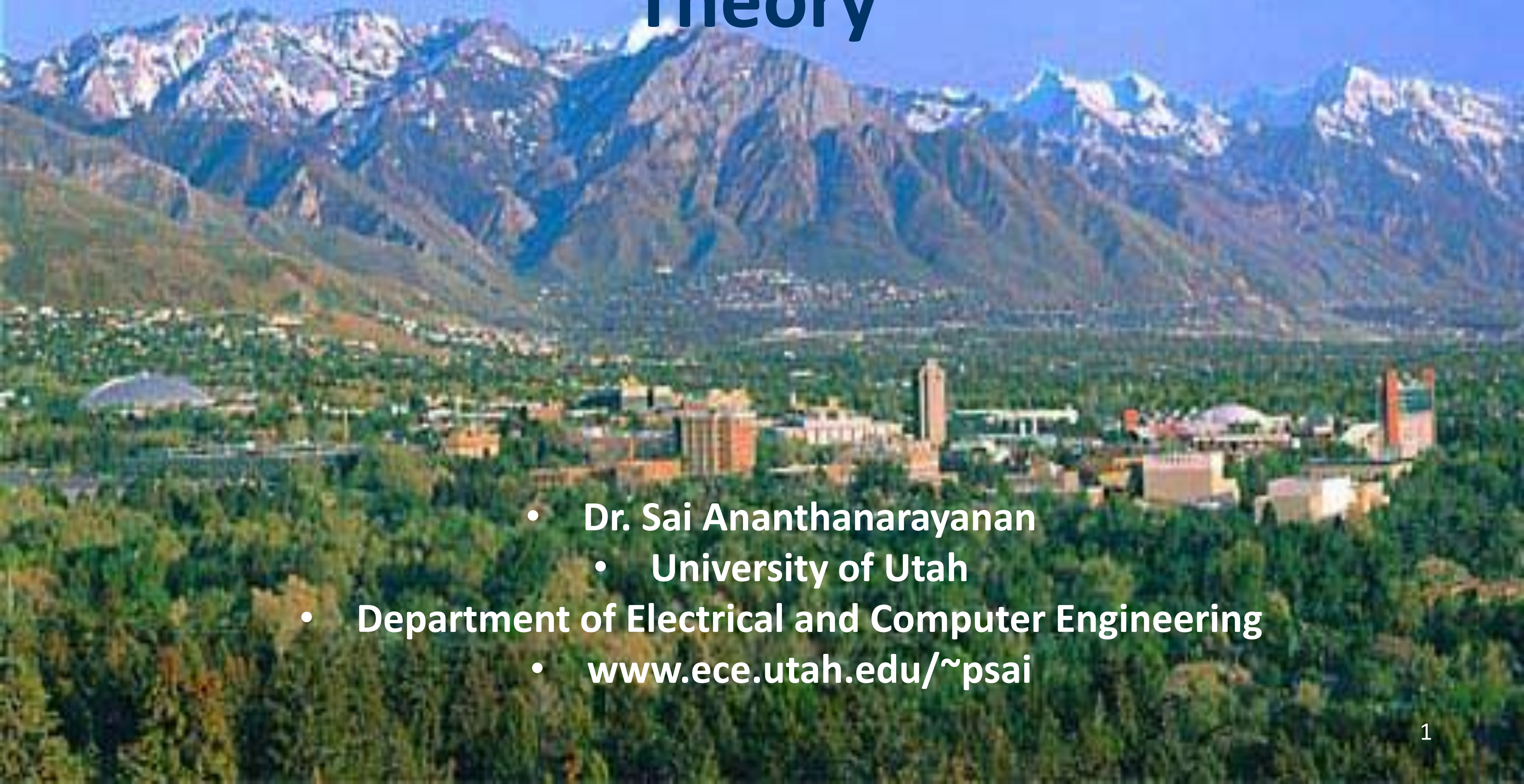
University of Utah
Advanced Electromagnetics
Equivalence Principle and Image Theory



