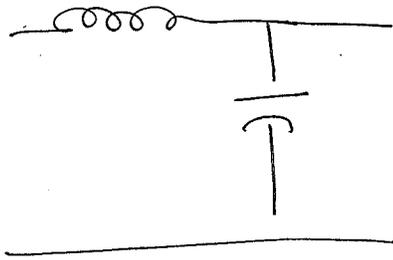Lecture-5

Q- Factor Analysis

L-Network

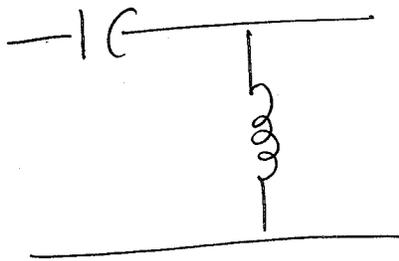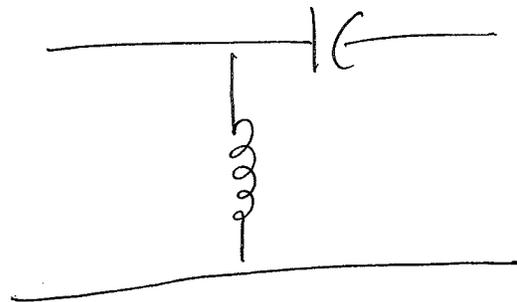Low-pass Config   if $R_g < R_L$ if $R_L < R_g$

or if we
have to
pass Dc
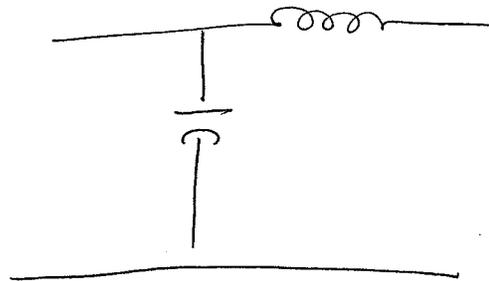
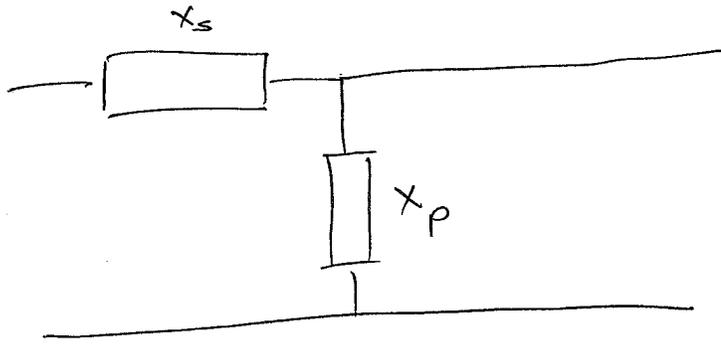High-pass Configuration

if $R_g \lessgtr R_L$ if $R_L < R_g$

eg:- Design a ckt to match a 100Ω Source to a 1000Ω load.
at $100MH_3$

Soln.

i) $R_L = 1000\Omega = R_P$

$R_g = 100\Omega = R_S$

X_s

X_p

Since $R_g < R_L$

③ Calculate Q

$$Q = \sqrt{\frac{R_P}{R_S} - 1} = \sqrt{\frac{1000}{100} - 1} = 3$$

$$Q_s = Q_p = Q = 3$$

④ Calculate X_p & X_s

$$X_s = Q_s R_s$$

$$= (3) \times 100$$

$$= 300\,\Omega \quad (\text{inductive})$$

$$X_p = \frac{R_p}{Q_p} = \frac{1000}{3} = 333\,\Omega \quad (\text{capacitive})$$

⑤ Calculate L & C

$$X_s = \omega L \qquad L = \frac{X_s}{\omega} = 477\,nH$$

$$X_p = \frac{1}{\omega C} = 4.8\,pF$$

L-network using smith chart

eg:- $Z_L = 200\,\Omega$ $Z_s = 20\,\Omega$ $\omega = 500m\,H_3$

$Q_L = ?$

$$Q_n = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{9} = 3$$
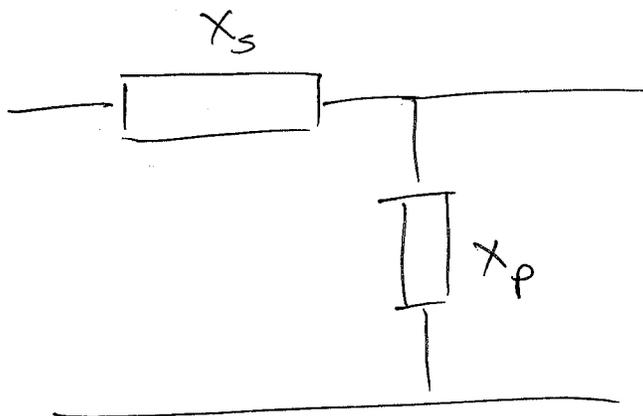
$$Q_L = \frac{Q_n}{2} = \frac{3}{2} = 1.5$$

$$BW = \frac{f_o}{Q_L} = \frac{500 \times 10^6}{1.5} = 333.33\,m\,H_3$$

ckt design

① $R_L = 200\,\Omega = R_p$

$R_g = 20\,\Omega = R_s$

②

$R_g < R_L$

③

$$Q = \sqrt{\frac{R_P}{R_S} - 1} = \sqrt{\frac{200}{20} - 1} = 3$$
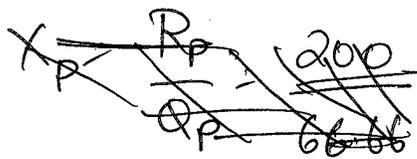
$$Q_S = Q_P = Q = 3$$

④ Calculate X_P & X_S

$$X_S = Q_S R_S$$

$$= 3 \times 20 = 60 \, \Omega$$

$$X_P = \frac{R_P}{Q_P} = \frac{200}{3} = 66.66 \, \Omega$$

⑤ Calculate L & C

$$X_S = \omega L \qquad L = \frac{X_S}{\omega} = 19.09 nH$$

$$X_P = \frac{R_P}{Q_P} = \frac{200}{66.66} \qquad X_P = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega X_P} \qquad = 4.7751 pF$$

The Complete Smith Chart
Black Magic Design

Steps for drawing
Q θ

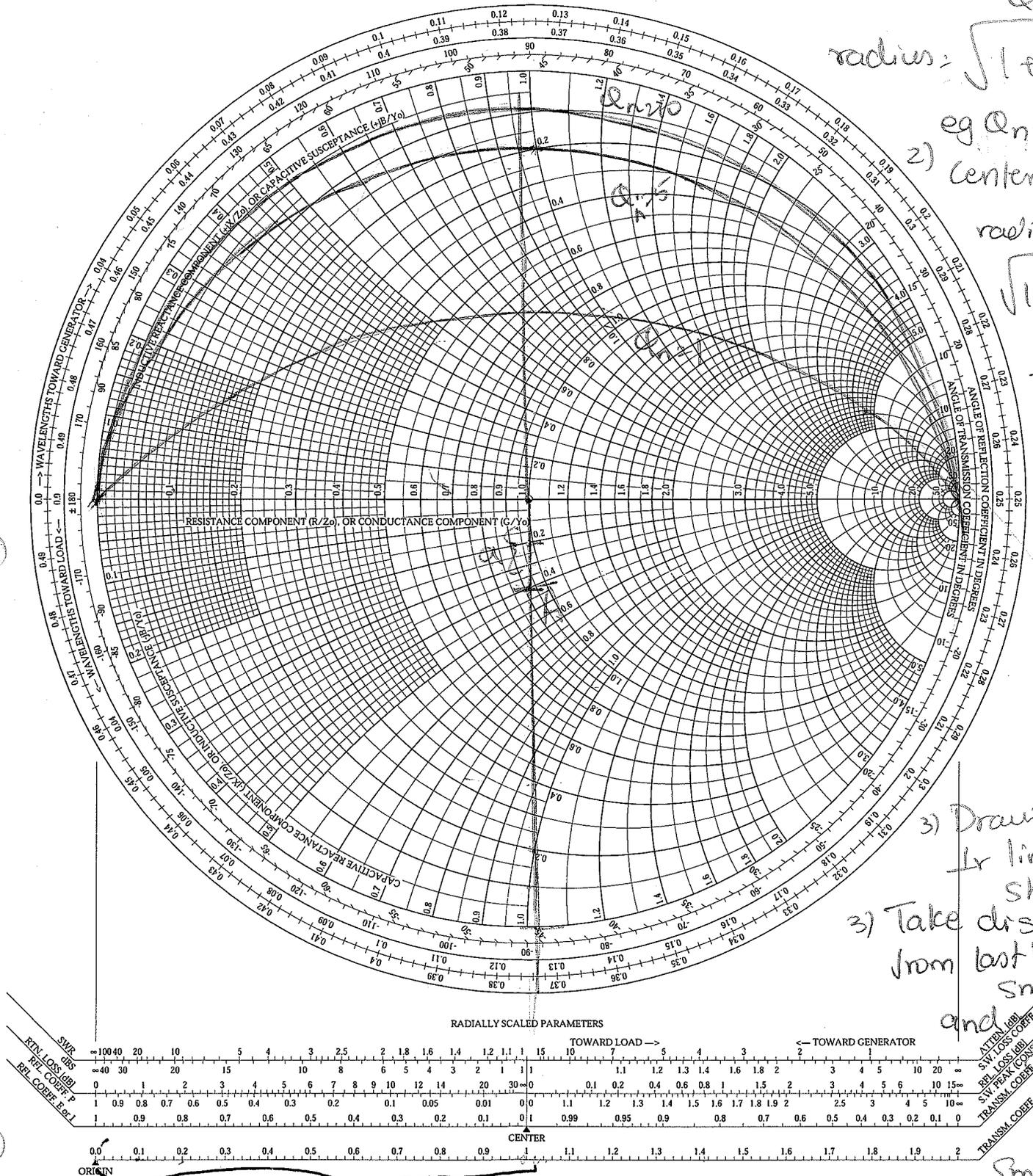1) Center $(0, \pm \frac{1}{Q_n})$

radius: $\sqrt{1 + \frac{1}{Q_n^2}}$

eg $Q_n = 5$

2) Center $(0, \pm 0.2)$

radius = $\sqrt{1 + \frac{1}{2\sigma}}$

$= 1.02$

RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)

INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)

CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)

→ WAVELENGTHS TOWARD GENERATOR →

← WAVELENGTHS TOWARD LOAD ←

ANGLE OF REFLECTION COEFFICIENT IN DEGREES

ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES

RADIALLY SCALED PARAMETERS

TOWARD LOAD →   ← TOWARD GENERATOR
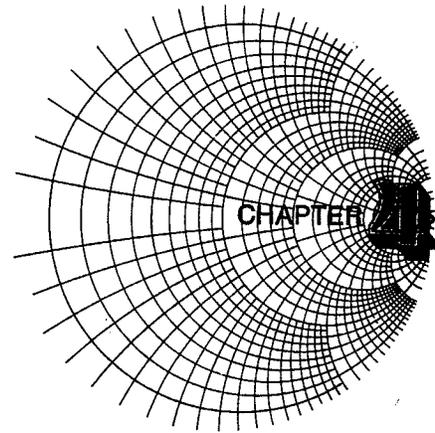
CENTER

ORIGIN

0.2

radius

3) Draw a ⊥r line as shown

3) Take distance from last line of smith chart and mark distance 'a' on smith chart

4) A is the location of the center.

5) Take the radius on compass & draw a θ with A as center

IMPEDANCE MATCHING

Impedance matching is often necessary in the design of rf circuitry to provide the maximum possible transfer of power between a source and its load. Probably the most vivid example of the need of such a transfer of power occurs in the front end of any sensitive receiver. Obviously, any *unnecessary* loss in a circuit that is already carrying extremely small signal levels simply cannot be tolerated. Therefore, in most instances, extreme care is taken during the initial design of such a front end to make sure that each device in the chain is matched to its load.

