Lecture 7

Stability Considerations

→ The stability of an amplifier, or its resistance to oscillate, is a very important consideration in a design and can be determined from the S-parameters, the matching networks, and the terminations.

→ Oscillations are possible when either the input or output port presents a negative resistance. This occurs when

$|\Gamma_{IN}| > 1$ or $|\Gamma_{OUT}| > 1$

For unilateral devices this means $|S_{11}| > 1$ or $|S_{22}| > 1$

We know that for unilateral transistor $S_{12} = 0$

&

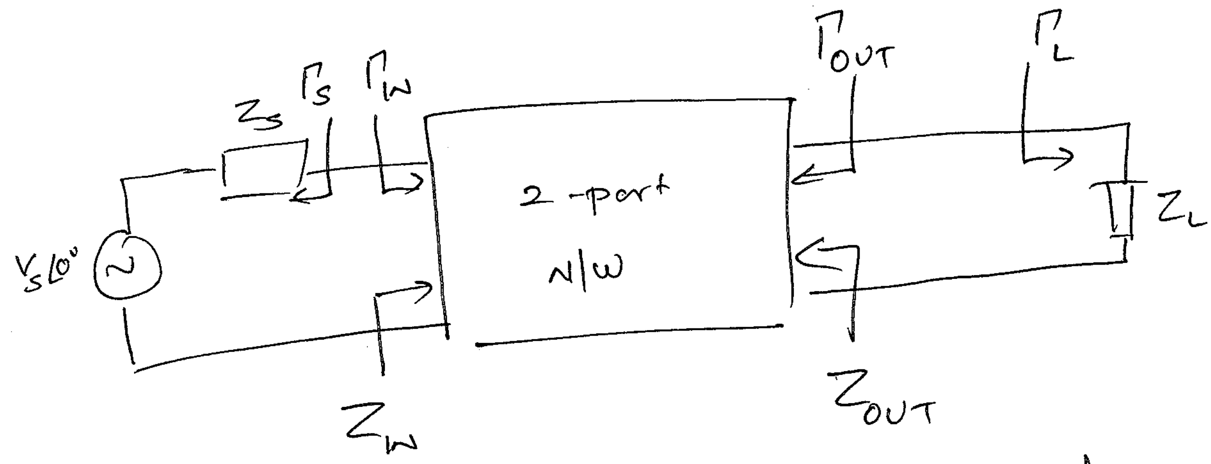
$$\Gamma_{IN} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} \quad \text{reduces to } S_{11}$$

$$\Gamma_{OUT} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S} \quad \text{reduces to } S_{22}$$

$\therefore |\Gamma_{IN}| = |S_{11}|$ & $|\Gamma_{OUT}| = |S_{22}|$

If $|S_{11}| > 1$ the transistor presents negative resistance at input & if $|S_{22}| > 1$ the transistor presents negative resistance at the output

Consider a two port N/w as shown in the figure below.



The two port N/w is said to be unconditionally stable at a given frequency if the real parts of Z_{IN} and Z_{OUT} are greater than zero for all passive load and source impedance.

The condition for unconditional stability at a given frequency are:

$$|\Gamma_S| < 1 \quad -(1)$$

$$|\Gamma_L| < 1 \quad -(2)$$

$$|\Gamma_{IN}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1 \quad -(3)$$

$$|\Gamma_{OUT}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1 \quad -(4)$$

all coefficients are normalized to the same characteristic impedance Z_0

Equation (1) & (2) state that the source and load are passive while (3) and (4) state that i/p & o/p impedance must also be passive (no negative resistance associated with their real parts)

When a two port N/w is potentially unstable there may be values of Γ_S and Γ_L for which the real parts of Z_{IN} and Z_{OUT} are positive. These values of Γ_S and Γ_L can be determined using the following graphical procedure.

① Regions where values of Γ_L and Γ_S produce $|\Gamma_{IN}|=1$ and $|\Gamma_{OUT}|=1$ are determined.

This is achieved by setting eqn (3) and (4) to 1 and solving for Γ_L and Γ_S.

