Review

Chapter-1

Maxwell's Equation

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} - \bar{M}_i \qquad \oint \bar{E}\cdot d\bar{l} = -\iint_S \bar{M}_i \cdot d\bar{s} - \frac{\partial}{\partial t}\iint_S \bar{B}\cdot d\bar{s}$$

$$\nabla \times \bar{H} = \bar{J}_c + \bar{J}_d + \bar{J}_i \qquad \oint \bar{H}\cdot d\bar{l} = \iint_S \bar{J}_i \cdot d\bar{s} + \iint_S \bar{J}_c \cdot d\bar{s} + \frac{\partial}{\partial t}\iint_S \bar{D}\cdot d\bar{s}$$

$$\left(\bar{J}_d = \frac{\partial \bar{D}}{\partial t}\right)$$

$$\nabla \cdot \bar{D} = \rho_v \qquad \oiint_S \bar{D}\cdot d\bar{s} = \iiint_V \rho_v \, dv$$

$$\nabla \cdot \bar{B} = 0 \qquad \oiint \bar{B}\cdot d\bar{s} = 0$$

$$\nabla \cdot \bar{J}_{ic} = -\frac{\partial \rho_v}{\partial t} \qquad \oiint_S \bar{J}_{ic}\cdot d\bar{s} = -\frac{\partial}{\partial t}\iiint_V \rho_v \, dv$$

$$\bar{D} = \varepsilon \bar{E} \qquad \bar{B} = \mu \bar{H} \qquad \bar{J}_c = \sigma \bar{E}$$

Boundary Conditions

Finite Conductivity Media

→ Tangential component of electric field are continuous

$$\mathcal{E}_{1t} = \mathcal{E}_{2t} \quad \underline{\text{or}} \quad \hat{n} \times (\bar{\mathcal{E}}_2 - \bar{\mathcal{E}}_1) = 0$$

$$\mathcal{H}_{1t} = \mathcal{H}_{2t} \quad \underline{\text{or}} \quad \hat{n} \times (\bar{\mathcal{H}}_2 - \bar{\mathcal{H}}_1) = 0$$

→ Normal components

$$\mathcal{D}_{2n} = \mathcal{D}_{1n} \quad \text{or} \quad \hat{n} \cdot (\bar{\mathcal{D}}_2 - \bar{\mathcal{D}}_1) = 0$$

$$\varepsilon_2 \mathcal{E}_{2n} = \varepsilon_1 \mathcal{E}_{1n} \Rightarrow \mathcal{E}_{1n} = \frac{\varepsilon_2}{\varepsilon_1} \mathcal{E}_{2n}$$

OR

$$\hat{n} \cdot (\varepsilon_2 \mathcal{E}_2 - \varepsilon_1 \mathcal{E}_1) = 0$$

$$\hat{n} \cdot (\bar{B}_2 - \bar{B}_1) = 0$$

$$\hat{n} \cdot (\mu_2 \bar{\mathcal{H}}_2 - \mu_1 \bar{\mathcal{H}}_1) = 0$$ [Normal component of magnetic field intensity are discontinuous]

Infinite Conductivity Media

$$\hat{n} \times (\bar{\mathcal{H}}_2 - \bar{\mathcal{H}}_1) = \bar{J}_s$$ ($\bar{J}_s \rightarrow$ surface electric current density)

If one of the two media is a perfect electric conductor, The above equation must be reduced to account for the presence of the conductor. lets assume medium 1 has infinite conductivity. Then $\mathcal{E}_1 = 0$

$$\boxed{\hat{n} \times \bar{\mathcal{E}}_2 = 0 \Rightarrow \mathcal{E}_{2t} = 0}$$

$$\nabla \times \mathcal{E}_1 = 0 = -\frac{\partial \bar{B}_1}{\partial t} \Rightarrow \bar{B}_1 = 0 \Rightarrow \bar{H}_1 = 0$$

as long as μ_1 is finite

$$\hat{n} \times \bar{\mathcal{H}}_2 = \bar{J}_s \Rightarrow \mathcal{H}_{2t} = J_s$$

$$\hat{n} \times (\bar{D}_2 - \bar{D}_1) = \rho_v$$

$$D_{2n} - D_{1n} = \rho_v$$

$$\hat{n} \cdot (\varepsilon_2 \bar{E}_2 - \varepsilon_1 \bar{E}_1) = \rho_v$$

- normal components of the electric field are discontinuous across a boundary along which a surface charge density resides.