In this chapter, then, we will study several methods of matching a given source to a given load. This will be done both numerically and with the aid of the Smith Chart and, in both cases, exact step-by-step procedures will be presented making any calculations as painless as possible.
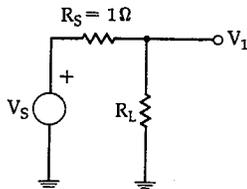
BACKGROUND

There is a well-known theorem which states that, for *dc circuits*, maximum power will be transferred from a source to its load if the *load resistance* equals the *source resistance*. A simple proof of this theorem is given by the calculations and the sketches shown in Fig. 4-1. In the calculation, for convenience, the source is normalized for a resistance of one ohm and a source voltage of one volt.

In dealing with ac or time-varying waveforms, however, that same theorem states that the maximum transfer of power, from a source to its load, occurs when the *load impedance* (Z_L) is equal to the *complex conjugate* of the *source impedance*. Complex conjugate simply refers to a complex impedance having the same *real part* with an opposite reactance. Thus, if the source impedance were $Z_s = R + jX$, then its complex conjugate would be $Z_s = R - jX$.

If you followed the mathematics associated with Fig. 4-1, then it should be obvious why maximum transfer of power does occur when the load impedance is the complex conjugate of the source. This is shown schematically in Fig. 4-2. The source (Z_s), with a series reactive component of $+jX$ (an inductor), is driving its complex conjugate load impedance con-

(A) Circuit.

(B) Graph.

Fig. 4-1. The power theorem.

Proof that P_{out} MAX occurs when $R_L = R_s$, in the circuit of Fig. 4-1A, is given by the formula:

$$V_1 = \frac{R_L}{R_S + R_L}(V_s)$$

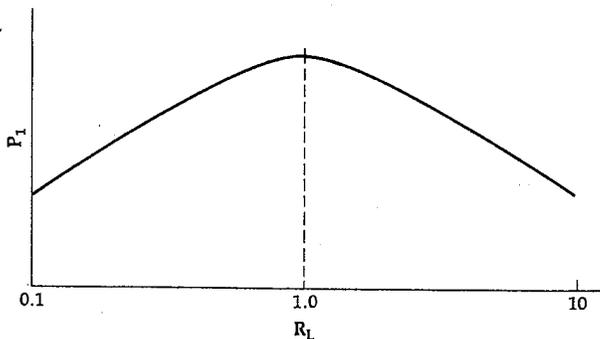Set $V_s = 1$ and $R_s = 1$, for convenience. Therefore,

$$V_1 = \frac{R_L}{1 + R_L}$$

Then, the power into R_L is:

$$P_1 = \frac{V_1^2}{R_L}$$

$$= \frac{\left(\dfrac{R_L}{1 + R_L}\right)^2}{R_L}$$

$$= \frac{R_L}{(1 + R_L)^2}$$

If you plot P_1 versus R_L, as in the preceding equation, the result is shown by the curve of the graph in Fig. 4-1B.

sisting of a $-jX$ reactance (capacitor) in series with R_L. The $+jX$ component of the source and the $-jX$ component of the load are in series and, thus, cancel each other, leaving only R_S and R_L, which are equal by definition. Since R_S and R_L are equal, maximum power transfer will occur. So when we speak of a source driving its complex conjugate, we are simply referring to a condition in which any *source* reactance is resonated with an equal and opposite *load* reactance; thus, leaving only equal resistor values for the source and the load terminations.

The primary objective in any impedance *matching* scheme, then, is to force a load impedance to "look like" the complex conjugate of the source impedance so that maximum power may be transferred to the load. This is shown in Fig. 4-3 where a load impedance of $2 -j6$ ohms is transformed by the impedance matching network to a value of $5 +j10$ ohms. Therefore, the source "sees" a load impedance of $5 +j10$ ohms, which just happens to be its complex conjugate. It should be noted here that because we are dealing with *reactances*, which are frequency dependent, the *perfect* impedance match can occur only at one frequency. That is the frequency at which the $+jX$ component exactly equals the $-jX$ component and, thus, cancellation or resonance occurs. At all other frequencies removed from the matching center frequency, the impedance match becomes progressively worse and eventually nonexistent. This can be a problem in broadband circuits where we would ideally like to provide a perfect match everywhere within the broad passband. There are methods, however, of increasing the bandwidth of the match and a few of these methods will be presented later in this chapter.

There are an infinite number of possible networks which could be used to perform the impedance matching function of Fig. 4-3. Something as simple as a 2-element LC network or as elaborate as a 7-element filter, depending on the application, would work equally well. The remainder of this chapter is devoted to providing you with an insight into a few of those infinite possibilities. After studying this chapter, you should be able to match almost any two complex loads with a minimum of effort.

THE L NETWORK

Probably the simplest and most widely used matching circuit is the L network shown in Fig. 4-4. This circuit receives its name because of the component orientation which resembles the shape of an L. As shown in the sketches, there are four possible arrangements of the two L and C components. Two of the arrangements (Figs. 4-4A and 4-4B) are in a low-pass configuration while the other two (Figs. 4-4C and 4-4D) are in a high-pass configuration. Both of these circuits should be recognized from Chapter 3.

Before we introduce equations which can be used to design the matching networks of Fig. 4-4, let's first
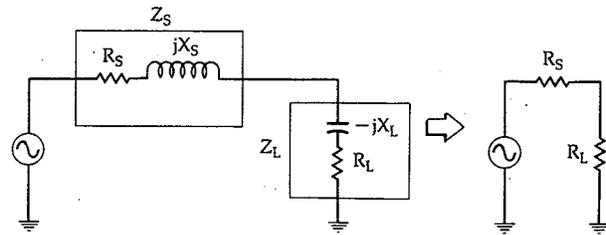

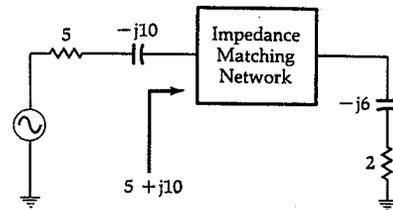Fig. 4-2. Source impedance driving its complex conjugate and the resulting equivalent circuit.

Fig. 4-3. Impedance transformation.

(A) Low-pass.   (B) Low-pass.
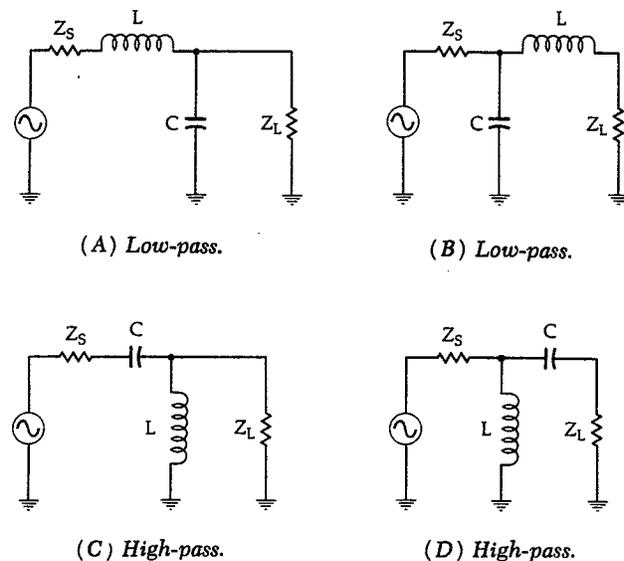
(C) High-pass.   (D) High-pass.
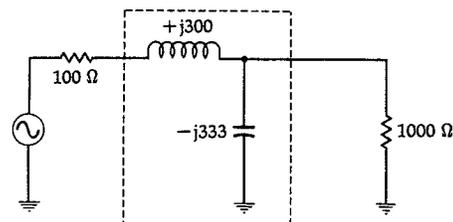
Fig. 4-4. The L network.

Fig. 4-5. Simple impedance-match network between a 100-ohm source and a 1000-ohm load.

analyze an existing matching network so that we can understand exactly how the impedance match occurs. Once this analysis is made, a little of the "black magic" surrounding impedance matching should subside.
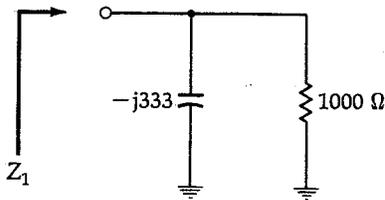
Fig. 4-6. Impedance looking into the parallel combination of R_L and X_c.
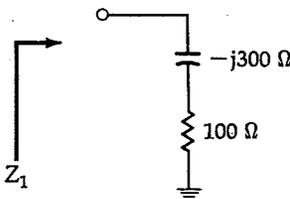

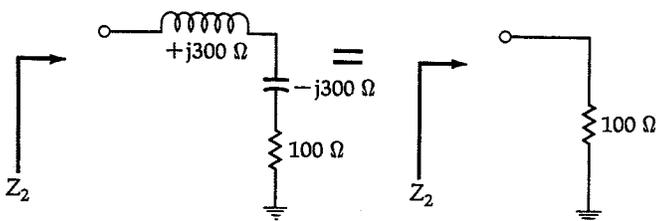Fig. 4-7. Equivalent circuit of Fig. 4-6.

Fig. 4-8. Completing the match.

Fig. 4-5 shows a simple L network impedance-matching circuit between a 100-ohm source and a 1000-ohm load. Without the impedance-matching network installed, and with the 100-ohm source driving the 1000-ohm load directly, about 4.8 dB of the available power from the source would be lost. Thus, roughly one-third of the signal *available* from the source is gone before we even get started. The impedance-matching network eliminates this loss and allows for maximum power transfer to the load. This is done by forcing the 100-ohm source to see 100 ohms when it looks into the impedance-matching network. But how?

If you analyze Fig. 4-5, the simplicity of how the match occurs will amaze you. Take a look at Fig. 4-6. The first step in the analysis is to determine what the load impedance actually looks like when the −j333-ohm capacitor is placed across the 1000-ohm load resistor. This is easily calculated by:

$$Z = \frac{X_c R_L}{X_c + R_L}$$

$$= \frac{-j333(1000)}{-j333 + 1000}$$

$$= 315 \ \angle{-71.58°}$$

$$= 100 - j300 \text{ ohms}$$

Thus, the parallel combination of the −j333-ohm ca-

pacitor and the 1000-ohm resistor *looks like* an impedance of 100 −j300 ohms. This is a *series* combination of a 100-ohm resistor and a −j300-ohm capacitor as shown in Fig. 4-7. Indeed, if you hooked a signal generator up to circuits that are similar to Figs. 4-6 and 4-7, you would not be able to tell the difference between the two as they would exhibit the same characteristics (except at dc, obviously).

Now that we have an *apparent* series 100 −j300-ohm impedance for a load, all we must do to complete the impedance match to the 100-ohm source is to add an equal and opposite (+j300 ohm) reactance in series with the network of Fig. 4-7. The addition of the +j300-ohm inductor causes cancellation of the −j300-ohm capacitor leaving only an apparent 100-ohm load resistor. This is shown in Fig. 4-8. Keep in mind here that the actual network topology of Fig. 4-5 has not changed. All we have done is to analyze small portions of the network so that we can understand the function of each component.

To summarize then, the function of the *shunt* component of the impedance-matching network is to transform a larger impedance down to a smaller value with a real part equal to the real part of the other terminating impedance (in our case, the 100-ohm source). The series impedance-matching element then resonates with or cancels any reactive component present, thus leaving the source driving an apparently equal load for optimum power transfer. So you see, the impedance match isn't "black magic" at all but can be completely explained every step of the way.