We get solutions for Γ_L and Γ_S on circles (called stability Cs) whose equations are given by

$$\left| \Gamma_L - \frac{(S_{22} - \Delta S_{11}^{*})^{*}}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12} \, S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \quad -(5)$$

$$\left| \Gamma_S - \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \quad -(6)$$

$$\Delta = S_{11} S_{22} - S_{12} S_{21}$$

The radii and center of the ⊙ where $|\Gamma_{IN}| = 1$ and $|\Gamma_{OUT}| = 1$ in the Γ_L and Γ_S plane are given as

Γ_L values for $|\Gamma_{IN}| = 1$ (output stability ⊙s

$$r_L = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \quad \text{(radius)} \quad -(7)$$

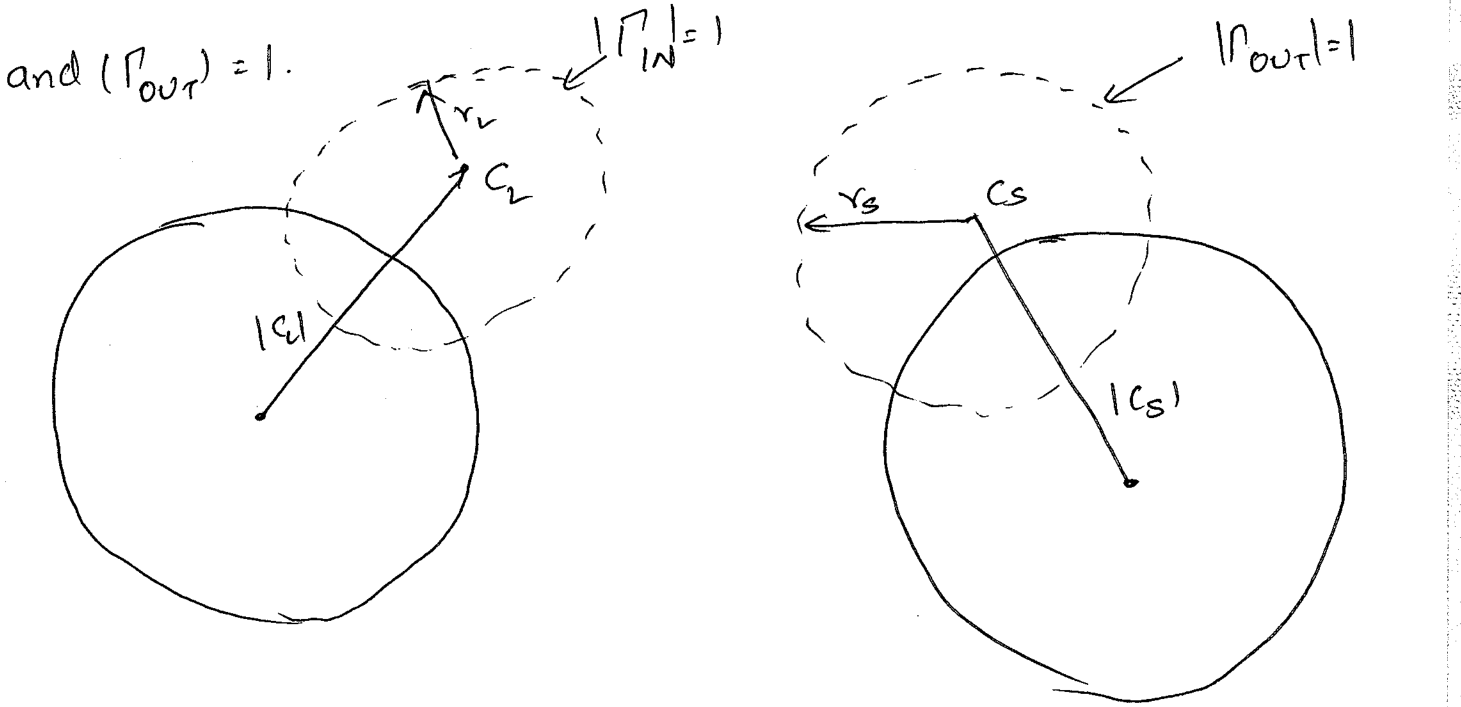
$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \quad \text{(center)} \quad -(8)$$

Γ_S values for $|\Gamma_{OUT}| = 1$ (Input stability ⊙s)

$$r_S = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \quad \text{(radius)} \quad -(9)$$

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \quad \text{(center)} \quad -(10)$$

The stability condition can be established by drawing the stability ⊙s on the smith chart and determining the set of values of Γ_L and Γ_S that produce $|\Gamma_{IN}|=1$ and $|\Gamma_{OUT}|=1$.



On one side of the stability ⊙ in the Γ_L plane we will have $|\Gamma_{IN}|<1$ and on other side $|\Gamma_{IN}|>1$.

