Operating and available Power Gain Circles.

Operating power gain Circles

The operating power gain circle for both unconditionally stable and potentially unstable transistors is simple and recommended for practical designs.

Unconditionally stable bilateral case.

$$G_p = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{\left(1-\left|\frac{S_{11}-\Delta \Gamma_L}{1-S_{22}\Gamma_L}\right|^2\right)|1-S_{22}\Gamma_L|^2}$$

$$= |S_{21}|^2 g_p$$

$$g_p = \frac{G_p}{|S_{21}|^2} = \frac{1-|\Gamma_L|^2}{1-|S_{11}|^2+|\Gamma_L|^2(|S_{22}|^2-|\Delta|^2)-2Re(\Gamma_L C_2)}$$

$$C_2 = S_{22}-\Delta S_{11}^*$$

The values of Γ_L that produce a constant value of g_p are shown to lie on a ⊙, known as an operating power-gain circles. The equation for an operating power-gain ⊙ in Γ_L plane, with g_p as a parameter is

$$|\Gamma_L - C_p| = r_p$$

$$C_p = \frac{g_p C_2^*}{1+g_p(|S_{22}|^2-|\Delta|^2)}$$

$$r_p = \frac{[1 - 2K|S_{12}S_{21}|g_p + |S_{12}S_{21}|^2 g_p^2]^{1/2}}{|1 + g_p(|S_{22}|^2 - |\Delta|^2)}$$

angle of the ⊙ is C_2^*

Maximum operating power gain occurs at the value of Γ_L when $g_p = 0$.

$$g_{p,max}^2 |S_{12}S_{21}|^2 - 2K|S_{12}S_{21}|g_{p,max} + 1 = 0$$

$$g_{p,max} = \frac{1}{|S_{12}S_{21}|}(K - \sqrt{K^2 - 1})$$

$$G_{p,max} = \frac{|S_{21}|}{|S_{12}|}(K - \sqrt{K^2-1})$$

Value of Γ_L that produces $G_{p,max}$ follows by substituting $g_p = g_{p,max}$ in C_p. This value of $C_{p,max}$ must be equal to Γ_{ML}.

$$\Gamma_{ML} = C_{p,max} = \frac{g_{p,max} C_2^*}{1 + g_{p,max}(|S_{22}|^2 - |\Delta|^2)}$$

Lowest value of g_p is zero which corresponds to $G_p = 0$ it occurs when $|\Gamma_L| = 1$ (operating power gain is zero when all the o/p power is reflected from the load).

For a given G_p, Γ_L is selected from constant operating power-gain circles. $G_{p,max}$ results when Γ_L is selected at a distance where $g_{p,max} = \dfrac{G_{p,max}}{|S_{21}|^2}$

The maximum o/p power results when a conjugate match is selected at the i/p (i.e $\Gamma_S = \Gamma_{IN}^*$)

when $\Gamma_S = \Gamma_{IN}^*$ i/p power is equal to maximum available i/p power.

Drawing constant operating power gain Os.

1) For given G_p calculate C_p & r_p
2) Select Desired Γ_L
3) For given Γ_L maximum o/p power is obtained with a conjugate match at the input - namely, $\Gamma_S = \Gamma_{IN}^*$
This value of Γ_S produces the transducer power gain $G_T = G_p$

eg.

Design the amplifier for operating power gain of 9 dB instead of $G_{T,max} = G_{P,max} = 11.38$

Soln

1) $|S_{21}|^2 = (2.058)^2 = 4.235$ or 6.27 dB.

$$g_p = \frac{G_p}{|S_{21}|^2} = \frac{7.94}{4.235} = 1.875$$

We know that $K = 1.504$

$|\Delta| = 0.3014$

$C_2 = S_{22} - \Delta S_{11}^* = 0.3911 \angle -103.9°$

$$r_p = \frac{[1 - 2K|S_{12}S_{21}|g_p + (S_{12}S_{21})^2 g_p^2]^{1/2}}{|1 + g_p(|S_{22}|^2 - |\Delta|^2)|}$$

$= \cancel{1.875}\ 0.431$

$$C_p = \frac{g_p C_2^*}{1 + g_p(|S_{22}|^2 - |\Delta|^2)}$$

$= 0.508 \angle 103.9°$

i/p VSWR is 1 since $\Gamma_S = \Gamma_{in}^*$

For o/p VSWR

$$|\Gamma_b| = \left| \frac{\Gamma_{out} - \Gamma_L^*}{1 - \Gamma_{out}\Gamma_L} \right| = 0.622$$

$$(VSWR)_{out} = \frac{1 + 0.622}{1 - 0.622}$$

$= 4.3$

Potentially unstable bilateral case

The design procedure for a given G_p is as follows.