Now, back to the *design* of the impedance-matching networks of Fig. 4-4. These circuits can be very easily designed using the following equations:

$$Q_s = Q_p = \sqrt{\frac{R_p}{R_s} - 1} \qquad \text{(Eq. 4-1)}$$

$$Q_s = \frac{X_s}{R_s} \qquad \text{(Eq. 4-2)}$$

$$Q_p = \frac{R_p}{X_p} \qquad \text{(Eq. 4-3)}$$

where, as shown in Fig. 4-9:
  Q_s = the Q of the series leg,
  Q_p = the Q of the shunt leg,
  R_p = the shunt resistance,
  X_p = the shunt reactance,
  R_s = the series resistance,
  X_s = the series reactance.

The quantities X_p and X_s may be either capacitive or inductive reactance but each must be of the opposite type. Once X_p is chosen as a capacitor, for example, X_s must be an inductor, and vice-versa. Example 4-1 illustrates the procedure.

DEALING WITH COMPLEX LOADS

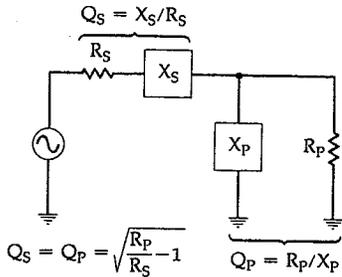The design of Example 4-1 was used for the simple case of matching two *real* impedances (pure resis-

$$Q_S = X_S/R_S$$

$$Q_S = Q_P = \sqrt{\frac{R_P}{R_S} - 1} \qquad Q_P = R_P/X_P$$

Fig. 4-9. Summary of the L-network design.

EXAMPLE 4-1

Design a circuit to match a 100-ohm source to a 1000-ohm load at 100 MHz. Assume that a dc voltage must also be transferred from the source to the load.

Solution

The need for a dc path between the source and load dictates the need for an inductor in the series leg, as shown in Fig. 4-4A. From Equation 4-1, we have:

$$Q_s = Q_p = \sqrt{\frac{1000}{100} - 1}$$

$$= \sqrt{9}$$

$$= 3$$

From Equation 4-2, we get:

$$X_s = Q_s R_s$$

$$= (3)(100)$$

$$= 300 \text{ ohms (inductive)}$$

Then, from Equation 4-3,

$$X_p = \frac{R_p}{Q_p}$$

$$= \frac{1000}{3}$$

$$= 333 \text{ ohms (capacitive)}$$

Thus, the component values at 100 MHz are:

$$L = \frac{X_s}{\omega}$$

$$= \frac{300}{2\pi(100 \times 10^6)}$$

$$= 477 \text{ nH}$$

$$C = \frac{1}{\omega X_p}$$

$$= \frac{1}{2\pi(100 \times 10^6)(333)}$$

$$= 4.8 \text{ pF}$$

This yields the circuit shown in Fig. 4-10. Notice that what you have done is to design the circuit that was previously given in Fig. 4-5 and, then, analyzed.
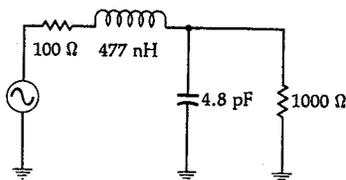

Fig. 4-10. Final circuit for Example 4-1.

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tances). It is very rare when such an occurrence actually exists in the real world. Transistor input and output impedances are almost always *complex*; that is they contain both resistive and reactive components (R ±jX). Transmission lines, mixers, antennas, and most other sources and loads are no different in that respect. Most will always have some reactive component which must be dealt with. It is, therefore, necessary to know how to handle these stray reactances and, in some instances, to actually put them to work for you.

There are two basic approaches in handling complex impedances:

1. Absorption—To actually absorb any stray reactances into the impedance-matching network itself. This can be done through prudent placement of each matching element such that element capacitors are placed in parallel with stray capacitances, and element inductors are placed in series with any stray inductances. The *stray* component values are then subtracted from the *calculated* element values leaving new element values (C', L'), which are smaller than the calculated element values.

2. Resonance—To resonate any stray reactance with an equal and opposite reactance at the frequency of interest. Once this is done the matching network design can proceed as shown for two pure resistances in Example 4-1.

Of course, it is possible to use both of the approaches outlined above at the same time. In fact, the majority of impedance-matching designs probably do utilize a little of both. Let's take a look at two simple examples to help clarify matters.

Notice that nowhere in Example 4-2 was a *conjugate* match even mentioned. However, you can rest assured that if you perform the simple analysis outlined in the previous section of this chapter, the impedance looking into the matching network, as seen by the source, will be 100 −j126 ohms, which is indeed the complex conjugate of 100 +j126 ohms.

Obviously, if the *stray* element values are larger than the calculated element values, absorption cannot take place. If, for instance, the *stray* capacitance of Fig. 4-11 were 20 pF, we could not have added a *shunt* element capacitor to give us the total needed shunt capacitance of 4.8 pF. In a situation such as this, when absorption is not possible, the concept of resonance coupled with absorption will often do the trick.

Examples 4-2 and 4-3 detail some very important concepts in the design of impedance-matching networks. With a little planning and preparation, the design of simple impedance-matching networks between complex loads becomes a simple number-crunching task using elementary algebra. Any stray reactances present in the source and load can usually be absorbed in the matching network (Example 4-2), or they can

EXAMPLE 4-2

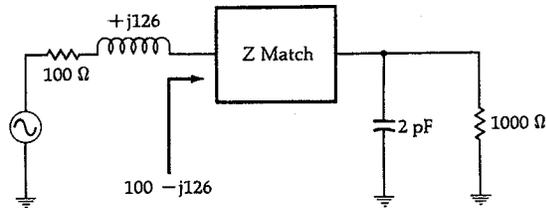Use the absorption approach to match the source and load shown in Fig. 4-11 (at 100 MHz).

Fig. 4-11. Complex source and load circuit for Example 4-2.

Solution

The first step in the design process is to totally ignore the reactances and simply match the 100-ohm real part of the source to the 1000-ohm real part of the load (at 100 MHz). Keep in mind that you would like to use a matching network that will place element inductances in series with stray inductance and element capacitances in parallel with stray capacitances. Thus, conveniently, the network circuit shown in Fig. 4-4A is again chosen for the design and, again, Example 4-1 is used to provide the details of the procedure. Thus, the calculated values for the network, if we ignore stray reactances, are shown in the circuit of Fig. 4-10. But, since the stray reactances really do exist, the design is not yet finished as we must now somehow absorb the stray reactances into the matching network. This is done as follows. At the load end, we need 4.8 pF of capacitance for the matching network. We already have a stray 2 pF available at the load so why not use it. Thus, if we use a 2.8-pF *element* capacitor, the *total* shunt capacitance becomes 4.8 pF, the design value. Similarly, at the source, matching network calls for a series 477-nH inductor. We already have a +j126-ohm, or 200-nH, inductor available in the source. Thus, if we use an actual element inductance of 477 nH — 200 nH = 277 nH, then the *total* series inductance will be 477 nH—which is the calculated design value. The final design circuit is shown in Fig. 4-12.
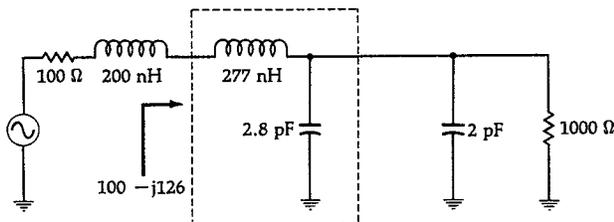

Fig. 4-12. Final design circuit for Example 4-2.

be resonated with an equal and opposite reactance, which is then absorbed into the network (Example 4-3).

THREE-ELEMENT MATCHING

Equation 4-1 reveals a potential disadvantage of the 2-element L networks described in the previous sections. It is a fact that once R_s and R_p, or the source and load impedance, are determined, the Q of the network is defined. In other words, with the L network, the designer does not have a choice of circuit Q and simply must take what he gets. This is, of course, usually the case because the source and load impedance are typically given in any design and, thus, R_p and R_s cannot be changed.

The lack of circuit-Q versatility in a matching network can be a hindrance, however, especially if a *narrow* bandwidth is required. The 3-element network overcomes this disadvantage and can be used for narrow-band high-Q applications. Furthermore, the designer can *select* any practical circuit Q that he wishes as long as it is *greater than* that Q which is possible with the L-matching network alone. In other words, the circuit Q established with an L-matching network is the *minimum* circuit Q available in the 3-element matching arrangement.

The 3-element network (shown in Fig. 4-17) is called a *Pi network* because it closely resembles the Greek letter π. Its companion network (shown in Fig. 4-18) is called a T network for equally obvious reasons.

The Pi Network

The Pi network can best be described as two "back-to-back" L networks that are both configured to match the load and the source to an invisible or "virtual" resistance located at the junction between the two networks. This is illustrated in Fig. 4-19. The significance of the negative signs for $-X_{s1}$ and $-X_{s2}$ is symbolic. They are used merely to indicate that the X_s values are the opposite type of reactance from X_{p1} and X_{p2}, respectively. Thus, if X_{p1} is a capacitor, X_{s1} must be an inductor, and vice-versa. Similarly, if X_{p2} is an inductor, X_{s2} must be a capacitor, and vice-versa. They do *not* indicate negative reactances (capacitors).

The design of each section of the Pi network proceeds exactly as was done for the L networks in the previous sections. The virtual resistance (R) must be smaller than either R_S or R_L because it is connected to the series arm of each L section but, otherwise, it can be any value you wish. Most of the time, however, R is defined by the desired loaded Q of the circuit that you specify at the beginning of the design process. For our purposes, the loaded Q of this network will be defined as:

$$Q = \sqrt{\frac{R_H}{R} - 1} \qquad (Eq. 4-4)$$

where,

R_H = the largest terminating impedance of R_s or R_L,
R = the virtual resistance.

Although this is not entirely accurate, it is a widely accepted Q-determining formula for this circuit, and is certainly close enough for most practical work. Example 4-4 illustrates the procedure.

Any of the networks in Fig. 4-21 will perform the impedance match between the 100-ohm source and the

EXAMPLE 4-3

Design an impedance matching network that will block the flow of dc from the source to the load in Fig. 4-13. The frequency of operation is 75 MHz. Try the resonant approach.
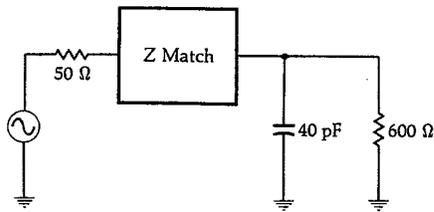

Fig. 4-13. Complex load circuit for Example 4-3.

Solution

The need to block the flow of dc from the source to the load dictates the use of the matching network of Fig. 4-4C. But, first, let's get rid of the stray 40-pF capacitor by resonating it with a shunt inductor at 75 MHz.

$$L = \frac{1}{\omega^2 C_{stray}}$$

$$= \frac{1}{[2\pi(75 \times 10^6)]^2(40 \times 10^{-12})}$$
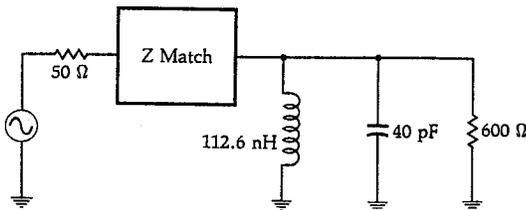
$$= 112.6 \text{ nH}$$

Fig. 4-14. Resonating the stray load capacitance.