In the Γ_S plane on one side of stability ⊙ boundary we will have $|\Gamma_{OUT}|<1$ and on othe side $|\Gamma_{OUT}|>1$

Next we determine which area on the Smith chart has the stable region. We have four cases.

1) when $Z_L = Z_0$ $\quad \Gamma_L = 0$ $\qquad \Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$|\Gamma_{IN}| = |S_{11}|$

If $|S_{11}| < 1$ $\quad |\Gamma_{IN}| < 1$ when $\Gamma_L = 0$

② If $|S_{11}| > 1$ $\quad |\Gamma_{IN}| > 1$ when $\Gamma_L = 0$.

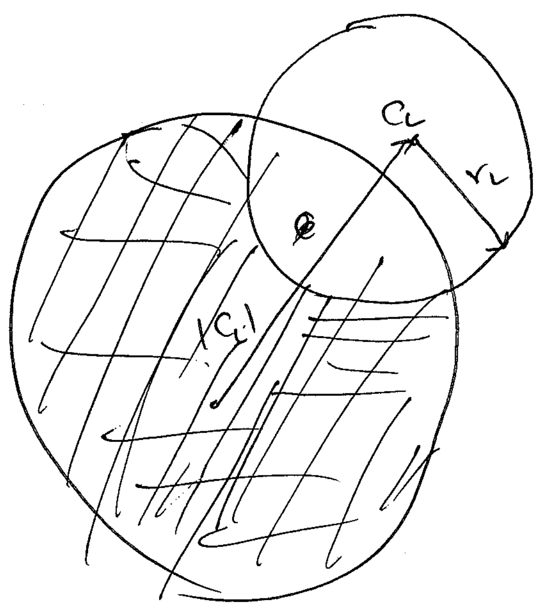
③ If $Z_S = Z_0$ $\quad \Gamma_S = 0$

$|\Gamma_{OUT}| = |S_{22}|$

If $|S_{22}| < 1$ $\quad |\Gamma_{OUT}| < 1$ when $\Gamma_S = 0$

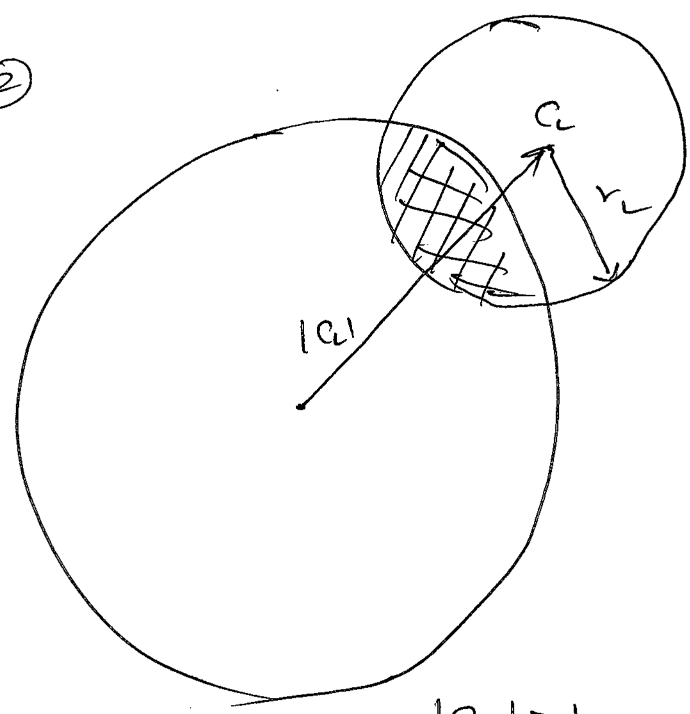
④ $|S_{22}| > 1$ $\quad |\Gamma_{OUT}| > 1$ when $\Gamma_S = 0$