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Electric Conductor

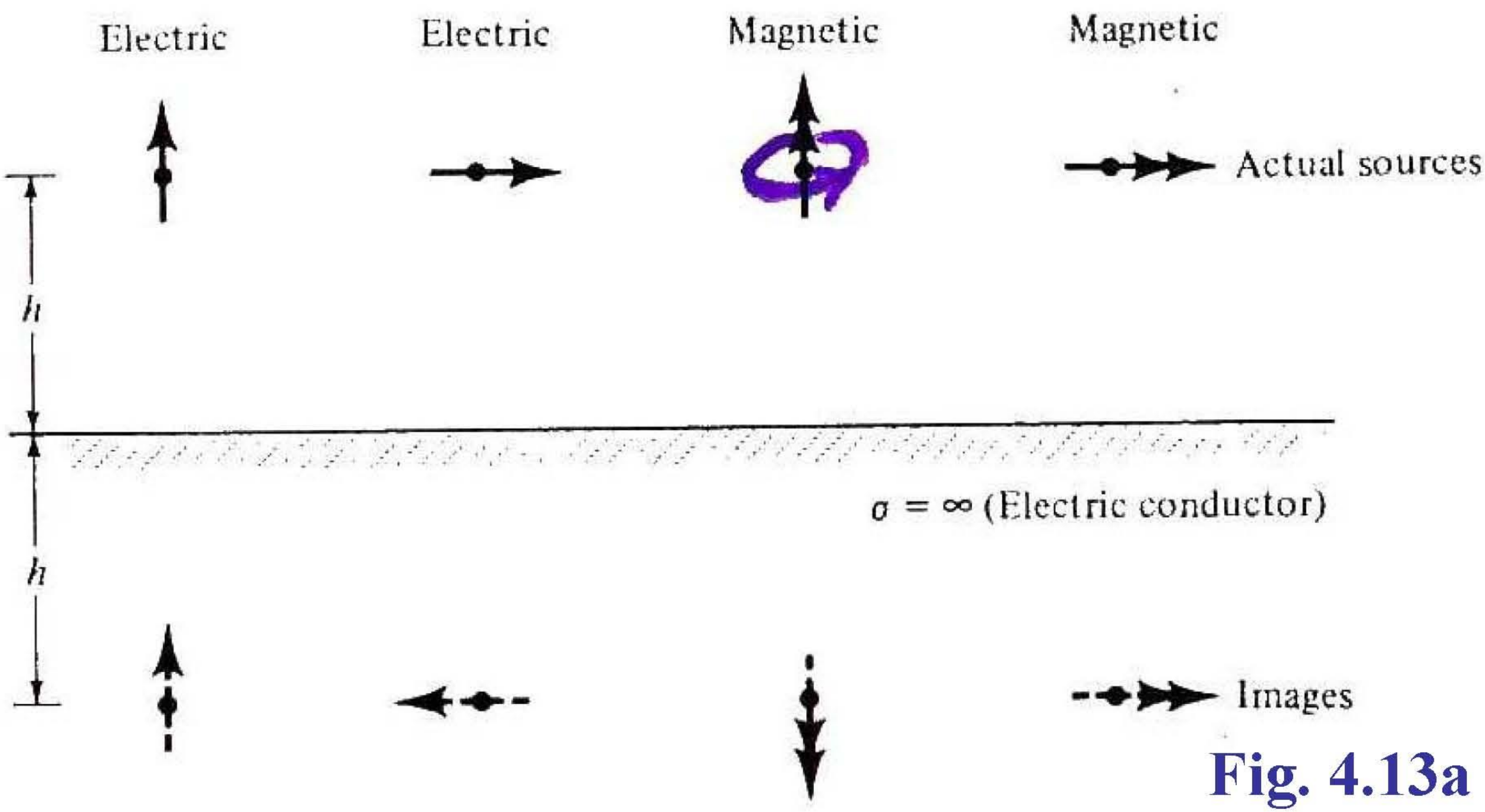


Fig. 4.13a

Magnetic Conductor

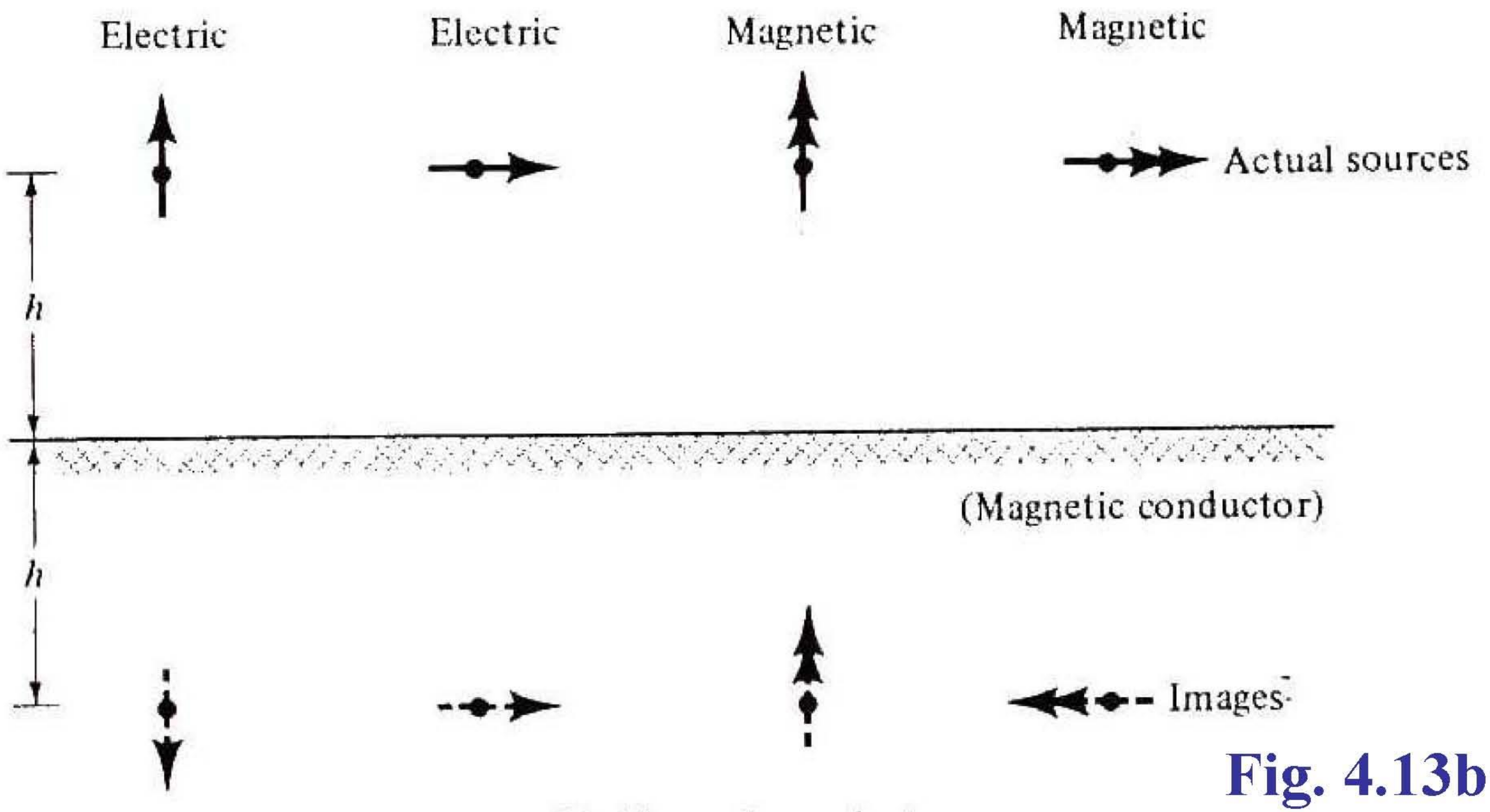