This leaves us with the circuit shown in Fig. 4-14. Now that we have eliminated the stray capacitance, we can proceed with matching the network between the 50-ohm load and the apparent 600-ohm load. Thus,

$$Q_s = Q_p = \sqrt{\frac{R_p}{R_s} - 1}$$

$$= \sqrt{\frac{600}{50} - 1}$$

$$= 3.32$$

$$X_s = Q_s R_s$$

$$= (3.32)(50)$$

$$= 166 \text{ ohms}$$

$$X_p = \frac{R_p}{Q_p}$$

$$= \frac{600}{3.32}$$

$$= 181 \text{ ohms}$$

Therefore, the element values are:

$$C = \frac{1}{\omega X_s}$$

$$= \frac{1}{2\pi(75 \times 10^6)(166)}$$

$$= 12.78 \text{ pF}$$

$$L = \frac{X_p}{\omega}$$

$$= \frac{181}{2\pi(75 \times 10^6)}$$
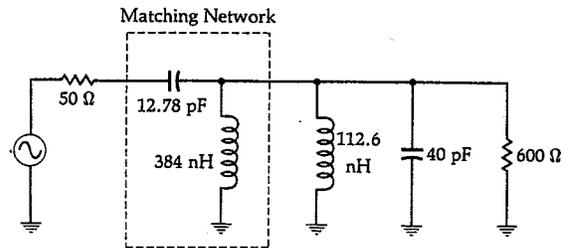
$$= 384 \text{ nH}$$

Fig. 4-15. The circuit of Fig. 4-14 after impedance matching.

These values, then, yield the circuit of Fig. 4-15. But notice that this circuit can be further simplified by simply replacing the two shunt inductors with a single inductor. Therefore,

$$L_{new} = \frac{L_1 L_2}{L_1 + L_2}$$

$$= \frac{(384)(112.6)}{384 + 112.6}$$

$$= 87 \text{ nH}$$

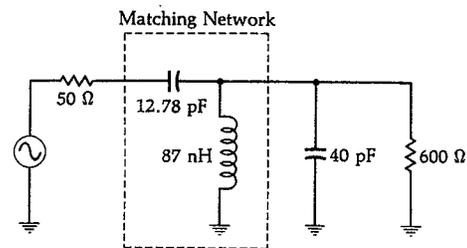The final circuit design appears in Fig. 4-16.

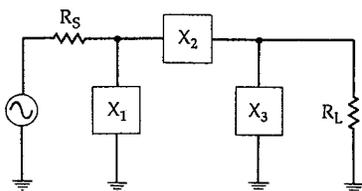Fig. 4-16. Final design circuit for Example 4-3.

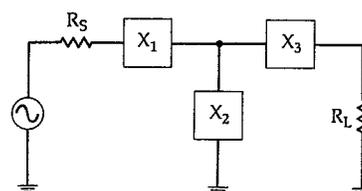Fig. 4-17. The three-element Pi network.

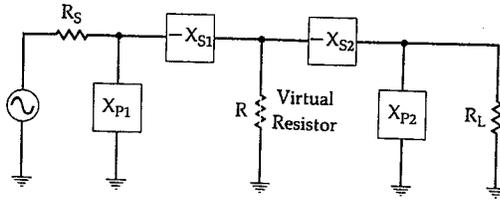Fig. 4-18. The three-element T network.

Fig. 4-19. The Pi network shown as two
back-to-back L networks.

1000-ohm load. The one that you choose for each particular application will depend on any number of factors including:

1. The elimination of stray reactances.
2. The need for harmonic filtering.
3. The need to pass or block dc voltage.

The T network

The design of the 3-element T network is exactly the same as for the Pi network except that with the T, you match the load and the source, through two L-type networks, to a virtual resistance which is *larger* than either the load or source resistance. This means that the two L-type networks will then have their shunt legs connected together as shown in Fig. 4-22.

The T network is often used to match two low-valued impedances when a high-Q arrangement is needed. The loaded Q of the T network is determined by the L section that has the highest Q. By definition, the L section with the highest Q will occur on the end which has the *smallest* terminating resistor. Remember, too, that each terminating resistor is in the *series* leg of each network. Therefore, the formula for determining the loaded Q of the T network is:

$$Q = \sqrt{\frac{R}{R_{small}} - 1} \qquad (Eq. 4-5)$$

where,

R = the virtual resistance,
R_{small} = the smallest terminating resistance.

This formula is exactly the same as the Q formula that was previously given for the Pi-type networks. However, since we have reversed or "flip-flopped" the L sections to produce the T network, we must also make sure that we redefine the Q formula to account for the new resistor placement, in relation to those L networks. In other words, Equations 4-4 and 4-5 are only special applications of the general formula that is given in Equation 4-1 (and repeated here for convenience).

$$Q = \sqrt{\frac{R_p}{R_s} - 1} \qquad (Eq. 4-1)$$

where,

R_p = the resistance in the shunt branch of the L network,
R_s = the resistance in the series branch of the L network.

So, try not to get confused with the different definitions of circuit Q. They are all the same.

Each L network is calculated in exactly the same manner as was given in the previous examples and, as we shall soon see, we will also end up with four possible configurations for the T network (Example 4-5).

LOW-Q OR WIDEBAND MATCHING NETWORKS

Thus far in this chapter we have studied: (1) the L network, which has a circuit Q that is automatically defined when the source and load impedances are set, and (2) the Pi and T networks, which allow us to select a circuit Q independent of the source and load impedances *as long as the Q chosen is larger than that which is available with the L network*. This seems to indicate, and rightfully so, that the Pi and T networks are great for narrow-band matching networks. But what if an impedance match is required over a fairly broad range of frequencies. How do we handle that? The answer is to simply use two L sections in still another configuration as shown in Fig. 4-25. Notice here that the virtual resistor is in the shunt leg of one L section and in the series leg of the other L section. We, therefore, have two *series-connected* L sections rather than the back-to-back configuration of the Pi and T networks. In this new configuration, the value of the virtual resistor (R) must be larger than the smallest termination impedance and, also, smaller than the largest termination impedance. Of course, any virtual resistance that satisfies these criteria may be chosen. The net result is a range of loaded-Q values that is *less than* the range of Q values obtainable from either a single L section, or the Pi and T networks previously described.

The maximum bandwidth (minimum Q) available from this network is obtained when the virtual resistor (R) is made equal to the geometric mean of the two impedances being matched.

$$R = \sqrt{R_S R_L} \qquad (Eq. 4-6)$$

The loaded Q of the network, for our purposes, is defined as:

$$Q = \sqrt{\frac{R}{R_{smaller}} - 1} = \sqrt{\frac{R_{larger}}{R} - 1} \qquad (Eq. 4-7)$$

where,

R = the virtual resistance,
$R_{smaller}$ = the smallest terminating resistance,
R_{larger} = the largest terminating resistance.

If even wider bandwidths are needed, more L networks may be cascaded with virtual resistances between each network. Optimum bandwidths in these cases are obtained if the ratios of each of the two succeeding resistances are equal:

$$\frac{R_1}{R_{smaller}} = \frac{R_2}{R_1} = \frac{R_3}{R_2} \cdots = \frac{R_{larger}}{R_n} \qquad (Eq. 4-8)$$

EXAMPLE 4-4

Using Fig. 4-19 as a reference, design four different Pi networks to match a 100-ohm source to a 1000-ohm load. Each network must have a loaded Q of 15.

Solution

From Equation 4-4, we can find the virtual resistance we will be matching.

$$R = \frac{R_H}{Q^2 + 1}$$

$$= \frac{1000}{226}$$

$$= 4.42 \text{ ohms}$$

To find X_{p2} we have:

$$X_{p2} = \frac{R_p}{Q_p}$$

$$= \frac{R_L}{Q}$$

$$= \frac{1000}{15}$$

$$= 66.7 \text{ ohms}$$

Similarly, to find X_{s2}:

$$X_{s2} = QR_{series}$$

$$= 15(R)$$

$$= (15)(4.42)$$

$$= 66.3 \text{ ohms}$$

This completes the design of the L section on the *load* side of the network. Note that R_{series} in the above equation was substituted for the virtual resistor R, which by definition is in the series arm of the L section.

The Q for the other L network is now defined by the ratio of R_s to R, as per Equation 4-1, where:

$$Q_1 = \sqrt{\frac{R_s}{R} - 1}$$

$$= \sqrt{\frac{100}{4.42} - 1}$$

$$= 4.6$$

Notice here that the source resistor is now considered to be in the shunt leg of the L network. Therefore, R_s is defined as R_p, and

$$X_{p1} = \frac{R_p}{Q_1}$$

$$= \frac{100}{4.6}$$

$$= 21.7 \text{ ohms}$$

Similarly,

$$X_{s1} = Q_1 R_{series}$$

$$= Q_1 R$$

$$= (4.6)(4.46)$$

$$= 20.51 \text{ ohms}$$

The actual network design is now complete and is shown in Fig. 4-20. Remember that the virtual resistor (R) is not really in the circuit and, therefore, is not shown. Reactances $-X_{s1}$ and $-X_{s2}$ are now in series and can simply be added together to form a single component.

So far in the design, we have dealt only with reactances and have not yet computed actual component values. This
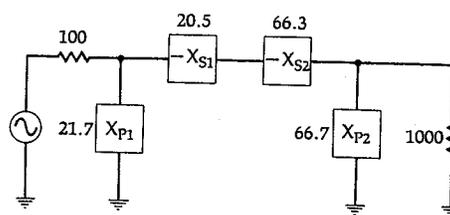

Fig. 4-20. Calculated reactances for Example 4-4.

is because of the need to maintain a general design approach so that four final networks can be generated quickly as per the problem statement.

Notice that X_{p1}, X_{s1}, X_{p2}, and X_{s2} can all be either capacitive or inductive reactances. The only constraint is that X_{p1} and X_{s1} are of opposite types, and X_{p2} and X_{s2} are of opposite types. This yields the four networks of Fig. 4-21 (the source and load have been omitted). Each component in Fig. 4-21 is shown as a reactance (in ohms). Therefore, to perform the transformation from the dual-L to the Pi network, the two series components are merely added if they are alike, and subtracted if the reactances are of opposite type. The final step, of course, is to change each reactance into a component value of capacitance and inductance at the frequency of operation.
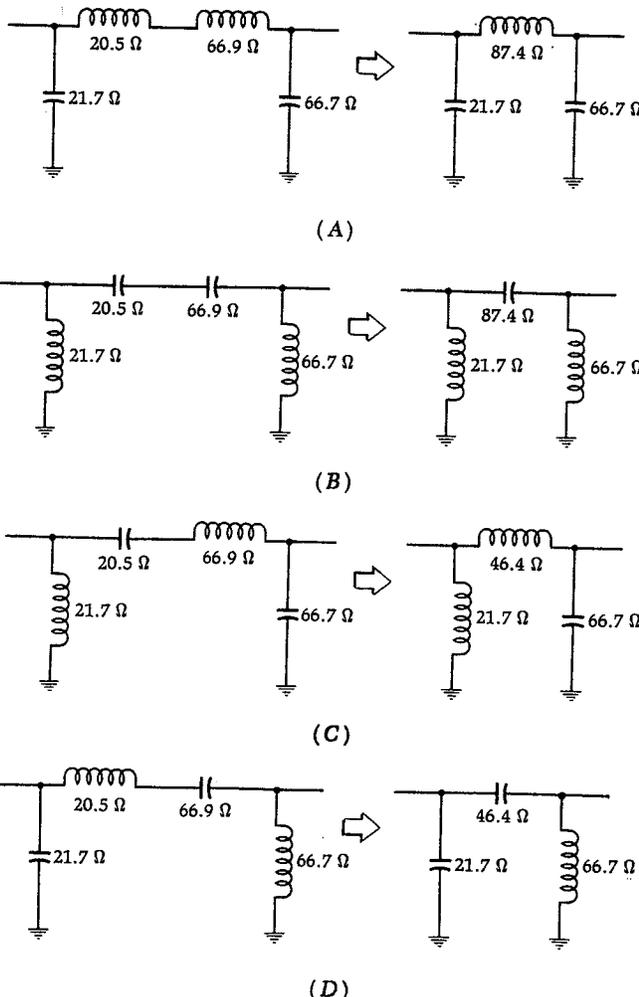

(A)

(B)

(C)

(D)

Fig. 4-21. The transformation from double-L to Pi networks.
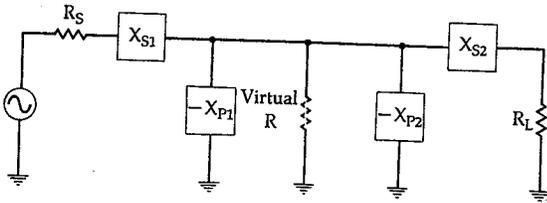
Fig. 4-22. The T network shown as two back-to-back L networks.

where,

$R_{smaller}$ = the smallest terminating resistance,
R_{larger} = the largest terminating resistance,
$R_1, R_2, \ldots R_n$ = virtual resistors.