① Draw constant power gain ⊙s using

$$C_p = \frac{g_p C_2^*}{1+g_p(|S_{22}|^2-|\Delta|^2)}$$

$$r_p = \frac{[1-2K|S_{12}S_{21}|g_p + |S_{12}S_{21}|^2 g_p^2]^{1/2}}{|1+g_p(|S_{22}|^2-|\Delta|^2)|}$$

② Draw o/p stability ⊙

$$r_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2-|\Delta|^2}\right|$$

$$C_L = \frac{(S_{22}-\Delta S_{11}^*)^*}{|S_{22}|^2-|\Delta|^2}$$

③ Select value of Γ_L that is in the stable region and not too close to the stability circle

④ Calculate Γ_{IN}

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

and determine if a conjugate match at the i/p is feasible.

↳ Draw i/p stability ⊙ calculating

$$r_S = \left|\frac{S_{12}S_{21}}{|S_{11}|^2-|\Delta|^2}\right|$$

$$C_S = \frac{(S_{11}-\Delta S_{22}^*)^*}{|S_{11}|^2-|\Delta|^2}$$

Determine $\Gamma_S = \Gamma_{IN}^*$ lies in the i/p stable region

5) If $\Gamma_S = \Gamma_{IN}^*$ is not in the stable region or is in the stable region but very close to the i/p stability c, value of Γ_S can be selected arbitrarily or a new value of G_p can be selected.

[Note - Choice of Γ_S affects the o/p power & VSWR)

-> Always Practical to keep value of G_p below the figure of merit value of G_{MSG}. The design for G_p lower than G_{MSG} can be performed with good stability and practical values of the input and output VSWR.

A design for a $G_p > G_{MSG}$ usually produces values of Γ_L and Γ_S close to the unstable regions and large values of the i/p & o/p VSWR.

In a potentially unstable situation with $K>1$ and $|\Delta|>1$ there is a $G_{p,min}$ just like $G_{T,min}$.

In a potentially unstable situation with $K>1$ & $|\Delta|>1$

$$g_{p,min} = \frac{1}{|S_{12}S_{21}|}\left(K+\sqrt{K^2-1}\right)$$

$$G_{p,min} = \frac{|S_{21}|}{|S_{12}|}\left(K+\sqrt{K^2-1}\right)$$

This equation gives the minimum value that G_p can have inside the stable region in a potentially unstable case with $K > 1$ and $|\Delta| > 1$

The value of Γ_L that produces $G_{p,min}$ denoted by $\Gamma_{L,min}$ is

$$\Gamma_{L,min} = \frac{g_{p,min} C_2^*}{1 + g_{p,min}(|S_{22}|^2 - |\Delta|^2)}$$

Example 3.7.2.

Step 1) Calculate K & Δ

$K = 0.4$ & $\Delta = 0.223 \angle 62.12^\circ$

$\because K < 1$ the transistor is potentially unstable.

2) Calculate $G_{MSG} = \frac{|S_{21}|}{|S_{12}|} = \frac{2.5}{0.08} = 31.25$ or $14.9\,dB$

3) We need G_p of $10\,dB$ draw $10\,dB$ operating power-gain circle. Calculate C_p & r_p

$C_p = 0.572 \angle 97.2^\circ$ $\quad r_p = 0.473$.

4) Draw o/p stability ⊙

Calculate r_L & C_L

$r_L = 0.34$ $\quad C_L = 1.18 \angle 97.2^\circ$

Since $|S_{11}| < 1$ the stable region is the region outside the stability ⊙.

5) choose Γ_L away from o/p stability ⊙

$\Gamma_L = 0.1\angle 97°$ $\quad Z_L = 50(0.96 + j0.19)$

Calculate

$$\Gamma_{IN}^{*} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_S = \Gamma_{IN}^{*} = 0.52\angle 179.32°$$

Available Power Gain

$$G_A = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{\left(1 - \left|\frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S}\right|^2\right)|1 - S_{11}\Gamma_S|^2} = |S_{21}|^2 g_a$$

where

$$g_a = \frac{G_A}{|S_{21}|^2} \qquad \frac{1 - |\Gamma_S|^2}{1 - |S_{22}|^2 + |\Gamma_S|^2(|S_{11}|^2 - |\Delta|^2) - 2\,\mathrm{Re}(\Gamma_S C_1)}$$

$$C_1 = S_{11} - \Delta S_{22}^{*}$$

~~Center~~ Available gain ⊙s.

$$C_a = \frac{g_a C_1^{*}}{1 + g_a(|S_{11}|^2 - |\Delta|^2)}$$

$$r_a = \frac{\left[1 - 2K|S_{12}S_{21}|g_a + |S_{12}S_{21}|^2 g_a^2\right]^{1/2}}{|1 + g_a(|S_{11}|^2 - |\Delta|^2)|}$$