Fig. 4.13b

Effect of Imperfectly Conducting, Flat Earth

- To improve the radiation efficiency at these frequencies, radial wires or metallic disks are sometimes placed on the ground to simulate a perfectly conducting ground plane.

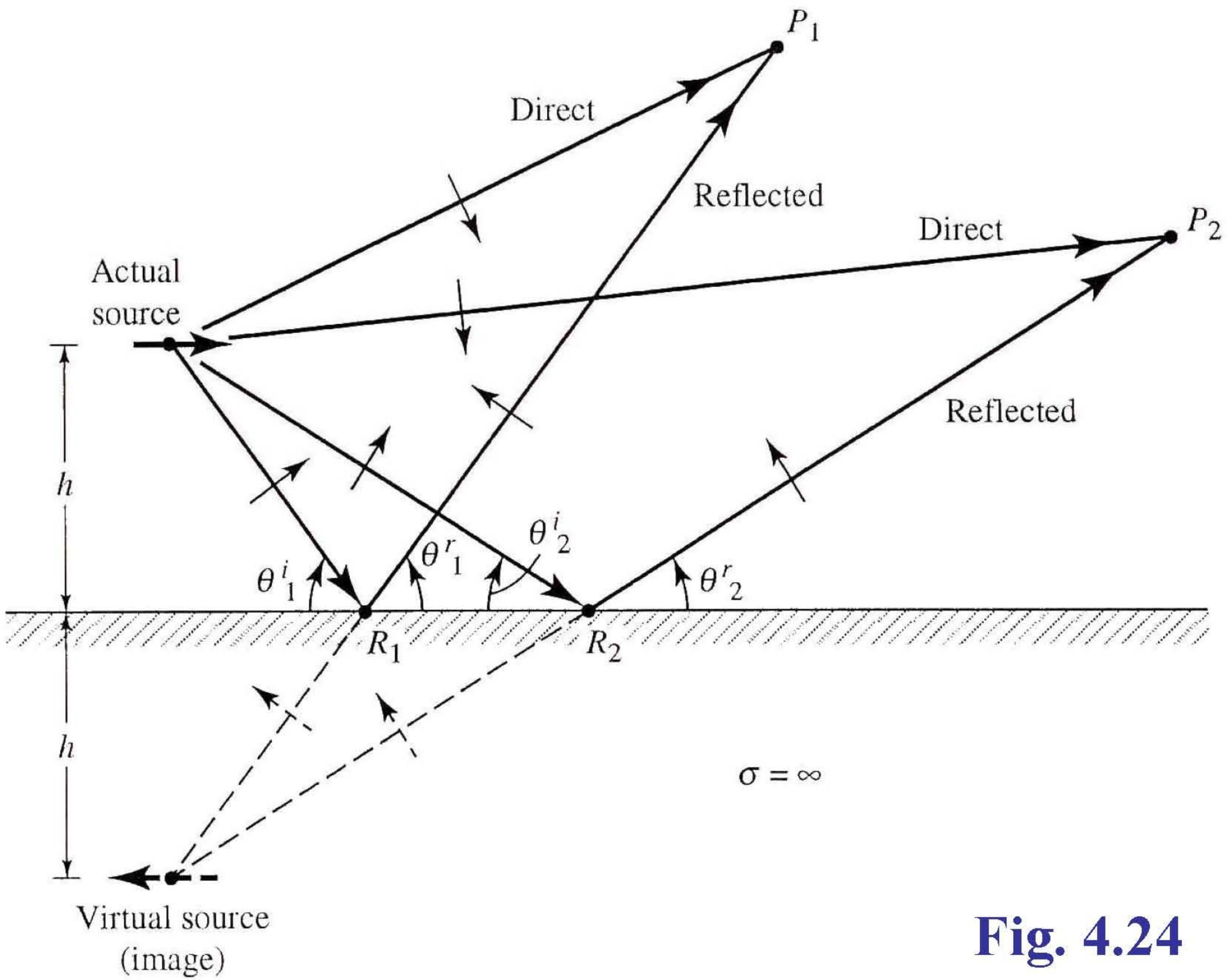
Examples of Antennas on Cellular and Cordless Telephones, Walkie-Talkies, and CB Radios