①



$|S_{11}| < 1$

②



$|S_{11}| > 1$

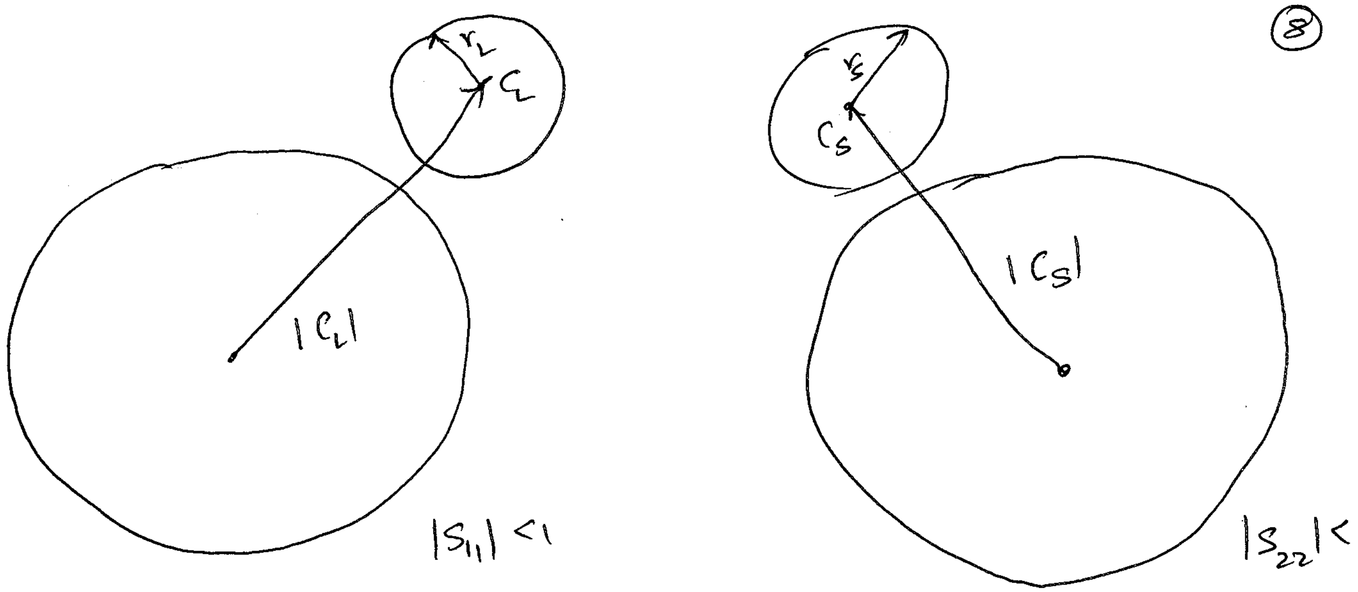
For unconditional stability any passive load or source in the network must produce a stable condition. From a graphical point of view, for $|S_{11}| < 1$ and $|S_{22}| < 1$ we want the stability Os to fall completely outside the Smith chart. The case in which the stability Os fall completely outside the Smith chart. The condition where stability Os fall completely outside the Smith chart is shown in figure below. The conditions for unconditional stability for all passive sources & loads can be expressed in the form

$$||C_L| - r_L| > 1 \quad \text{for } |S_{11}| < 1$$

&

$$||C_S| - r_S| > 1 \quad \text{for } |S_{22}| < 1$$

If $|S_{11}| > 1$ or $|S_{22}| > 1$ the n/w cannot be unconditionally stable because the termination $\Gamma_L = 0$ or $\Gamma_S = 0$ will produce $|\Gamma_{IN}| > 1$ or $|\Gamma_{OUT}| > 1$



Necessary and Sufficient Conditions for a two-port n/w to be Unconditionally Stable.

Using (1) & (4)

$$K > 1$$

$$1 - |S_{11}|^2 > |S_{12} S_{21}| \quad -(1)$$

$$1 - |S_{22}|^2 > |S_{12} S_{21}| \quad -(2)$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 |S_{12} S_{21}|} \quad -(3)$$

$$\Delta = S_{11} S_{22} - S_{12} S_{21} \quad -(4)$$

From (1) and (2)

$$2 - |S_{11}|^2 - |S_{22}|^2 > 2|S_{12}S_{21}| \qquad -(5)$$

Since

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| \le |S_{11}S_{22}| + |S_{12}S_{21}| \qquad -(6)$$

we use (6) to obtain

$$|\Delta| < |S_{11}S_{22}| + 1 - \frac{1}{2}|S_{11}|^2 - \frac{1}{2}|S_{22}|^2$$

$$|\Delta| < 1 - \frac{1}{2}(|S_{11}| - |S_{22}|)^2$$

or simply

$$|\Delta| < 1$$

Hence Necessary and sufficient condition is.

$$K > 1 \qquad -(7)$$

&

$$|\Delta| < 1 \qquad -(8)$$

There is one more way:

$$K > 1 \quad \&$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0 \qquad -(9)$$