- If either of the media is a perfect electric conductor.

$$\hat{n} \cdot \bar{D}_2 = \rho_v \Rightarrow D_{2n} = \rho_v$$

$$\cancel{\hat{n} \cdot D} \quad \hat{n} \cdot \bar{E}_2 = \frac{\rho_v}{\varepsilon_2} \Rightarrow E_{2n} = \frac{\rho_v}{\varepsilon_2}$$

Sources Along Boundary

$$-\hat{n} \times (\bar{E}_2 - \bar{E}_1) = \bar{M}_s$$

$$\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s$$

$$\hat{n} \cdot (\bar{D}_2 - \bar{D}_1) = \rho_v$$

$$\hat{n} \cdot (\bar{B}_2 - \bar{B}_1) = 0$$

Conservation of Energy Differential

$$\nabla \cdot (\bar{\mathcal{E}} \times \bar{\mathcal{H}}) + \bar{\mathcal{H}} \cdot (\bar{\mathcal{M}}_i + \bar{\mathcal{M}}_d) + \bar{\mathcal{E}} \cdot (\bar{\mathcal{J}}_i + \bar{\mathcal{J}}_c + \bar{\mathcal{J}}_d) = 0$$

integral form

$$\oint\!\!\oint (\bar{\mathcal{E}} \times \bar{\mathcal{H}}) \cdot d\bar{s} + \iiint_v [\bar{\mathcal{H}} \cdot (\bar{\mathcal{M}}_i + \bar{\mathcal{M}}_d) + \bar{\mathcal{E}} \cdot (\bar{\mathcal{J}}_i + \bar{\mathcal{J}}_c + \bar{\mathcal{J}}_d)] dv = 0$$

Time Harmonic Fields

$$\bar{\mathcal{E}}(x, y, z; t) = Re[\bar{E}(x, y, z) e^{j\omega t}]$$

$$\bar{\mathcal{H}}(x, y, z; t) = Re[\bar{H}(x, y, z) e^{j\omega t}]$$

Average Power Density

$$\mathcal{S}_{av} = \bar{S} = \frac{1}{2} Re[\bar{E} \times H^*]$$

$$P_s = P_e + P_d + j 2\omega (\bar{W}_m - \bar{W}_e)$$

$$P_s = -\frac{1}{2} \iiint_v (\bar{H}^* \cdot \bar{M}_i + \bar{E} \cdot \bar{J}_i^*) dv = \text{supplied complex Power (W)}$$

$$P_e = \oint\!\!\oint_S \left(\frac{1}{2} \bar{E} \times \bar{H}^*\right) \cdot d\bar{s} = \text{exiting complex power (W)}$$

$$P_d = \frac{1}{2} \iiint_v \sigma |\bar{E}|^2 dv = \text{dissipated real power (W)}$$

$$\bar{W}_m = \iiint_v \frac{1}{4} \mu |\bar{H}|^2 dv = \text{Time average magnetic energy}$$

$$\bar{W}_e = \iiint_v \frac{1}{4} \varepsilon |\bar{E}|^2 dv = \text{Time average electric energy}$$

Chapter 2

- Formation of electric dipoles is usually referred as orientational polarization

Dielectrics:

The dipole moment is given as $dp_i = Q \ell_i$

Total Dipole Moment $\bar{P}_t = \sum_{i=1}^{N_e \Delta v} d\vec{p}_i$

The polarization is given by

$$\bar{P} = \lim_{\Delta v \to 0} \left[\frac{1}{\Delta v} \vec{P}_t \right]$$

↳ dipole moment per unit volume.

For average dipole moment

$$d\vec{p}_i = d\vec{p}_{av} \cancel{= Q\Delta}$$

&

$$\bar{P} = N_e Q \ell_{av}$$

Dipole or Orientational Polarization

↳ Evident in material that, in the absence of an applied field and owing to their structure posses permanent dipole moments ~~and~~ that are randomly oriented. When an electric field is applied these dipoles tend to allign with the applied fields.

Ionic or Molecular Polarization

↳ Evident in materials that possess +ve and -ve ions and that tend to displace themselves when electric field is applied.