This is shown in Fig. 4-26.

The design procedure for these wideband matching networks is precisely the same as was given for the previous examples. To design for a specific low Q, simply solve Equation 4-7 for R to find the virtual

---

EXAMPLE 4-5

Using Fig. 4-22 as a reference, design four different networks to match a 10-ohm source to a 50-ohm load. Each network is to have a loaded Q of 10.

Solution

Using Equation 4-5, we can find the virtual resistance we need for the match.

$$R = R_{small}(Q^2 + 1)$$
$$= 10(101)$$
$$= 1010 \text{ ohms}$$

From Equation 4-2:

$$X_{s1} = QR_s$$
$$= 10(10)$$
$$= 100 \text{ ohms}$$

From Equation 4-3:

$$X_{p1} = \frac{R}{Q}$$
$$= \frac{1010}{10}$$
$$= 101 \text{ ohms}$$

Now, for the L network on the load end, the Q is defined by the virtual resistor and the load resistor. Thus,

$$Q_2 = \sqrt{\frac{R}{R_L} - 1}$$
$$= \sqrt{\frac{1010}{50} - 1}$$
$$= 4.4$$

Therefore,

$$X_{p2} = \frac{R}{Q_2}$$
$$= \frac{1010}{4.4}$$
$$= 230 \text{ ohms}$$
$$X_{s2} = Q_2 R_L$$
$$= (4.4)(50)$$
$$= 220 \text{ ohms}$$

The network is now complete and is shown in Fig. 4-23 without the virtual resistor.

The two shunt reactances of Fig. 4-23 can again be combined to form a single element by simply substituting a value that is equal to the combined equivalent parallel reactance of the two.

The four possible T-type networks that can be used for matching the 10-ohm source to the 50-ohm load are shown in Fig. 4-24.
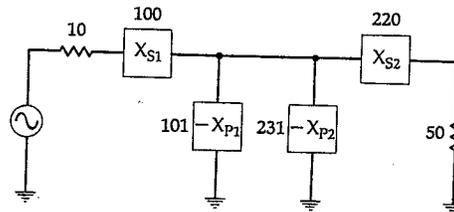

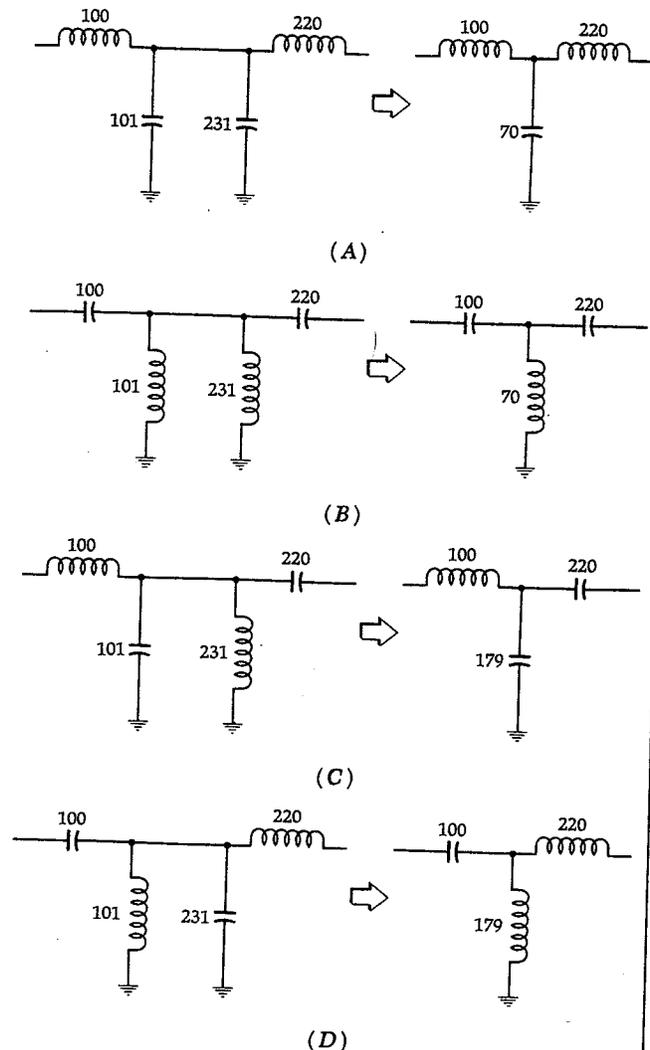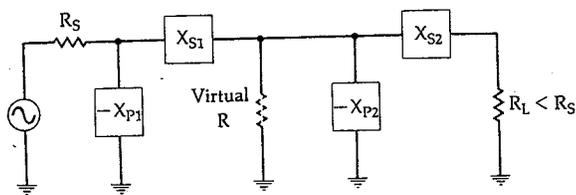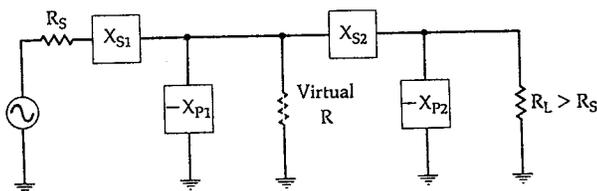Fig. 4-23. The calculated reactances of Example 4-5.

(A)

(B)

(C)

(D)

Fig. 4-24. The transformation of circuits from double-L to T-type networks.

(A) R in shunt leg.

(B) R in series leg.

Fig. 4-25. Two series-connected L networks for lower Q applications.
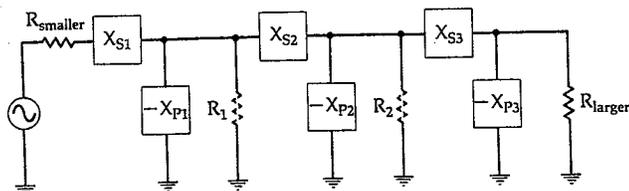

Fig. 4-26. Expanded version of Fig. 4-25 for even wider bandwidths.

resistance needed. Or, to design for an optimally wide bandwidth, solve Equation 4-6 for R. Once R is known, the design is straightforward.

THE SMITH CHART

Probably one of the most useful graphical tools available to the rf circuit designer today is the Smith Chart shown in Fig. 4-27. The chart was originally conceived back in the Thirties by a Bell Laboratories engineer named Phillip Smith, who wanted an easier method of solving the tedious repetitive equations that often appear in rf theory. His solution, appropriately named the Smith Chart, is still widely in use.

At first glance, a Smith Chart appears to be quite complex. Indeed, why would anyone of sound mind even care to look at such a chart? The answer is really quite simple; once the Smith Chart and its uses are understood, the rf circuit designer's job becomes much less tedious and time consuming. Very lengthy complex equations can be solved graphically on the chart in seconds, thus lessening the possibility of errors creeping into the calculations.

Smith Chart Construction

The mathematics behind the construction of a Smith Chart are given here for those that are interested. It is important to note, however, that you do not *need* to know or understand the mathematics surrounding the actual construction of a chart as long as you

understand what the chart represents and how it can be used to your advantage. Indeed, there are so many uses for the chart that an entire volume has been written on the subject. In this chapter, we will concentrate mainly on the Smith Chart as an impedance matching tool and other uses will be covered in later chapters. The mathematics follow.

The reflection coefficient of a load impedance when given a source impedance can be found by the formula:

$$\rho = \frac{Z_s - Z_L}{Z_s + Z_L} \qquad \text{(Step 1)}$$

In normalized form, this equation becomes:

$$\rho = \frac{Z_0 - 1}{Z_0 + 1} \qquad \text{(Step 2)}$$

where Z_0 is a complex impedance of the form $R + jX$.

The polar form of the reflection coefficient can also be represented in rectangular coordinates:

$$\rho = p + jq$$

Substituting into Step 2, we have:

$$p + jq = \frac{R + jX - 1}{R + jX + 1} \qquad \text{(Step 3)}$$

If we solve for the real and imaginary parts of $p + jq$, we get:

$$p = \frac{R^2 - 1 + X^2}{(R + 1)^2 + X^2} \qquad \text{(Step 4)}$$

and,

$$q = \frac{2x}{(R + 1)^2 + X^2} \qquad \text{(Step 5)}$$

Solve Step 5 for X:

$$X = \left(\frac{p(R + 1)^2 - R^2 + 1}{1 - p} \right)^{1/2} \qquad \text{(Step 6)}$$

Then, substitute Step 6 into Step 5 to obtain:

$$\left(p - \frac{R}{R + 1} \right)^2 + q^2 = \left(\frac{1}{R + 1} \right)^2 \qquad \text{(Step 7)}$$

Step 7 is the equation for a family of circles whose centers are at:

$$p = \frac{R}{R + 1}$$

$$q = 0$$

and whose radii are equal to:

$$\frac{1}{R + 1}$$

These are the constant resistance circles, some of which are shown in Fig. 4-28A.

Similarly, we can eliminate R from Steps 4 and 5 to obtain:

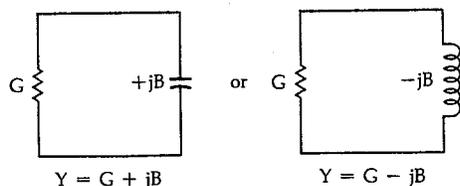$$(p - 1)^2 + \left(q - \frac{1}{X} \right)^2 = \left(\frac{1}{X} \right)^2 \qquad \text{(Step 8)}$$

Fig. 4-34. Circuit representation for admittance.

tion on the chart. For instance, rather than locating a point 180° away from our original starting point, why not just rotate the chart itself 180° while fixing the starting point in space? The result is the same, and it can be read directly off of the rotated chart without performing a single construction. This is shown in Fig. 4-36 (Smith Chart Form ZY-01-N)* where the rotated chart is shown in black. Notice that the impedance plotted (solid lines on the red coordinates) is located at $Z = 1 + j1$ ohms, and the reciprocal of that (the admittance) is shown by dotted lines on the black coordinates as $Y = 0.5 - j0.5$. Keep in mind that because we have rotated the chart 180° to obtain the admittance coordinates, the upper half of the admittance chart represents *negative susceptance* (−jB) which is *inductive*, while the lower half of the admittance chart represents a *positive susceptance* (+jB) which is *capacitive*. Therefore, nothing has been lost in the rotation process.

The chart shown in Fig. 4-36, containing the superimposed impedance and admittance coordinates, is an extremely useful version of the Smith Chart and is the one that we will use throughout the remainder of the book. But first, let's take a closer look at the admittance coordinates alone.

Admittance Manipulation on the Chart

Just as the impedance coordinates of Figs. 4-32 and 4-33 were used to obtain a visual indication of what occurs when a *series* reactance is added to an *impedance*, the admittance coordinates provide a visual indication of what occurs when a *shunt* element is added to an *admittance*. The addition of a shunt capacitor is shown in Fig. 4-37. Here we begin with an admittance of $Y = 0.2 - j0.5$ mho and add a shunt capacitor with a susceptance (reciprocal of reactance) of +j0.8 mho. Mathematically, we know that parallel susceptances are simply added together to find the equivalent susceptance. When this is done, the result becomes:

$$Y = 0.2 - j0.5 + j0.8$$
$$= 0.2 + j0.3 \text{ mho}$$

If this point is plotted on the admittance chart, we quickly recognize that all we have done is to move along a constant conductance circle (G) *downward* (clockwise) a distance of $jB = 0.8$ mho. In other words,

the real part of the admittance has not changed, only the imaginary part has. Similarly, as Fig. 4-38 indicates, adding a shunt inductor to an admittance moves the point along a constant conductance circle upward (counterclockwise) a distance (−jB) equal to the value of its susceptance.

