: the baseband pulse $h(t)$ is now the impulse response of the filter with transfer function $|G(f)|^2 = |P(f)G(f)|^2$. This impulse response is the *autocorrelation* of the impulse response of the combined transmitter-channel filter $\tilde{G}(f)$,

$$h(t) = \int_{-\infty}^{\infty} \tilde{g}^*(s)\tilde{g}(s+t)\,ds \tag{4.23}$$

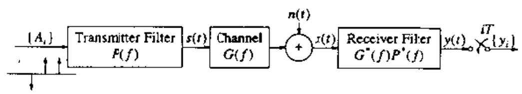

FIGURE 4.7 Baseband PAM mode with a matched filter at the receiver.

With a matched filter at the receiver, the equivalent discrete-time transfer function is

$$\begin{aligned} H_{eq}(e^{j2\pi fT}) &= \frac{1}{T}\sum_k \left|\tilde{G}\left(f-\frac{k}{T}\right)\right|^2 \\ &= \frac{1}{T}\sum_k \left|P\left(f-\frac{k}{T}\right)G\left(f-\frac{k}{T}\right)\right|^2 \end{aligned} \tag{4.24}$$

which relates the sequence of transmitted symbols $\{A_k\}$ to the sequence of received samples $\{y_k\}$ in the absence of noise. Note that $H_{eq}(e^{j2\pi fT})$ is positive, real valued, and an even function of f. If the channel is bandlimited to twice the Nyquist bandwidth, then $H(f) = 0$ for $|f| > 1/T$, and the Nyquist condition is given by Eq. (4.14) where $H(f) = |G(f)P(f)|^2$. The aliasing sum in Eq. (4.10b) can therefore be described as a folding operation in which the channel response $|H(f)|^2$ is folded around the Nyquist frequency $1/(2T)$. For this reason, $H_{eq}(e^{j2\pi fT})$ with a matched receiver filter is often referred to as the folded channel spectrum.

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4.4 Eye Diagrams

ie. y(t) contains all possible combinations of bit patterns

One way to assess the severity of distortion due to intersymbol interference in a digital communications system is to examine the **eye diagram**. The eye diagram is illustrated in Figs. 4.8a and 4.8b, for a raised cosine pulse shape with 25% excess bandwidth and an ideal bandlimited channel. Figure 4.8a shows the data signal at the receiver

Shankar (3.29c)

$$y(t) = \sum_i A_i h(t - iT) + \tilde{n}(t) \tag{4.25}$$

where $h(t)$ is given by Eq. (4.17), $\alpha = 1/4$, each symbol A_i is independently chosen from the set $\{\pm 1, \pm 3\}$, where each symbol is equally likely, and $\tilde{n}(t)$ is bandlimited white Gaussian noise. (The received SNR is 30 dB.) The eye diagram is constructed from the time-domain data signal $y(t)$ as follows (assuming nominal sampling times at $kT, k = 0, 1, 2, \ldots$):

1. Partition the waveform $y(t)$ into successive segments of length T starting from $t = T/2$.
2. Translate each of these waveform segments $[y(t), (k+1/2)T \le t \le (k+3/2)T, k = 0, 1, 2, \ldots]$ to the interval $[-T/2, T/2]$, and superimpose.

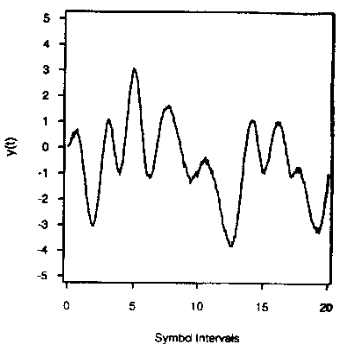

Figure 4.8a Received signal $y(t)$.

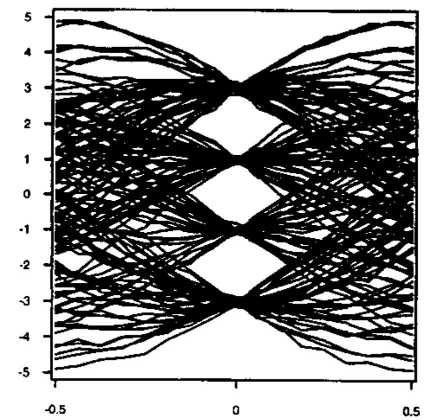

Figure 4.8b Eye diagram for received signal shown in Fig. 4.8a

The resulting picture is shown in Fig. 4.8b for the $y(t)$ shown in **Fig.** 4.8a. (Partitioning $y(t)$ into successive segments of length iT, $i > 1$, is also possible. This would result in i successive eye diagrams.) The number of eye openings is one less than the number of transmitted signal levels. In practice, the eye diagram is easily viewed on an oscilloscope by applying the received waveform $y(t)$ to the vertical deflection plates of the oscilloscope and applying a sawtooth waveform at the symbol rate $1/T$ to the horizontal deflection plates. This causes successive symbol intervals to be translated into one interval on the oscilloscope display.

Each waveform segment $y(t)$, $(k+1/2)T \le t \le (k+3/2)T$, depends on the particular sequence of channel symbols surrounding A_k. The number of channel symbols that affects a particular waveform segment depends on the extent of the intersymbol interference, shown in Eq. (4.6). This, in turn, depends on the duration of the impulse response $h(t)$. For example, if $h(t)$ has most of its energy in the interval $0 < t < mT$, then each waveform segment depends on approximately m symbols. Assuming binary transmission, this implies that there are a total of 2^m waveform segments that can be superimposed in the eye diagram. (It is possible that only one sequence of channel symbols causes significant intersymbol interference, and this sequence occurs with very low probability.) In current digital wireless applications the impulse response typically spans only a few symbols.