Electronic Polarization

↳ Exists when an applied electric field displaces the electric cloud center of an atom relative to the center of the nucleus.

$$\bar{D}_0 = \varepsilon_0 \bar{E}_a$$

electric flux density

applied electric field.

$$\bar{D} = \varepsilon_0 \bar{E}_a + \bar{P}$$

$$\bar{P} = \varepsilon_0 \chi_e \bar{E}_a$$

$$\chi_e = \frac{1}{\varepsilon_0} \frac{\bar{P}}{\bar{E}_a}$$

$$\bar{D} = \varepsilon_0 \bar{E}_a + \varepsilon_0 \chi_e \bar{E}_a$$
$$= \varepsilon_0 (1 + \chi_e) \bar{E}_a$$

|||ly

$$\bar{B} = \mu_0 (\bar{H}_a + \bar{m})$$

$$\bar{M} = \chi_m \bar{H}_a$$

$$\bar{B} = \mu_0 (1 + \chi_m) \bar{H}_a$$

The magnetic current density $\bar{J}_m$ is related to magnetic polarization vector $\bar{m}$ as

$$\bar{J}_m = \bar{\nabla} \times \bar{m}$$

$$\bar{I}_m = \iint_S \bar{J}_m \cdot d\bar{s} = \iint_{S_0} (\nabla \times \bar{m}) \cdot d\bar{s}$$

CHAPTER 3

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} - \bar{M}_i$$

$$\nabla \times \bar{H} = \bar{J}_i + \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}$$

Vector Wave equation

$$\nabla^2 \bar{E} = \nabla \times \bar{M}_i + \mu \frac{\partial \bar{J}_i}{\partial t} + \frac{1}{\varepsilon} \nabla q_{ev} + \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla^2 \bar{H} = \nabla \times \bar{J}_i + \sigma \bar{M}_i + \frac{1}{\mu} \nabla q_{mv} + \varepsilon \frac{\partial \bar{M}_i}{\partial t} + \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2}$$

For Source free region $\bar{J}_i = q_{ev} = \bar{M}_i = q_{mv} = 0$

The above equations reduce to

$$\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla^2 \bar{H} = \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2}$$

For lossless medium $\sigma = 0$

$$\nabla^2 \bar{E} = \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} \qquad \nabla^2 \bar{H} = \mu\varepsilon \frac{\partial^2 \bar{H}}{\partial t^2}$$

Time Harmonic form

$$\nabla^2 \bar{E} = j\omega\mu\sigma \bar{E} - \omega^2 \mu\varepsilon \bar{E}$$
$$= \gamma^2 \bar{E}$$

$$\nabla^2 \bar{H} = j\omega\mu\sigma\bar{H} - \omega^2\mu\varepsilon\bar{H}$$

$$= \gamma^2 \bar{H}$$

For lossless media

$$\nabla^2 \bar{E} = -\omega^2\mu\varepsilon\bar{E} = -\beta^2\bar{E}$$

$$\nabla^2 \bar{H} = -\omega^2\mu\varepsilon\bar{H} = -\beta^2\bar{H}$$

$$\beta^2 = \omega^2\mu\varepsilon$$

<u>Rectangular coordinate System</u>

$$\nabla^2 \bar{E}_s(x,y,z) + \beta^2\bar{E} = \nabla^2(\hat{a}_x E_x + \hat{a}_y E_y + \hat{a}_z E_z) + \beta^2(\hat{a}_x E_x + \hat{a}_y E_y + \hat{a}_z E_z) = 0$$

<u>Solution</u>

Traveling wave	$e^{-j\beta x}$	$+x$ travel
	$e^{j\beta x}$	$-x$ travel
Standing wave	$\cos(\beta x)$	$\pm x$
	$\sin(\beta x)$	$\pm x$
Attenuating traveling wave	$e^{-\gamma x} = e^{-\alpha x} e^{-j\beta x}$	$+x$ travel
	$e^{\gamma x} = e^{\alpha x} e^{j\beta x}$	$-x$ travel

Cylindrical Coordinate System

Traveling waves

$$H_m^{(1)}(\beta\rho) = \bar{J}_m(\beta\rho) + j\,\bar{Y}_m(\beta\rho) \qquad \text{`}-\rho\text{' travel}$$

$$H_m^{(2)}(\beta\rho) = \bar{J}_m(\beta\rho) - j\,\bar{Y}_m(\beta\rho) \qquad \text{`}+\rho\text{' travel}$$

Standing wave

$$\bar{J}_m(\beta\rho) \qquad \text{for } \pm\rho$$

$$\bar{Y}_m(\beta\rho) \qquad \text{for } \pm\rho$$

Spherical Coordinate System

Traveling wave

$$h_n^{(1)}(\beta r) = j_n(\beta r) + j\,y_n(\beta r) \qquad \text{for `}-r\text{' travel}$$

$$h_n^{(2)}(\beta r) = j_n(\beta r) - j\,y_n(\beta r) \qquad \text{for `}+r\text{' travel}$$