Fig. 4.22

Triangular Array Of Linear Dipoles For Wireless Mobile Communication Base Stations

Fig. 4.23

Horizontal Polarization



P_1
Direct
Reflected
P_2
Direct
Actual
source
Reflected
h
θ^i_2
θ^r_1
θ^i_1
θ^r_2
R_1
R_2
h
$\sigma = \infty$
Virtual source
(image)

Fig. 4.24

Horizontal Electric Dipole Above Ground Plane

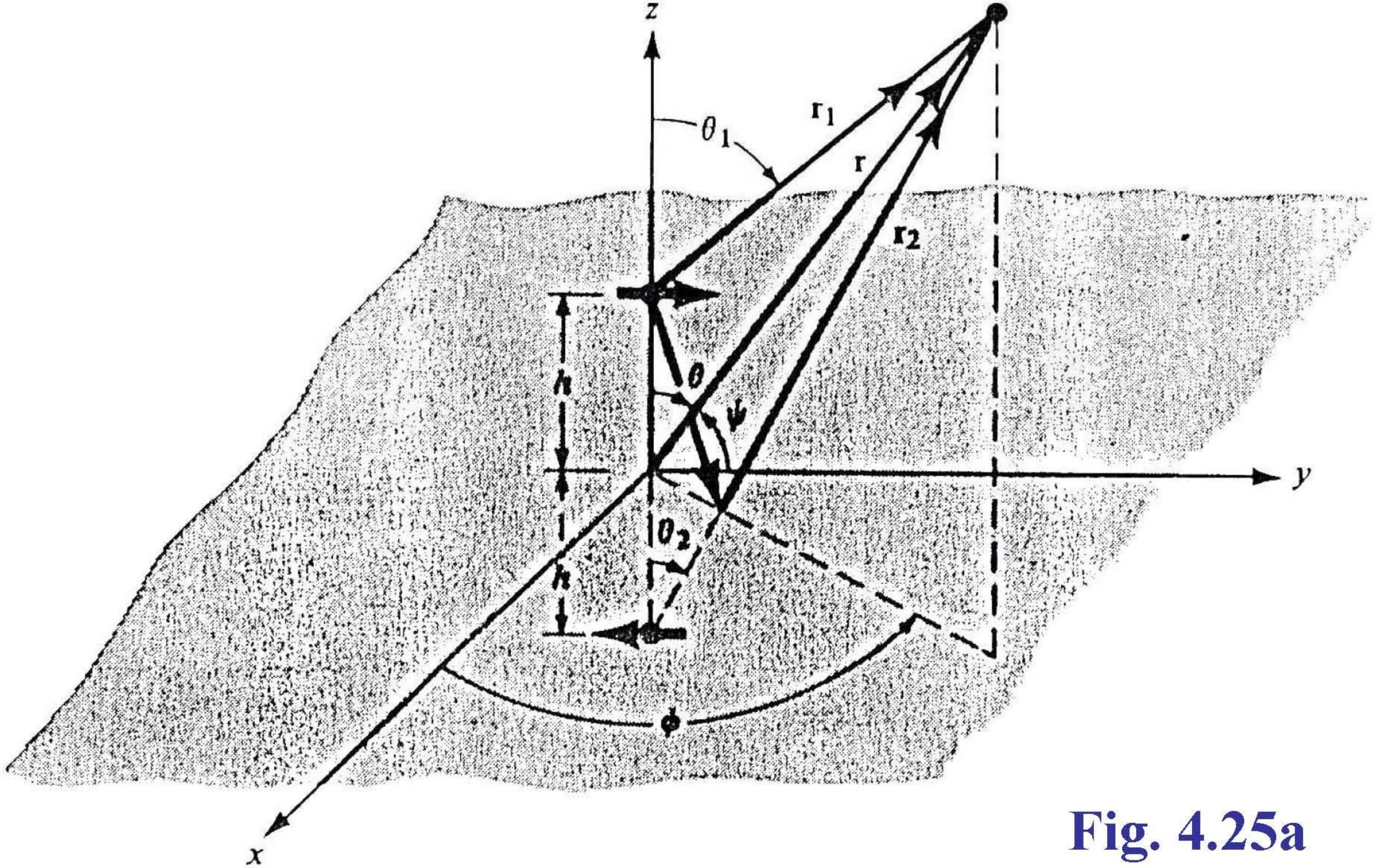


Fig. 4.25a

Horizontal Electric Dipole Above an Infinite Perfect Electric Conductor

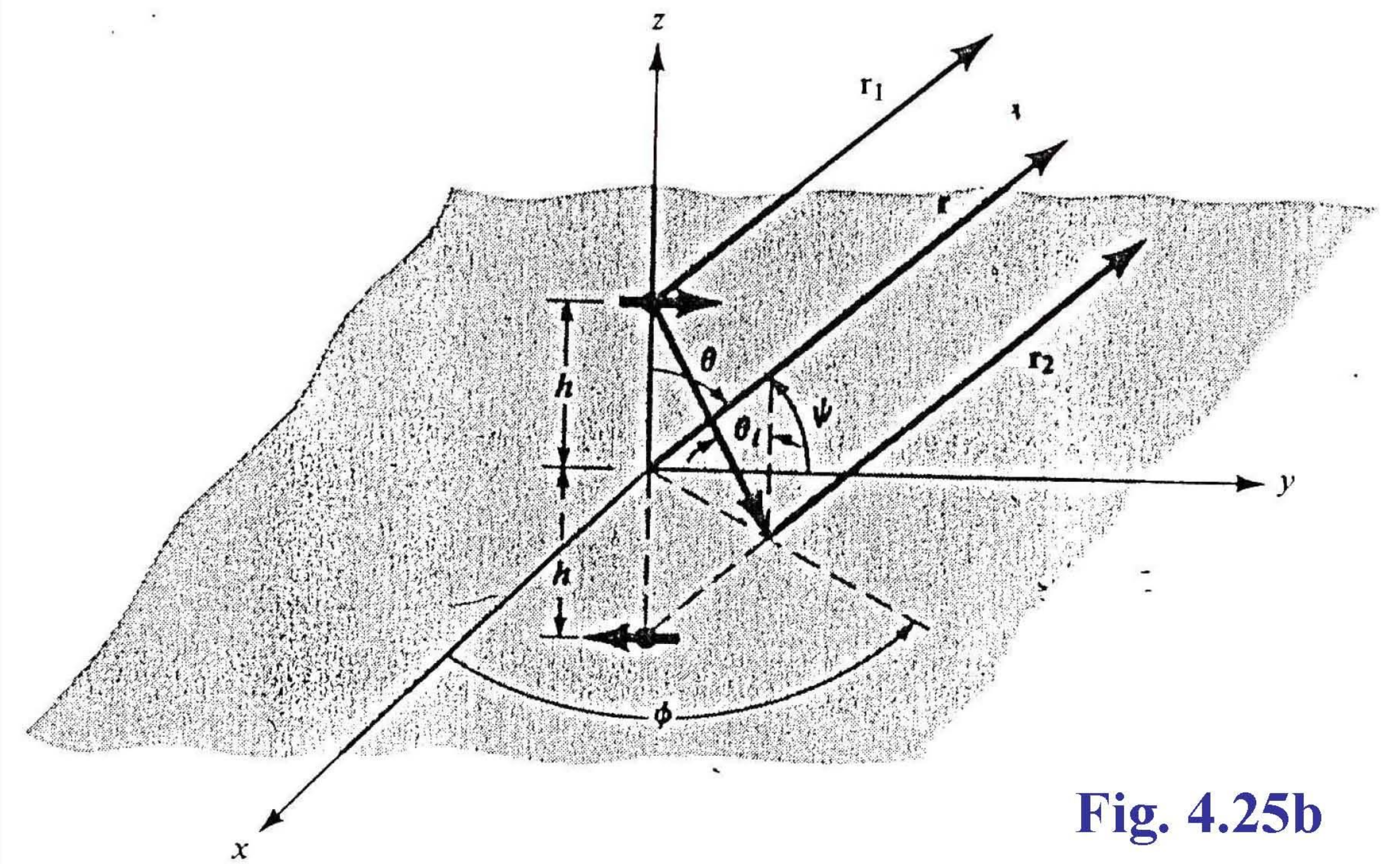


Fig. 4.25b

Lossy Surface

Modeling of the Effect of Earth on Antenna Systems

- An obstacle that is almost always present in an antenna system is the earth.
- The earth is NOT
  - perfectly conducting
  - flat

Effect of Imperfectly Conducting, Flat Earth

- The assumption of a flat earth is a good engineering approximation for observation angles greater than 3 degrees above the horizon.
- Antenna characteristics (especially radiation efficiency) at LF and MF (below 3 MHz) are profoundly and adversely affected by the lossy earth.

Effect of Imperfectly Conducting, Flat Earth

- To improve the radiation efficiency at these frequencies, radial wires or metallic disks are sometimes placed on the ground to simulate a perfectly conducting ground plane.

Constitutive Parameters of Ground

$$\varepsilon_0, \mu_0, \sigma = 0$$

$$\varepsilon_1, \mu_1, \sigma_1$$

Constitutive Parameters of Ground

$$\varepsilon_1, \mu_1, \sigma_1$$

or

$$\dot{\varepsilon}_1, \mu_1$$

Could also include magnetic conductivity, if necessary.