If we again superimpose the impedance and admittance coordinates and combine Figs. 4-32, 4-33, 4-37, and 4-38 for the general case, we obtain the useful chart shown in Fig. 4-39. This chart graphically illustrates the direction of travel, along the impedance and admittance coordinates, which results when the particular type of component that is indicated is added to an existing impedance or admittance. A simple example should illustrate the point (Example 4-6).

IMPEDANCE MATCHING ON THE SMITH CHART

Because of the ease with which series and shunt components can be added in ladder-type arrangements on the Smith Chart, while easily keeping track of the impedance as seen at the input terminals of the structure, the chart seems to be an excellent candidate for an impedance-matching tool. The idea here is simple. Given a load impedance and given the impedance that the source would like to see, simply plot the load impedance and, then, begin adding series and shunt elements on the chart until the desired impedance is achieved—just as was done in Example 4-6.

Two-Element Matching

Two-element matching networks are mathematically very easy to design using the formulas provided in earlier sections of this chapter. For the purpose of illustration, however, let's begin our study of a Smith Chart impedance-matching procedure with the simple network given in Example 4-7.

To make life much easier for you as a Smith Chart user, the following equations may be used. For a series-C component:

$$C = \frac{1}{\omega XN} \qquad \text{(Eq. 4-11)}$$

For a series-L component:

$$L = \frac{XN}{\omega} \qquad \text{(Eq. 4-12)}$$

For a shunt-C component:

$$C = \frac{B}{\omega N} \qquad \text{(Eq. 4-13)}$$

For a shunt-L component:

$$L = \frac{N}{\omega B} \qquad \text{(Eq. 4-14)}$$

where,
  $\omega = 2\pi f$,
  X = the reactance as read from the chart,

NAME	TITLE	DWG. NO.
SMITH CHART FORM 82-BSPR (9-66)	KAY ELECTRIC COMPANY, PINE BROOK, N.J., © 1966. PRINTED IN U.S.A.	DATE

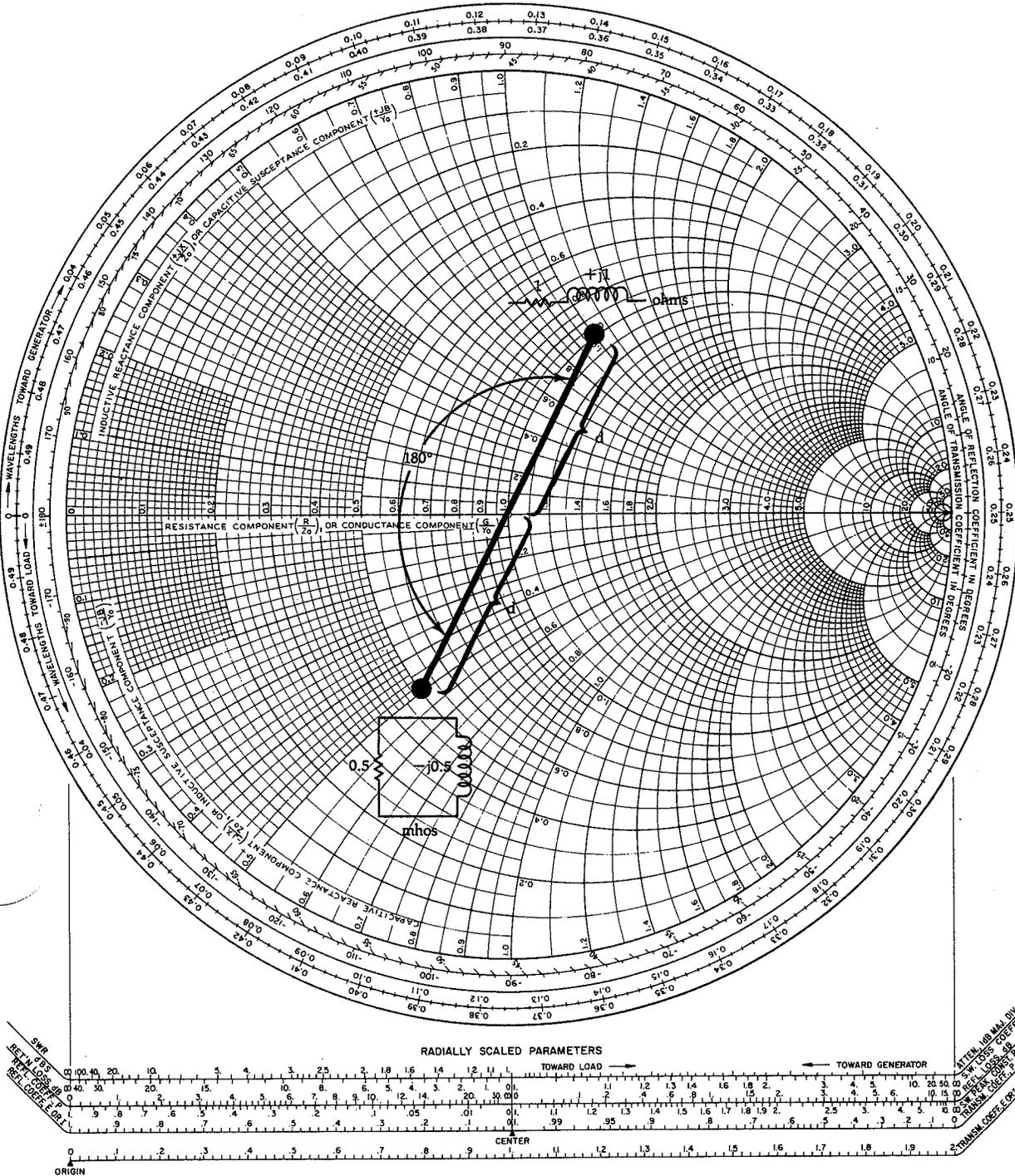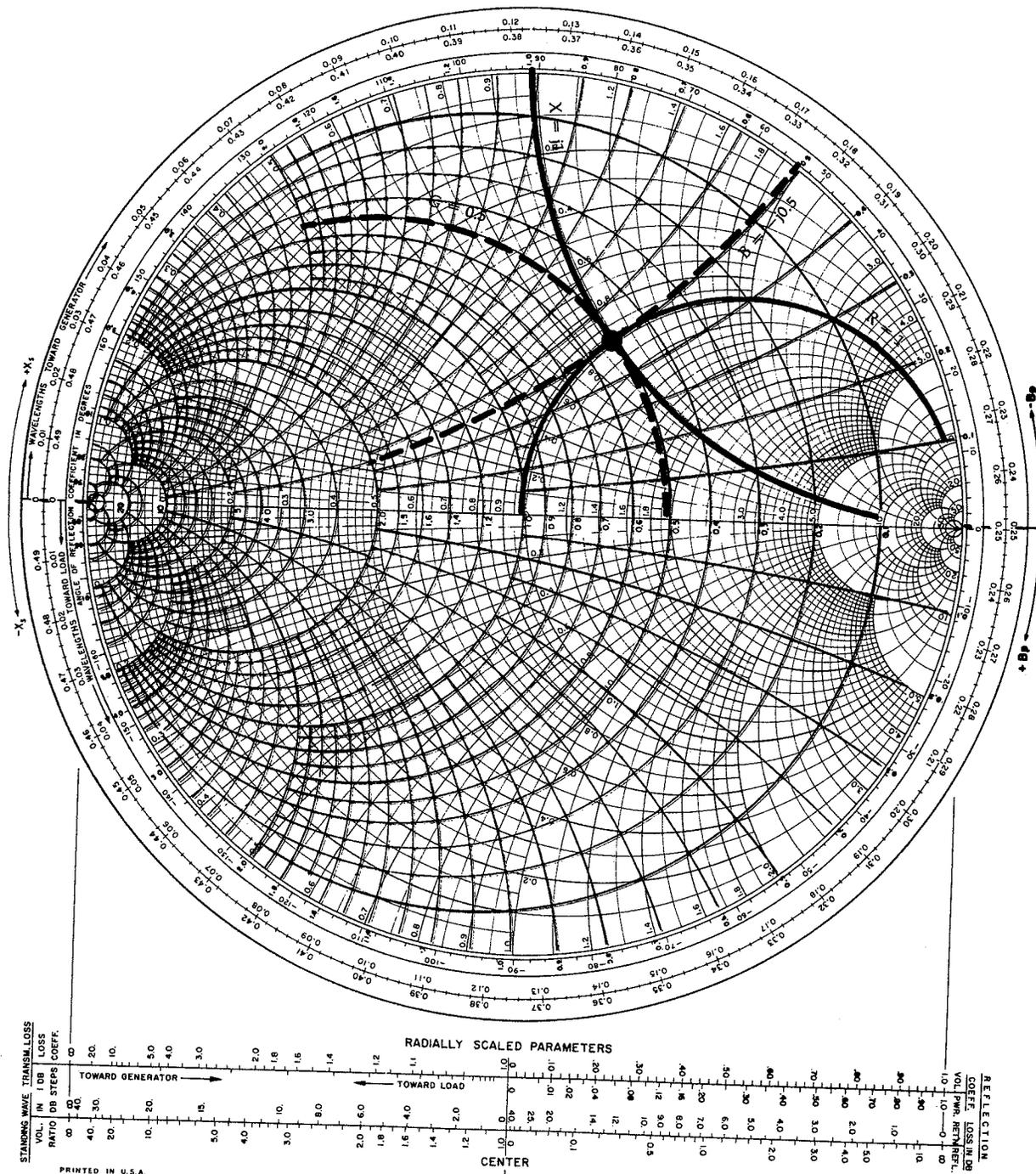## IMPEDANCE OR ADMITTANCE COORDINATES

RADIALLY SCALED PARAMETERS

A MEGA-CHART

Fig. 4-35. Impedance-admittance conversion on the Smith Chart.

NAME	TITLE		DWG. NO.
SMITH CHART FORM ZY-01-N	ANALOG INSTRUMENTS COMPANY, NEW PROVIDENCE, N.J. 07974		DATE

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

RADIALLY SCALED PARAMETERS

TOWARD GENERATOR →   ← TOWARD LOAD

CENTER

PRINTED IN U.S.A.