Standing wave

$$j_n(\beta r) \qquad \text{for } \pm r$$

$$y_n(\beta r) \qquad \text{for } \pm r$$

$$j_n(\beta r) = \sqrt{\frac{\pi}{2\beta r}}\;\bar{J}_{n+\frac{1}{2}}(\beta r)$$

$$y_n(\beta r) = \sqrt{\frac{\pi}{2\beta r}}\;\bar{Y}_{n+\frac{1}{2}}(\beta r)$$

$$h_n^{(1)}(\beta r) = \sqrt{\frac{\pi}{2\beta r}}\;\bar{H}_{n+\frac{1}{2}}^{(1)}(\beta r)$$

$$h_n^{(2)}(\beta r) = \sqrt{\frac{\pi}{2\beta r}}\;\bar{H}_{n+\frac{1}{2}}^{(2)}(\beta r)$$

Polarization

General Polarization Equation

$$\bar{\mathcal{E}} = \hat{a}_x \mathcal{E}_x + \hat{a}_y \mathcal{E}_y = Re\left[\hat{a}_x E_x^+ e^{j(\omega t - \beta z)} + \hat{a}_y E_y^+ e^{j(\omega t-\beta z)}\right]$$

$$= \hat{a}_x E_{x_o}^+ \cos(\omega t - \beta z + \phi_x) + \hat{a}_y E_{y_o}^+ \cos(\omega t - \beta z + \phi_y)$$

$$\bar{\mathcal{H}} = \hat{a}_y \frac{E_{x_o}^+}{\eta} \cos(\omega t - \beta z + \phi_x) - \hat{a}_x \frac{E_{y_o}^+}{\eta} \cos(\omega t - \beta z + \phi_y)$$

$$E(z) = \hat{a}_x E_{x_o}^+ e^{j\phi_x} e^{-j\beta z} + \hat{a}_y E_{y_o}^+ e^{j\phi_y} e^{-j\beta z}$$

$$= E_{x_o}^+ e^{j\phi_x}\left[\hat{a}_x + \hat{a}_y \frac{E_{y_o}^+}{E_{x_o}^+} e^{j(\phi_y - \phi_x)}\right] e^{-j\beta z}$$

Chapter 6

$$\bar{E} = -j\omega \bar{A} - \frac{j}{\omega \mu \varepsilon} \nabla(\nabla \cdot \bar{A}) - \frac{1}{\varepsilon} \nabla \times \bar{F}$$

$$\bar{H} = \frac{1}{\mu} \nabla \times \bar{A} - j\omega \bar{F} - \frac{j}{\omega \mu \varepsilon} \nabla(\nabla \cdot \bar{F})$$

$$\bar{E} = \hat{a}_x \left[-j\omega \bar{A}_x - \frac{j}{\omega\mu\varepsilon}\left(\frac{\partial^2 \bar{A}_x}{\partial x^2} + \frac{\partial^2 \bar{A}_y}{\partial x \partial y} + \frac{\partial^2 \bar{A}_z}{\partial x \partial z} \right) - \frac{1}{\varepsilon}\left(\frac{\partial \bar{F}_z}{\partial y} - \frac{\partial \bar{F}_y}{\partial z} \right) \right]$$

$$+ \hat{a}_y \left[-j\omega \bar{A}_y - \frac{j}{\omega\mu\varepsilon}\left(\frac{\partial^2 \bar{A}_x}{\partial x \partial y} + \frac{\partial^2 \bar{A}_y}{\partial y^2} + \frac{\partial^2 \bar{A}_z}{\partial y \partial z} \right) - \frac{1}{\varepsilon}\left(\frac{\partial \bar{F}_x}{\partial z} - \frac{\partial \bar{F}_z}{\partial x} \right) \right]$$

$$+ \hat{a}_z \left[-j\omega \bar{A}_z - \frac{j}{\omega\mu\varepsilon}\left(\frac{\partial^2 \bar{A}_x}{\partial x \partial z} + \frac{\partial^2 \bar{A}_y}{\partial y \partial z} + \frac{\partial^2 \bar{A}_z}{\partial z^2} \right) - \frac{1}{\varepsilon}\left(\frac{\partial \bar{F}_y}{\partial x} - \frac{\partial \bar{F}_x}{\partial y} \right) \right]$$

$$\bar{H} = \hat{a}_x \left[-j\omega \bar{F}_x - \frac{j}{\omega\mu\varepsilon}\left(\frac{\partial^2 \bar{F}_x}{\partial x^2} + \frac{\partial^2 \bar{F}_y}{\partial x \partial y} + \frac{\partial^2 \bar{F}_z}{\partial x \partial z} \right) + \frac{1}{\mu}\left(\frac{\partial \bar{A}_z}{\partial y} - \frac{\partial \bar{A}_y}{\partial z} \right) \right]$$