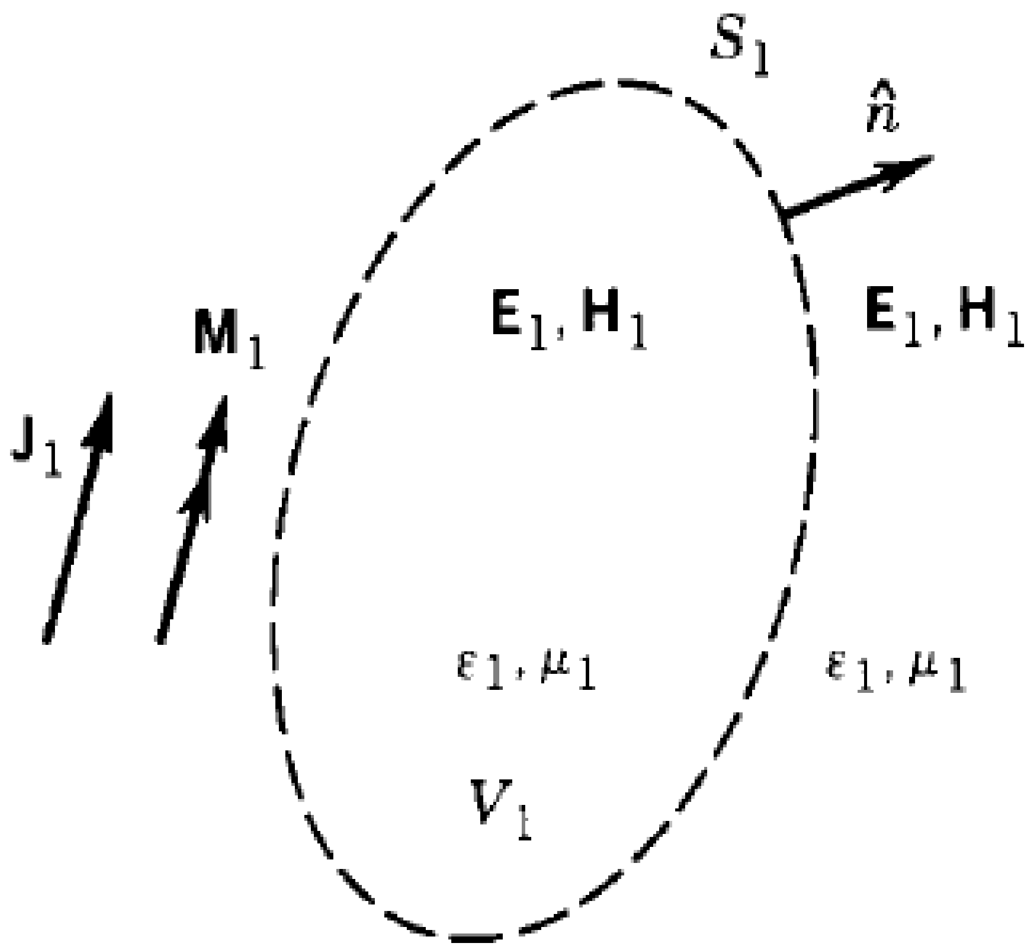
where

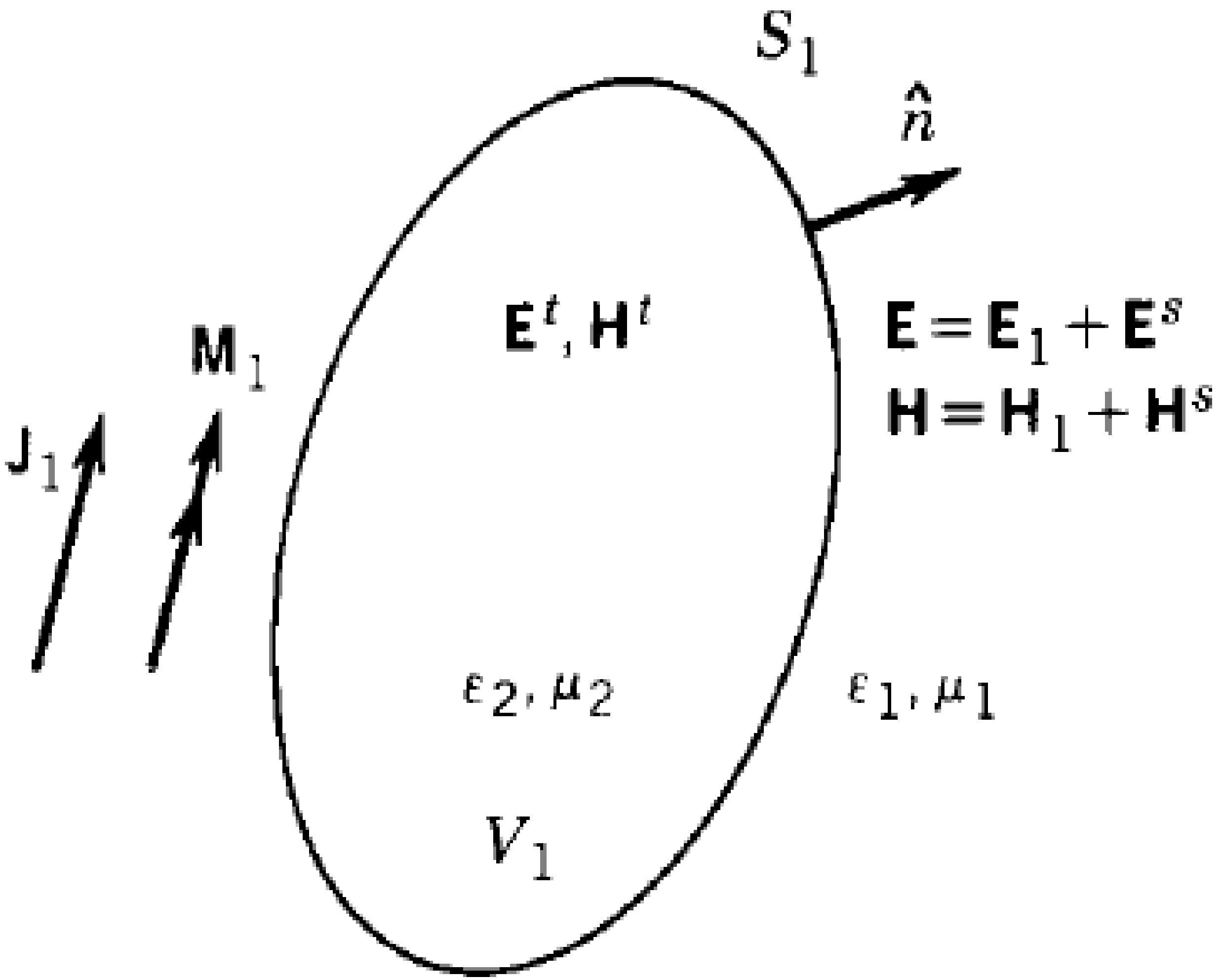
$$\dot{\varepsilon}_1 = \varepsilon_1' - j\varepsilon_1'' = \varepsilon_1 - j\frac{\sigma_1}{\omega}$$