3.3.1

① $f = 500\,MHz$

The Complete Smith Chart

Black Magic Design

② Calculate K & Δ

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 0.482$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$= 0.221\angle{-123^\circ}$$

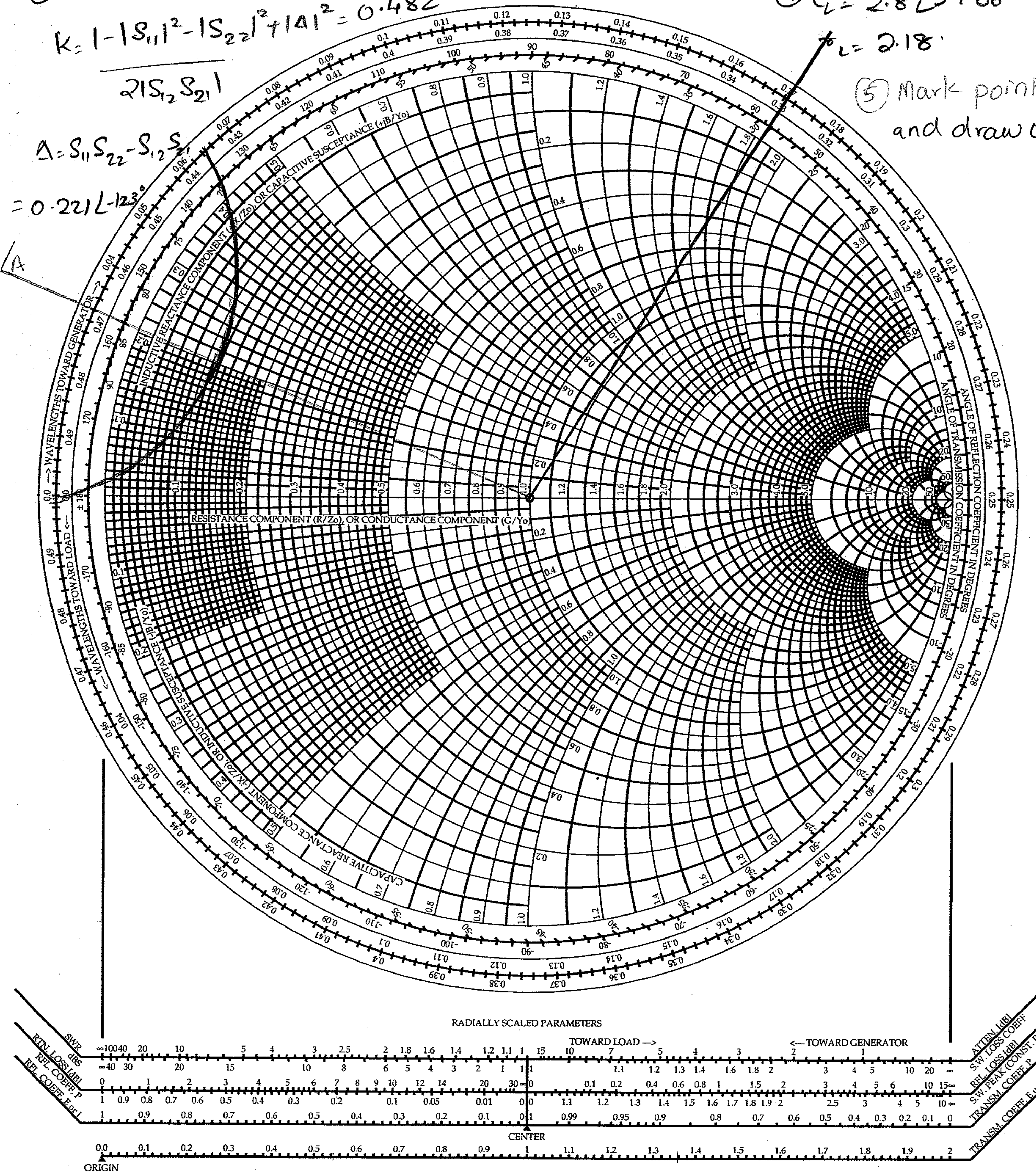
③ $C_S = 1.36\angle 157.6^\circ$

$r_s = 0.558$

④ $C_L = 2.8\angle 57.86^\circ$

$r_L = 2.18$

⑤ Mark point A and draw ⑥



eg 3.3.1

Unilateral Case :-

For unilateral case $S_{12}=0$

$\Gamma_{IN}=S_{11}$ $\Gamma_{OUT}=S_{22}$

Hence we have unconditional stability if $|S_{11}|<1$ and $|S_{22}|<1$ for all passive source and load terminations.