COPYRIGHT 1970 BY ANALOG INSTRUMENTS COMPANY, NEW PROVIDENCE, N.J.
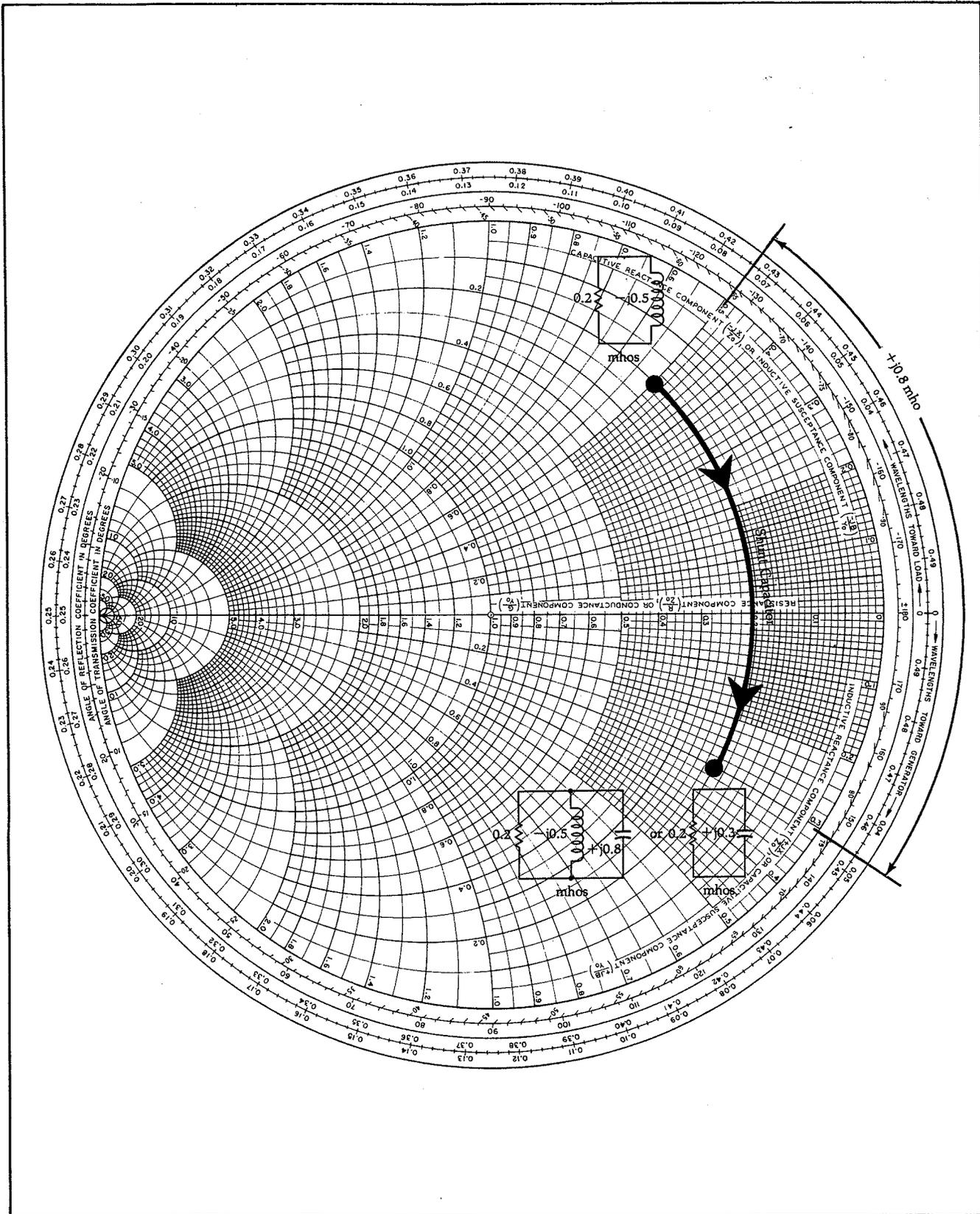
Fig. 4-36. Superimposed admittance coordinates.

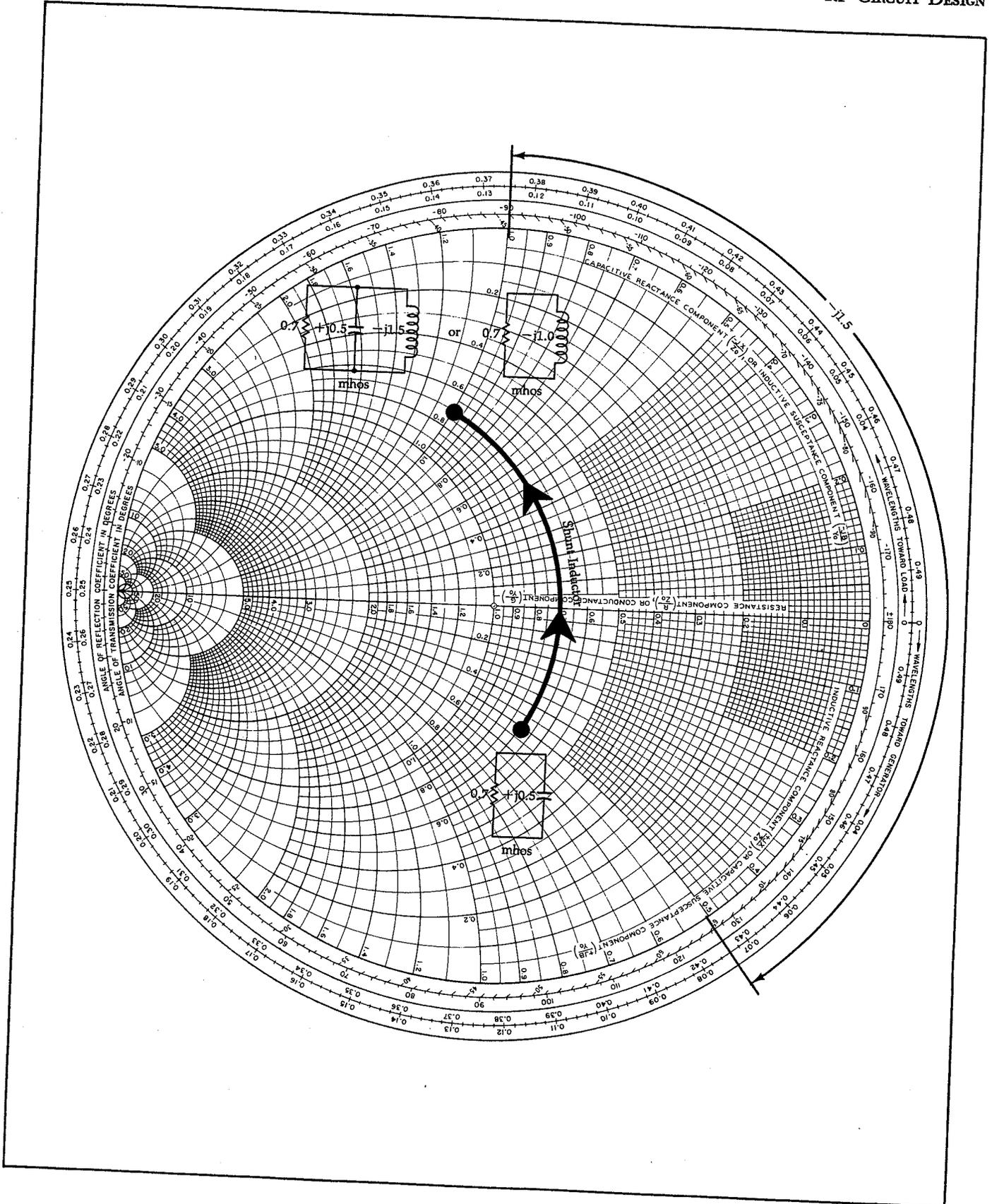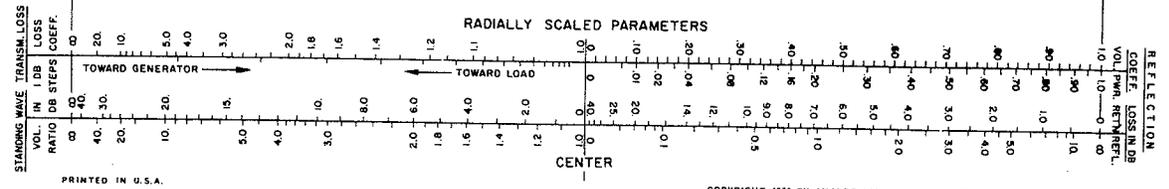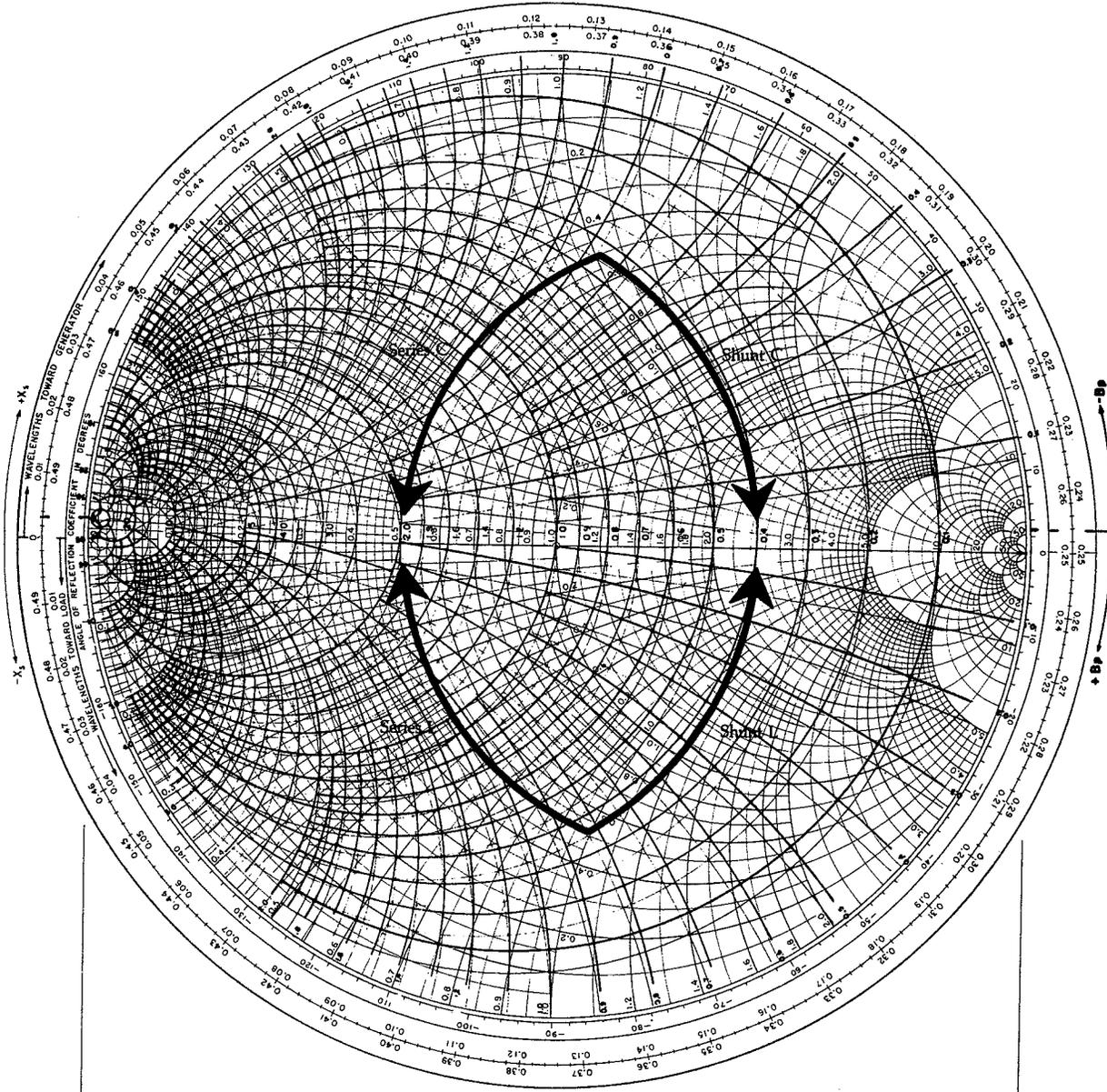Fig. 4-37. Addition of a shunt capacitor.

Fig. 4-38. Addition of a shunt inductor.

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

Fig. 4-39. Summary of component addition on a Smith Chart.

B = the susceptance as read from the chart,

N = the number used to normalize the original impedances that are to be matched.

If you use the preceding equations, you will never have to worry about changing susceptances into reactances before unnormalizing the impedances. The equations take care of both operations. The only thing you have to do is read the value of susceptance (for shunt components) or reactance (for series components) directly off of the chart, plug this value into the equation used, and wait for your actual component values to pop out.

Three-Element Matching

In earlier sections of this chapter, you learned that the only real difference between two-element and three-element matching is that with three-element matching, you are able to choose the loaded Q for the network. That was easy enough to do in a mathematical-design approach due to the virtual resistance concept. But how can circuit Q be represented on a Smith Chart?

