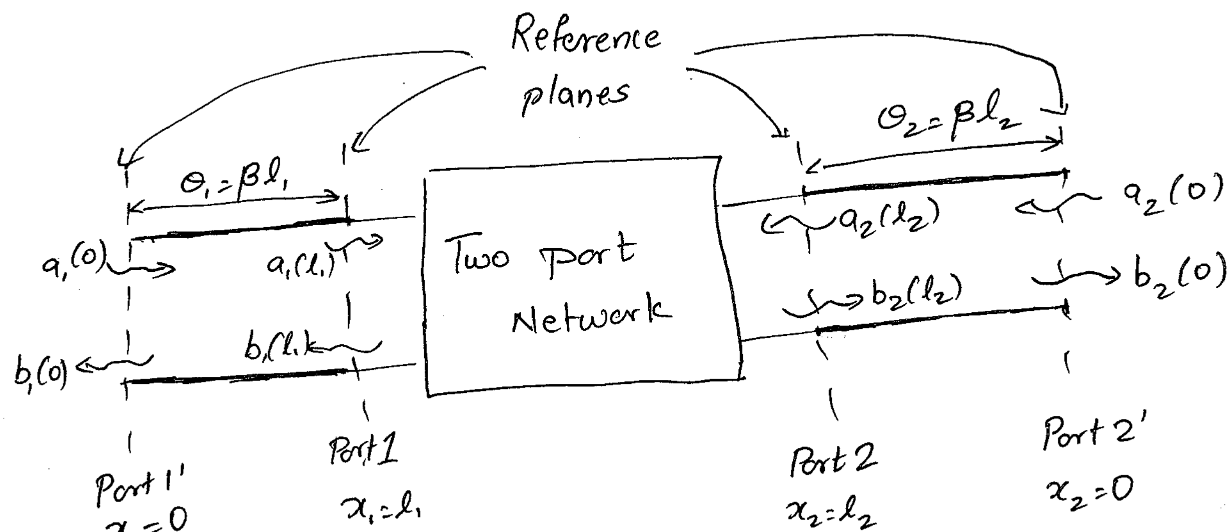
Lecture - 1 (Part 2)

Shifting Reference Planes

Consider the two-port network as shown in the figure below



We often need to attach transmission lines to the two-port network.

Since S parameters are measured using traveling waves, we need to specify the positions where the measurements are made.

These positions are called *reference planes*

S parameters at reference planes located at port 1' and port 2' and relate them to the S parameters at port 1 and port 2 of the two port network

At the reference planes at port 1 and port 2, the scattering matrix can be written as:

$$\begin{bmatrix} b_1(l_1) \\ b_2(l_2) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1(l_1) \\ a_2(l_2) \end{bmatrix} \quad - (1)$$

and at port 1' and port 2' as

$$\begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \begin{bmatrix} S_{11}' & S_{12}' \\ S_{21}' & S_{22}' \end{bmatrix} \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix} \quad - (2)$$

The two port Network with transmission lines when viewed from port 1' and port 2' constitute a new two port N/w whose S parameters are defined by (2).

θ_1 & θ_2 are the electrical lengths of the transmission lines between the primed and unprimed reference planes.

$$b_1(l_1) = b_1(0)\, e^{j\theta_1}$$

$$a_1(l_1) = a_1(0)\, e^{-j\theta_1}$$

$$b_2(l_2) = b_2(0)\, e^{j\theta_2}$$

$$a_2(l_2) = a_2(0)\, e^{-j\theta_2}$$

where the factor $e^{\pm j\theta}$ accounts for the phase difference of the waves at the different reference planes

$$\left.\begin{array}{ll} b_1(l_1) = b_1(0)\, e^{j\beta l_1} & a_1(l_1) = a_1(0)\, e^{-j\beta l_1} \\ b_2(l_2) = b_2(0)\, e^{j\beta l_2} & a_2(l_2) = a_2(0)\, e^{-j\beta l_2} \end{array}\right\} - (3)$$

Substituting (3) in (1) we get

$$\begin{bmatrix} b_1(0)\, e^{j\beta l_1} \\ b_2(0)\, e^{j\beta l_2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1(0)\, e^{-j\beta l_1} \\ a_2(0)\, e^{-j\beta l_2} \end{bmatrix}$$

$$\begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \begin{bmatrix} S_{11} e^{-j2\beta l_1} & S_{12} e^{-j(\beta l_1 + \beta l_2)} \\ S_{21} e^{-j(\beta l_1 + \beta l_2)} & S_{22} e^{-j2\beta l_2} \end{bmatrix} \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix} \quad - (4)$$