$$+ \hat{a}_y \left[-j\omega \bar{F}_y - \frac{j}{\omega\mu\varepsilon}\left(\frac{\partial^2 \bar{F}_x}{\partial x \partial y} + \frac{\partial^2 \bar{F}_y}{\partial y^2} + \frac{\partial^2 \bar{F}_z}{\partial y \partial z} \right) + \frac{1}{\mu}\left(\frac{\partial \bar{A}_x}{\partial z} - \frac{\partial \bar{A}_z}{\partial x} \right) \right]$$

$$+ \hat{a}_z \left[-j\omega \bar{F}_z - \frac{j}{\omega\mu\varepsilon}\left(\frac{\partial^2 \bar{F}_x}{\partial x \partial z} + \frac{\partial^2 \bar{F}_y}{\partial y \partial z} + \frac{\partial^2 \bar{F}_z}{\partial z^2} \right) + \frac{1}{\mu}\left(\frac{\partial \bar{A}_y}{\partial x} - \frac{\partial \bar{A}_x}{\partial y} \right) \right]$$

For TEM mode

1) $\bar{A}_z \neq 0$ & $\bar{F}_z \neq 0$

2) $\bar{A}_z \neq 0$ & $\bar{F}_z = 0$

3) $\bar{A}_z = 0$ & $\bar{F}_z \neq 0$

TE mode

$\bar{F}_z \neq 0 \quad \bar{E}_z = 0$

TM mode

$\bar{A}_z \neq 0 \quad H_z = 0$

$$\bar{A} = \frac{\mu}{4\pi} \iint_S \bar{J}_s(x', y', z') \frac{e^{-j\beta R}}{R} \, ds'$$

$$\bar{F} = \frac{\varepsilon}{4\pi} \iint_S \bar{M}_s(x', y', z') \frac{e^{-j\beta R}}{R} \, ds'$$

$(x', y', z') \rightarrow$ source

$(x, y, z) \rightarrow$ far field location.

Steps to solve if the current density is given

$\underline{I}_e(z') = \hat{a}_z I_e$

Step 1

$$\bar{A}(x, y, z) = \frac{\mu}{4\pi} \int_{-\ell/2}^{\ell/2} \hat{a}_z \underline{I}_e \frac{e^{-j\beta R}}{R} \, dz'$$

2) At far field

$R = r$

& phase $R = r - r' \cos\psi$

Step 3

Find $\bar{E}$ & $\bar{H}$

Radiation Problem

If the $\bar{E}_a$ & $\bar{H}_a$ at the source is provided then do the following steps

1) Calculate $\bar{J}_s$ & $\bar{M}_s$ using the equation

$$\bar{J}_s = \hat{n} \times \bar{H}_a$$

$$\bar{M}_s = -\hat{n} \times \bar{E}_a$$

2) Calculate $N_\theta, N_\phi, L_\theta, L_\phi$

$$N_\theta = \iint_s (J_x \cos\theta \cos\phi + J_y \cos\theta \sin\phi - J_z \sin\theta) e^{j\beta r' \cos\psi} ds'$$

$$N_\phi = \iint_s (-J_x \sin\phi + J_y \cos\phi) e^{j\beta r' \cos\psi} ds'$$

$$L_\theta = \iint_s (M_x \cos\theta \cos\phi + M_y \cos\theta \sin\phi - M_z \sin\theta) e^{j\beta r' \cos\psi} ds'$$

$$L_\phi = \iint_s (-M_x \sin\phi + M_y \cos\phi) e^{+j\beta r' \cos\psi} ds'$$

3) Calculate $\bar{E}$ & $\bar{H}$

$$E_r \simeq 0$$

$$E_\theta \simeq \frac{-j\beta e^{-j\beta r}}{4\pi r}\left(L_\phi + \eta N_\theta\right)$$

$$E_\phi \simeq \frac{j\beta e^{-j\beta r}}{4\pi r}\left(L_\theta - \eta N_\phi\right)$$

$$H_r \simeq 0$$

$$H_\theta \simeq \frac{j\beta e^{-j\beta r}}{4\pi r}\left(N_\phi - \frac{L_\theta}{\eta}\right)$$

$$H_\phi \simeq \frac{-j\beta e^{-j\beta r}}{4\pi r}\left(N_\theta + \frac{L_\phi}{\eta}\right)$$

For a Scattering Problem

Step 1

Calculate the total Electric and magnetic field on the surface of the aperture due to an incident & reflected wave.

Step 2

Follow the steps similar to the radiation problem.

THANK YOU

BEST OF LUCK