As you have seen before, in earlier chapters, the Q of a series-impedance circuit is simply equal to the ratio of its reactance to its resistance. Thus, any point on a Smith Chart has a Q associated with it. Alternately, if you were to specify a certain Q, you could find an infinite number of points on the chart that could satisfy that Q requirement. For example, the following impedances located on a Smith Chart have a Q of 5:

$$R + jX = 1 \pm j5$$
$$= 0.5 \pm j2.5$$
$$= 0.2 \pm j1$$
$$= 0.1 \pm j0.5$$
$$= 0.05 \pm j0.25$$

These values are plotted in Fig. 4-45 and form the arcs shown. Thus, any impedance located on these arcs must have a Q of 5. Similar arcs for other values of Q can be drawn with the arc of infinite Q being located along the perimeter of the chart and the Q = 0 arc (actually a straight line) lying along the pure resistance line located at the center of the chart.

The design of high-Q three-element matching networks on a Smith Chart is approached in much the same manner as in the mathematical methods presented earlier in this chapter. Namely, one branch of the network will determine the loaded Q of the circuit, and it is this branch that will set the characteristics of the rest of the circuit.

The procedure for designing a three-element impedance-matching network for a specified Q is summarized as follows:

1. Plot the constant-Q arcs for the specified Q.

2. Plot the load impedance and the complex conjugate of the source impedance.

3. Determine the end of the network that will be used to establish the loaded Q of the design. For T networks, the end with the *smaller* terminating resistance determines the Q. For Pi networks, the end with the *larger* terminating resistor sets the Q.

4. For T networks:

$$R_s > R_L$$

EXAMPLE 4-6

What is the impedance looking into the network shown in Fig. 4-40? Note that the task has been simplified due to the fact that shunt susceptances are shown rather than shunt reactances.
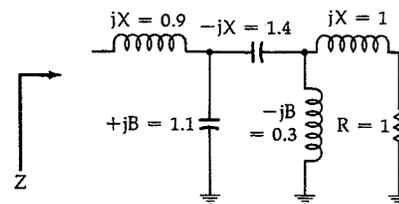

Fig. 4-40. Circuit for Example 4-6.

Solution

This problem is very easily handled on a Smith Chart and not a single calculation needs to be performed. The solution is shown in Fig. 4-42. It is accomplished as follows.

First, break the circuit down into individual branches as shown in Fig. 4-41. Plot the impedance of the series RL branch where $Z = 1 + j1$ ohm. This is point A in Fig. 4-42. Next, following the rules diagrammed in Fig. 4-39, begin adding each component back into the circuit—one at a time. Thus, the following constructions (Fig. 4-42) should be noted:
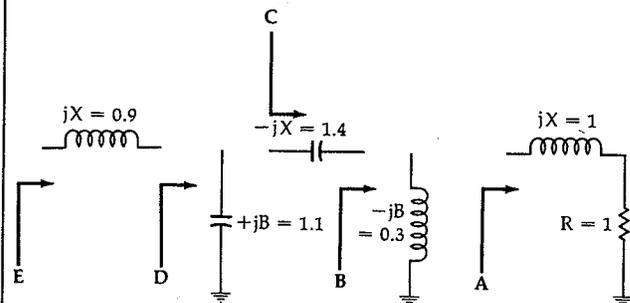

Fig. 4-41. Circuit is broken down into individual branch elements.

Arc AB = shunt L = $-jB$ = 0.3 mho
Arc BC = series C = $-jX$ = 1.4 ohms
Arc CD = shunt C = $+jB$ = 1.1 mhos
Arc DE = series L = $+jX$ = 0.9 ohm

The impedance at point E (Fig. 4-42) can then be read directly off of the chart as $Z = 0.2 + j0.5$ ohm.

EXAMPLE 4-6—Cont.

NAME	TITLE		
SMITH CHART FORM ZY-01-N	ANALOG INSTRUMENTS COMPANY, NEW PROVIDENCE, N.J. 07974	DWG. NO.	
		DATE	

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

RADIALLY SCALED PARAMETERS
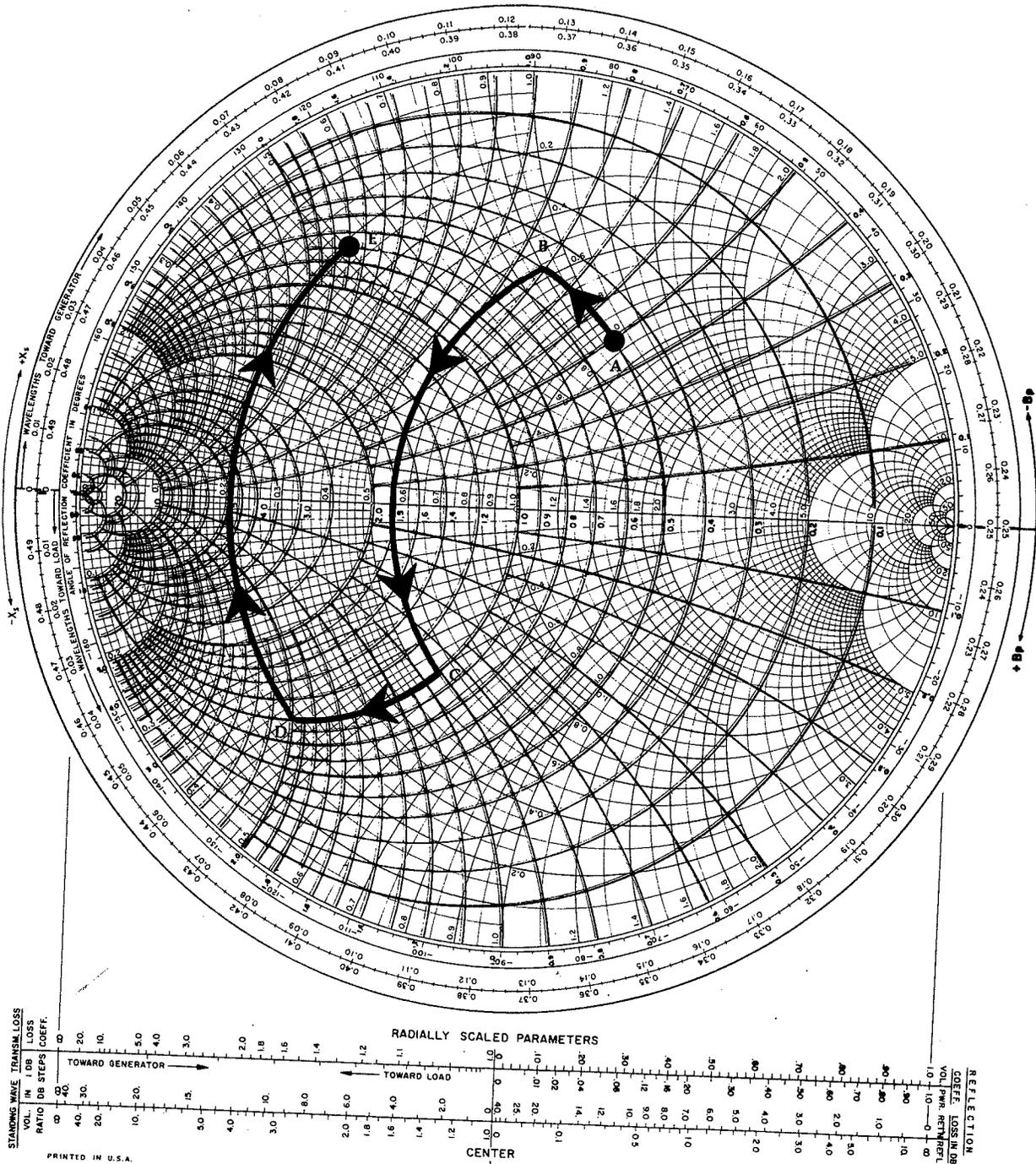
TOWARD GENERATOR →

← TOWARD LOAD

CENTER

Fig. 4-42. Smith Chart solution for Example 4-6.

Move from the load along a constant-R circle (series element) and intersect the Q curve. The length of this move determines your first element. Then, proceed from this point to Z_s^* ($Z_s^* = Z_s$ conjugate) in two moves—first with a shunt and, then, with a series element.

$$R_s < R_L$$

Find the intersection (I) of the Q curve and the source impedance's R = constant circle, and plot that point. Move *from the load impedance* to point I with two elements—first, a series element and, then, a shunt element. Move from point I to Z_s^* along the R = constant circle with another series element.

5. For Pi networks:

$$R_S > R_L$$

Find the intersection (I) of the Q curve and the source impedance's G = constant circle, and plot that point. Move from the load impedance to point

I with two elements—first, a shunt element and, then, a series element. Move from point I to Z_s^* along the G = constant circle with another shunt element.

$$R_S < R_L$$

Move from the load along a constant G circle (shunt element) and intersect the Q curve. The length of this move determines your first element. Then, proceed from this point to Z_s^* in two moves—first, with a series element and, then, with a shunt element.

The above procedures might seem complicated to the neophyte but remember that we are only forcing the constant-resistance or constant-conductance arc, located between the Q-determining termination and the specified-Q curve, to be one of our matching elements. An example may help to clarify matters (Example 4-8).

Multielement Matching

In multielement matching networks where there is no Q constraint, the Smith Chart becomes a veritable

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EXAMPLE 4-7

Design a two-element impedance-matching network on a Smith Chart so as to match a 25 − j15-ohm source to a 100 − j25-ohm load at 60 MHz. The matching network must also act as a low-pass filter between the source and the load.

Solution

Since the source is a complex impedance, it wants to "see" a load impedance that is equal to its complex conjugate (as discussed in earlier sections of this chapter). Thus, the task before us is to force the 100 − j25-ohm load to look like an impedance of 25 + j15 ohms.

Obviously, the source and load impedances are both too large to plot on the chart, so normalization is necessary. Let's choose a convenient number (N = 50) and divide all impedances by this number. The results are 0.5 + j0.3 ohm for the impedance the source would like to see and 2 − j0.5 ohms for the actual load impedance. These two values are easily plotted on the Smith Chart, as shown in Fig. 4-44, where, at point A, Z_L is the *normalized* load impedance and, at point C, Z_s^* is the *normalized* complex conjugate of the source impedance.

The requirement that the matching network also be a low-pass filter forces us to use some form of series-L, shunt-C arrangement. The only way we can get from the impedance at point A to the impedance at point C and still fulfill this requirement is along the path shown in Fig. 4-44. Thus, following the rules of Fig. 4-39, the arc AB of Fig. 4-44 is a shunt capacitor with a value of +jB = 0.73 mho. The arc BC is a series inductor with a value of +jX = 1.2 ohms.

The shunt capacitor as read from the Smith Chart is a susceptance and can be changed into an equivalent reactance by simply taking the reciprocal.

$$X_C = \frac{1}{+jB}$$

$$= \frac{1}{j0.73 \text{ mho}}$$

$$= -j1.37 \text{ ohms}$$

To complete the network, we must now unnormalize all impedance values by *multiplying* them by the number N = 50—the value originally used in the normalization process. Therefore:

$$X_L = 60 \text{ ohms}$$
$$X_C = 68.5 \text{ ohms}$$

The component values are:

$$L = \frac{X_L}{\omega}$$

$$= \frac{60}{2\pi(60 \times 10^6)}$$

$$= 159 \text{ nH}$$

$$C = \frac{1}{\omega X_C}$$

$$= \frac{1}{2\pi(60 \times 10^6)(68.5)}$$
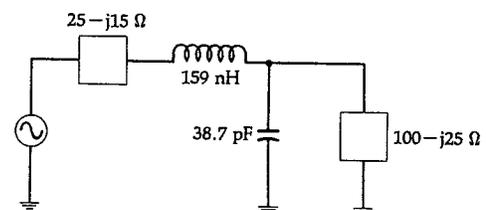
$$= 38.7 \text{ pF}$$

The final circuit is shown in Fig. 4-43.

Fig. 4-43. Final circuit for Example 4-7.

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