With $S_{12}=0$ $K=\infty$ $\Delta=S_{11}S_{22}$ &

$$1-|S_{11}|^2-|S_{22}|^2+|S_{11}S_{22}|^2>0$$

or

$$(1-|S_{11}|^2)(1-|S_{22}|^2)>0$$

The preceding inequality requires that $|S_{11}|<1$ & $|S_{22}|<1$ for unconditional stability of a unilateral two-port.

In potentially unstable situation the real part of input and output impedances can be negative for some source & load reflection coefficients. In this case selecting Γ_S and Γ_L in the stable region produces a stable operation.

Even if Γ_L and Γ_S produces $|\Gamma_{IN}|>1$ or $|\Gamma_{OUT}|>1$ the circuit can be made stable if the total i/p & o/p loop resistance is positive. The circuit is stable if

$$Re\,(Z_S+Z_{IN}) \geq 0$$

$$Re\,(Z_L+Z_{OUT}) > 0$$

A potentially unstable transistor can be unconditionally stable by either resistively loading the transistor or by adding negative feedback. These techniques are not recommended in narrow band amplifiers because of the resulting degradation in power gain, noise figure & VSWRs.

eg. S parameter of a properly biased BJT are found at 1 GHz as follows. $S_{11}=0.6\angle{-155^\circ}$ $S_{22}=0.48\angle{-20^\circ}$, $S_{12}=0$ & $S_{21}=6\angle{180^\circ}$

Determine the maximum gain possible with this transistor and design an RF circuit that can provide this gain.

Solution

1) Check Stability

$$K = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = \infty$$

because $S_{12} = 0$

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| = |S_{11}S_{22}| = 0.2909$$

$\cancel{\text{1}}$ $K > 1$ $|\Delta| < 1$

Since both conditions are satisfied, the transistor is unconditionally stable.

2) The maximum possible power gain is

$$G_{TU} = \frac{1-|\Gamma_S|^2}{(1-S_{11}\Gamma_S)^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

&

$$G_{TU\max} = \frac{1-|S_{11}^*|^2}{|1-|S_{11}|^2|^2} |S_{21}|^2 \frac{1-|S_{22}^*|^2}{|1-|S_{22}|^2|^2}$$