The eye diagram has the following important features which measure the performance of a digital communications system.

4.4.1 Vertical Eye Opening

The vertical openings at any time t_0, $-T/2 \le t_0 \le T/2$, represent the separation between signal levels with worst-case intersymbol interference, assuming that $y(t)$ is sampled at times $t = kT + t_0$, $k = 0, 1, 2, \ldots$. It is possible for the intersymbol interference to be large enough so that this vertical opening between some, or all, signal levels disappears altogether. In that case, the eye is said to be closed. Otherwise, the eye is said to be open. A closed eye implies that if the estimated bits are obtained by thresholding the samples $y(kT)$, then the decisions will depend primarily on the intersymbol interference rather than on the desired symbol. The probability of error will, therefore, be close to 1/2. Conversely, wide vertical spacings between signal levels imply a large degree of immunity to additive noise. In general, $y(t)$ should be sampled at the times $kT + t_0$, $k = 0, 1, 2, \ldots$, where t_0 is chosen to maximize the vertical eye opening.

4.4.2 Horizontal Eye Opening

The width of each opening indicates the sensitivity to timing offset. Specifically, a very narrow eye opening indicates that a small timing offset will result in sampling where the eye is closed. Conversely, a wide horizontal opening indicates that a large timing offset can be tolerated, although the error probability will depend on the vertical opening.

4.4.3 Slope of the Inner Eye

The slope of the inner eye indicates sensitivity to timing jitter or variance in the timing offset. Specifically, a very steep slope means that the eye closes rapidly as the timing offset increases. In this case, a significant amount of jitter in the sampling times significantly increases the probability of error.

The shape of the eye diagram is determined by the pulse shape. In general, the faster the baseband pulse decays, the wider the eye opening. For example, a rectangular pulse produces a box-shaped eye diagram (assuming binary signalling). The minimum bandwidth pulse shape Eq. (4.12) produces an eye diagram which is closed for all t except for $t = 0$. This is because, as shown earlier, an arbitrarily small timing offset can lead to an intersymbol interference term that is arbitrarily large, depending on the data sequence.

4.5 Partial-Response Signalling

To avoid the problems associated with Nyquist signalling over an ideal bandlimited channel, bandwidth and/or power efficiency must be compromised. Raised cosine pulses compromise

bandwidth efficiency to gain robustness with respect to timing errors. Another possibility is to introduce a controlled amount of intersymbol interference at the transmitter, which can be removed at the receiver. This approach is called **partial-response (PR) signalling**. The terminology reflects the fact that the sampled system impulse response does not have the full response given by the Nyquist condition Eq. (4.7).

To illustrate PR signalling, suppose that the Nyquist condition Eq. (4.7) is replaced by the condition

$$h_k = \begin{cases} 1 & k = 0, 1 \\ 0 & \text{all other } k \end{cases} \tag{4.26}$$

The kth received sample is then

$$y_k = A_k + A_{k-1} + \tilde{n}_k \tag{4.27}$$

so that there is intersymbol interference from one neighboring transmitted symbol. For now we focus on the spectral characteristics of PR signalling and defer discussion of how to detect the transmitted sequence $\{A_k\}$ in the presence of intersymbol interference. The equivalent discrete-time transfer function in this case is the discrete Fourier transform of the sequence in Eq. (4.26),

$$\begin{aligned} H_{\text{eq}}(e^{j2\pi fT}) &= \frac{1}{T}\sum_k H\left(f + \frac{k}{T}\right) \\ &= 1 + e^{-j2\pi fT} = 2e^{-j\pi fT}\cos(\pi fT) \end{aligned} \tag{4.28}$$

As in the full-response case, for Eq. (4.28) to be satisfied, the *minimum* bandwidth of the channel $G(f)$ and transmitter filter $P(f)$ is $W = 1/(2T)$. Assuming $P(f)$ has this minimum bandwidth implies

$$H(f) = \begin{cases} 2Te^{-j\pi fT}\cos(\pi fT) & |f| < 1/(2T) \\ 0 & |f| > 1/(2T) \end{cases} \tag{4.29a}$$

and

$$h(t) = T\{\ \text{sinc}\ (t/T) +\ \text{sinc}\ [(t-T)/T]\} \tag{4.29b}$$

where sinc $x = (\sin \pi x)/(\pi x)$. This pulse is called a *duobinary* pulse and is shown along with the associated $H(f)$ in Fig. 4.9. [Notice that $h(t)$ satisfies Eq. (4.26).] Unlike the ideal bandlimited frequency response, the transfer function $H(f)$ in Eq. (4.29a) is continuous and is, therefore, easily approximated by a physically realizable filter. Duobinary PR was first proposed by Lender, [7], and later generalized by Kretzmer, [6].

The main advantage of the duobinary pulse Eq. (4.29b), relative to the minimum bandwidth pulse Eq. (4.12), is that signalling at the Nyquist symbol rate is feasible with zero excess bandwidth. Because the pulse decays much more rapidly than a Nyquist pulse, it is robust with respect to timing errors. Selecting the transmitter and receiver filters so that the overall system response is duobinary is appropriate in situations where the channel frequency response $G(f)$ is near zero or has a rapid rolloff at the Nyquist band edge $f = 1/(2T)$.