Comparing (4) and (0)

$$\begin{bmatrix} S_{11}' & S_{12}' \\ S_{21}' & S_{22}' \end{bmatrix} = \begin{bmatrix} S_{11} e^{-j2\beta l_1} & S_{12} e^{-j(\beta l_1 + \beta l_2)} \\ S_{21} e^{-j(\beta l_1 + \beta l_2)} & S_{22} e^{-j2\beta l_2} \end{bmatrix}$$

and

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} S_{11}' e^{j2\beta l_1} & S_{12}' e^{j(\beta l_1 + \beta l_2)} \\ S_{21}' e^{j(\beta l_1 + \beta l_2)} & S_{22}' e^{j2\beta l_2} \end{bmatrix}$$

Properties of Scattering Parameters

Two important properties

1) Matrix $[S]$ is symmetrical

$[S]^t = [S]$ $\quad$ $[S]^t \rightarrow$ Transpose matrix of $[S]$

&

$S_{ij} = S_{ji}$

2) Matrix $[S]$ is unitary

$$[S]^a = [S^*]^t = [S]^{-1}$$

$[S]^a$ is adjoint matrix of $[S]$

$[S^*]^t$ is conjugate of $[S]^t$

$[S]^{-1}$ is inverse matrix of $[S]$

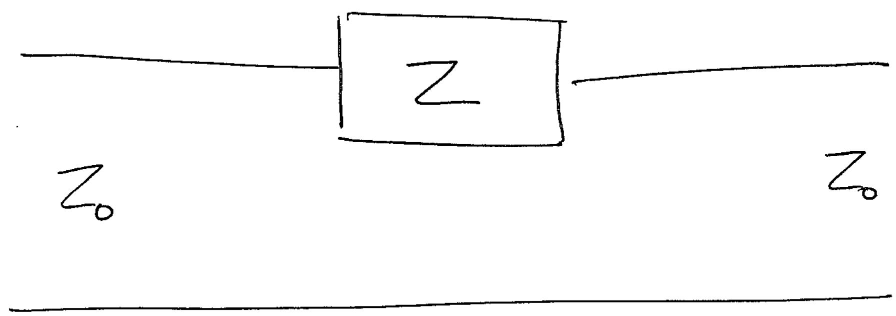
$$\sum_{i=1}^{N} S_{ij} S_{ik}^* = \delta_{jk} = \begin{cases} 1 & \text{for } j = k \\ 0 & \text{otherwise} \end{cases}$$

Therefore

$$\sum_{i=1}^{N} S_{ij} S_{ij}^* = \sum_{i=1}^{N} |S_{ij}|^2 = 1 \qquad j = 1, 2, 3 \cdots N$$

example

Find S parameters of a series impedance Z connected between the two ports as shown in the figure.



<u>Soln</u>

As explained earlier we can write

$$S_{11} = \Gamma_1 \Big|_{a_2=0} = \frac{(Z+Z_0) - Z_0}{(Z+Z_0) + Z_0} = \frac{Z}{Z+2Z_0}$$

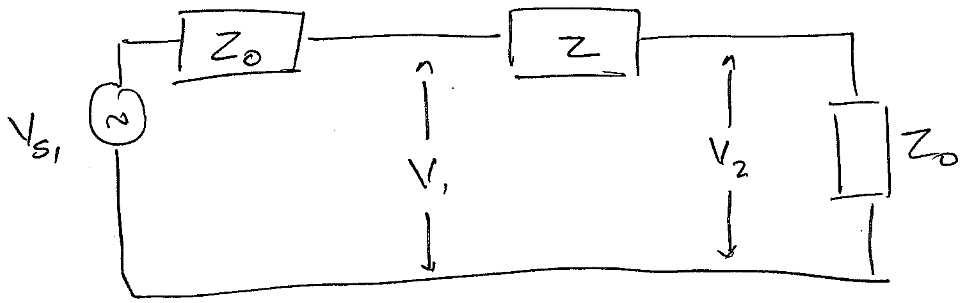
Similarly when $\Gamma_s = 0$ (ie port 1 is terminated by a matched load)

$$S_{22} = \Gamma_2 \Big|_{a_1=0} = \frac{(Z+Z_0) - Z_0}{(Z+Z_0) - Z_0} = \frac{Z}{Z+2Z_0}$$

S_{21} and S_{12} can be determined using

$$S_{21} = \frac{2V_2}{V_{s1}} \sqrt{\frac{Z_{01}}{Z_{02}}} \quad \& \quad S_{12} = \frac{2V_1}{V_{s2}} \sqrt{\frac{Z_{02}}{Z_{01}}}$$