$$= 73.9257$$

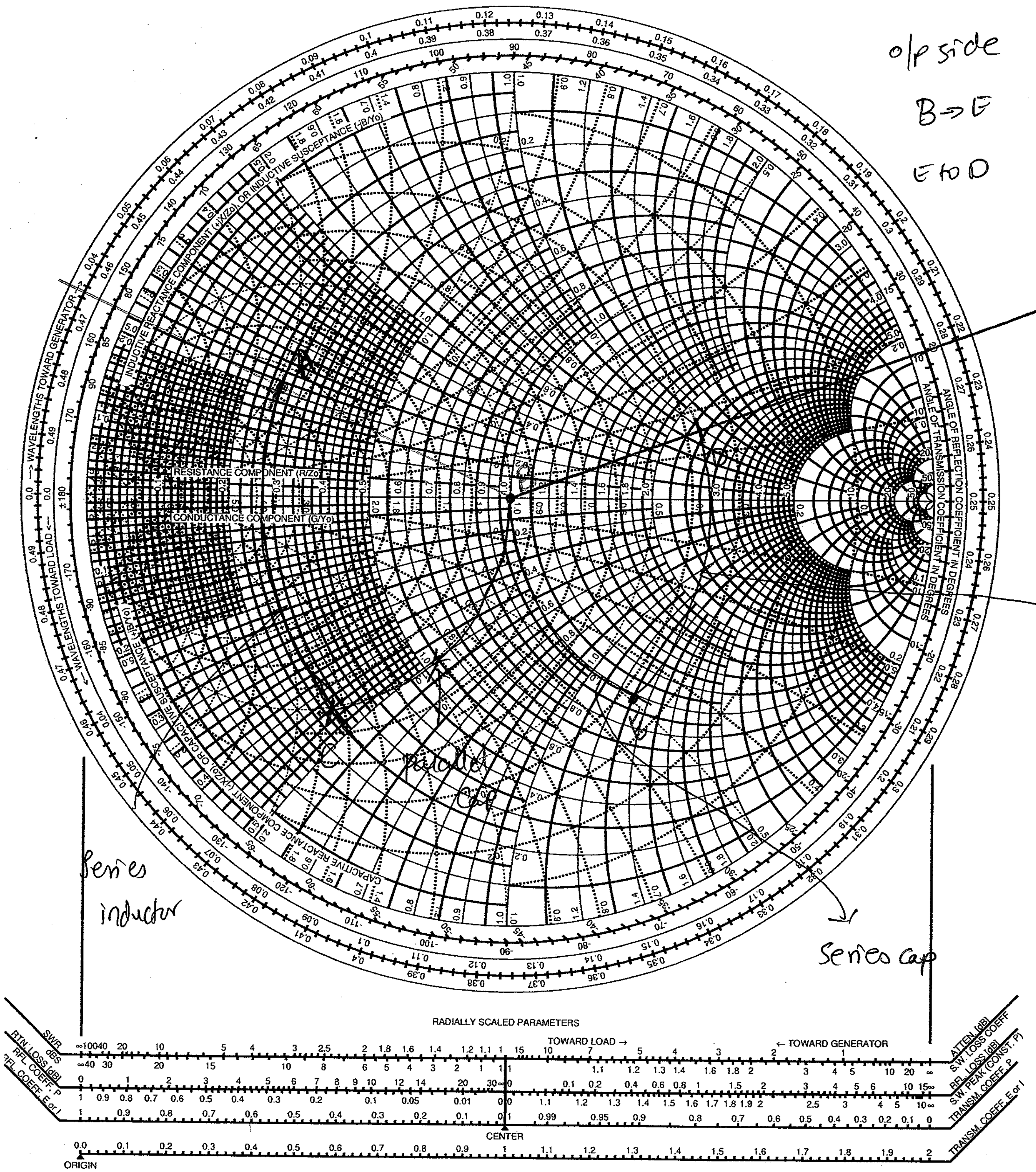
$$G_{TU\max} = 10\log_{10}(73.9257)\ \text{dB} = 18.688\ \text{dB}$$

$\Gamma_S = 0.60\angle 155^\circ$

The Complete Smith Chart (ZY)

i/p side
B-C &
C to A

o/p side
B→E
E to D



parallel inductor

For maximum unilateral power gain

$\Gamma_S = S_{11}^* = 0.606 \angle 155^\circ$ and $\Gamma_L = S_{22}^* = 0.48 \angle 20^\circ$

See Smith Chart

Constant Gain ○'s :- Unilateral Case

if $S_{12} = 0$

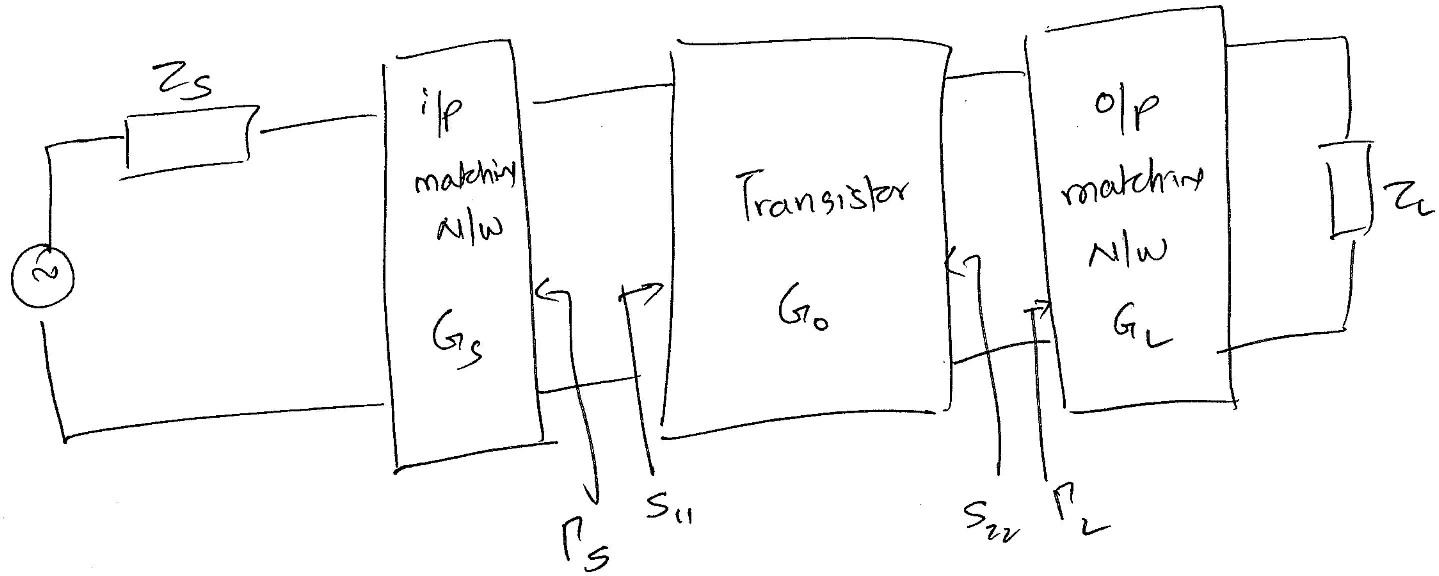
$\Gamma_{IN} = S_{11}$ $\Gamma_{OUT} = S_{22}$

transistor scattering parameters.

$$G_{TU} = \underbrace{\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}}_{\text{depends on transistor } S_{11} \text{ and Source reflection coefficient}} \; |S_{21}|^2 \; \frac{1-|\Gamma_L|^2}{\underbrace{|1-S_{22}\Gamma_L|^2}_{S_{22} \text{ parameter of transistor \& load reflection coefficient}}}$$

$$G_{TU} = G_S \, G_0 \, G_L$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} \qquad G_0 = |S_{21}|^2 \qquad G_L = \frac{1-|\Gamma_L|^2}{(1-S_{22}\Gamma_L)^2}$$



G_S, G_L - gain or loss produced by matching or mismatching between ~~Γ_S & S_{11}~~ i/p & o/p ckts.

$G_S \rightarrow$ degree of matching or mismatching between Γ_S & S_{11}

$$G_{TU}(dB) = G_S(dB) + G_0(dB) + G_L(dB)$$

If we optimize Γ_S and Γ_L to provide maximum gain to G_S & G_L we refer to the gain as maximum unilateral transducer power gain, $G_{TU,max}$

For unilateral unconditionally stable transistor (ie for $|S_{11}| < 1$ and $|S_{22}| < 1$) the maximum values of G_S and G_L are obtained when

$$\Gamma_S = S_{11}^* \quad \text{and} \quad \Gamma_L = S_{22}^*$$

$$G_{S,max} = \frac{1}{1-|S_{11}|^2}$$

$$G_{L,max} = \frac{1}{1-|S_{22}|^2}$$

$$G_{TU,max} = \frac{1}{1-|S_{11}|^2} |S_{21}|^2 \frac{1}{1-|S_{22}|^2}$$

We know that in the unilateral case $\Gamma_{IN} = S_{11}$ and and $\Gamma_{OUT} = S_{22}$, The maximomum value of G_{TU} occurs when $\Gamma_S = S_{11}^* = \Gamma_{IN}^*$ and $\Gamma_L = S_{22}^* = \Gamma_{OUT}^*$ and this is epual to maximum value of power gain G_p & available power gain G_A . ie $G_{TU,max}$

$= G_{PU,max} = G_{AU,max}$

The general expressions for G_S & G_L are written as.

$$G_i = \frac{1-|\Gamma_i|^2}{|1-S_{ii}\Gamma_i|^2} \qquad \begin{array}{l} i = S \text{ with } ii = 11 \\ i = L \text{ with } ii = 22 \end{array}$$

Unconditional stable case, $|S_{ii}| < 1$

$$G_i = \frac{1 - |\Gamma_i|^2}{|1 - S_{ii}\Gamma_i|^2}$$

Max is optained when $\Gamma_i = S_{ii}^*$

$$G_{i,max} = \frac{1 - |S_{ii}^*|^2}{|1 - S_{ii} S_{ii}^*|^2}$$

$$= \frac{1}{1 - |S_{ii}|^2}$$

The terminations that produce $G_{i,max}$ are called the optimum terminations

G_i has a maximum value of zero when $|\Gamma_i| = 1$. Other values of Γ_i produce values of G_i between 0 & ~~G_i~~ $G_{i,max}$

$$0 \le G_i \le G_{i,max}.$$

The values of Γ_i that produce a constant gain G_i will be shown to lie on a ⊙ in the Smith chart called the constant gain circles. (constant G_S & G_L ⊙s)