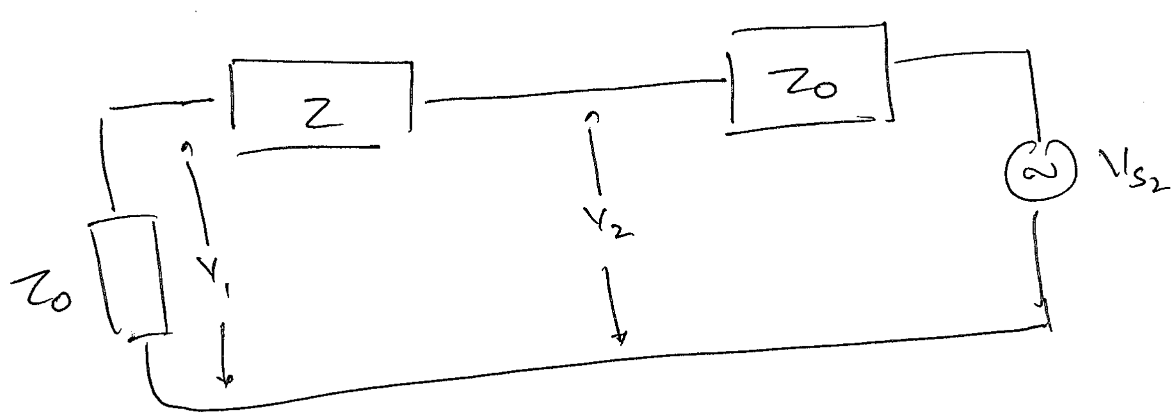
~~we~~ For calculating S_{21} we connect a voltage source at port 1 (V_{S_1}) while port 2 is terminated by Z_0. The source impedance is assumed to be Z_0



$$V_2 = V_{S_1} \frac{Z_0}{Z+2Z_0}$$

$$S_{21} = \frac{2V_2}{V_{S_1}} = \frac{2Z_0}{Z+2Z_0} = \frac{2Z_0+Z-Z}{Z+2Z_0} = 1 - \frac{Z}{Z+2Z_0}$$

For calculating S_{12}, we connect a voltage source V_{S_2} at port 2 while port 1 is terminated by Z_0



Using the voltage divider again, we get

$$V_1 = \frac{Z_0}{Z+2Z_0} V_{S2}$$

$$S_{12} = \frac{2V_1}{V_{S2}} = \frac{2Z_0}{Z+2Z_0} = 1 - \frac{Z}{Z+2Z_0}$$

$\therefore$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \Gamma_1 & 1-\Gamma_1 \\ 1-\Gamma_1 & \Gamma_1 \end{bmatrix}$$

where

$$\Gamma_1 = \frac{Z}{Z+2Z_0}$$

Looking at the matrix we can say that $S_{11} = S_{22}$.

This is because the given two port N/w is symmetrical.

If $S_{12} = S_{21}$ this means the Network is Reciprocal

If the series Z is the above example is purely

reactive we have

$$\Gamma_1 = \frac{jX}{jX + 2Z_0}$$

Therefore

$$|S_{11}|^2 + |S_{21}|^2 = \left|\frac{jX}{jX+2Z_0}\right|^2 + \left|1 - \frac{jX}{jX+2Z_0}\right|^2$$

$$= \frac{X^2}{X^2+(2Z_0)^2} + \frac{(2Z_0)^2}{X^2+(2Z_0)^2} = 1$$

III^{ly} $\qquad |S_{12}|^2 + |S_{22}|^2 = 1$

On the other hand

$$S_{11}S_{12}^* + S_{21}S_{22}^* = \frac{jX}{jX+2Z_0} \times \frac{2Z_0}{-jX+2Z_0} + \frac{2Z_0}{jX+2Z_0} \times \frac{-jX}{-jX+2Z_0}$$

$$= 0$$

and

$$S_{12}S_{11}^* + S_{22}S_{21}^* = \frac{2Z_0}{jX+2Z_0} \times \frac{-jX}{-jX+2Z_0} + \frac{jX}{jX+2Z_0} \times \frac{2Z_0}{-jX+2Z_0}$$

$$= 0$$

The characteristics of the scattering matrix can be summarized as:

$$\sum S_{ij} S_{ik}^* = \delta_{jk} = \begin{cases} 1 & \text{if } j=k \\ 0 & \text{otherwise} \end{cases} \quad \text{— unitary matrix}$$