Effect of Imperfectly Conducting, Flat Earth

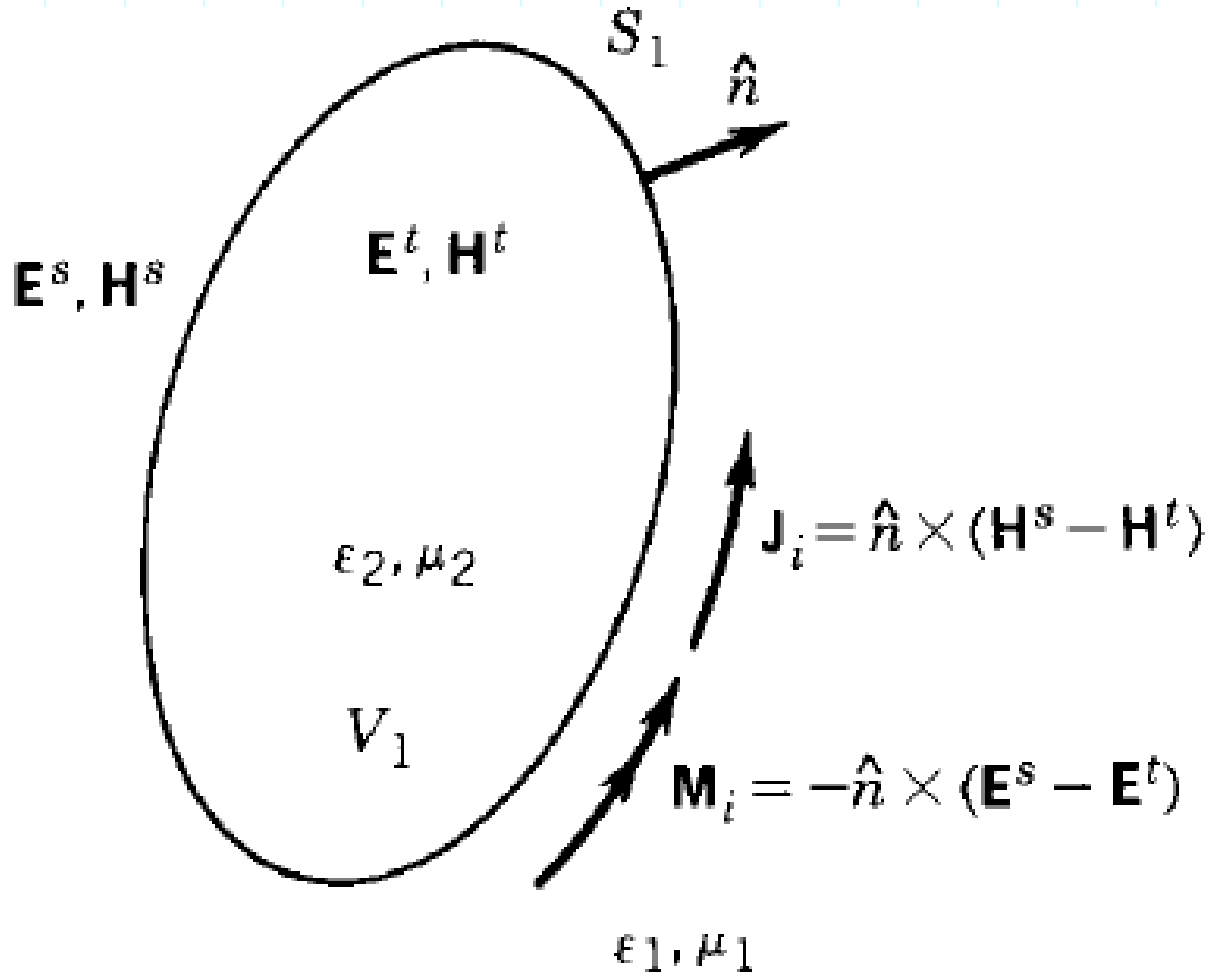
- For horizontal polarization:
  1. The phase of the reflection coefficient (R_h) is exactly 180° for PEC ground planes and near 180° for non-PEC ground planes.
- For vertical polarization:
  1. R_v = +1 for PEC ground plane, but is very different for non-PEC ground plane.
  2. For $\theta_i = 90°$, R_v = -1 for non-PEC ground plane. This causes a null in the pattern which does not occur for PEC ground plane.

Induction Theorem





S_1
$\hat{n}$
$\mathbf{E}^t, \mathbf{H}^t$
$\mathbf{M}_1$
$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}^s$
$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}^s$
$\mathbf{J}_1$
ε_2, μ_2
ε_1, μ_1
V_1


S_1
$\hat{n}$
$\mathbf{E}^s, \mathbf{H}^s$
$\mathbf{E}^t, \mathbf{H}^t$
$\mathbf{J}_i = \hat{n} \times (\mathbf{H}^s - \mathbf{H}^t)$
ϵ_2, μ_2
V_1
$\mathbf{M}_i = -\hat{n} \times (\mathbf{E}^s - \mathbf{E}^t)$
ϵ_1, μ_1

$$\mathbf{J}_i = \hat{n} \times (\mathbf{H}^s - \mathbf{H}^t)$$

$$\mathbf{M}_i = -\hat{n} \times (\mathbf{E}^s - \mathbf{E}^t)$$

$$\mathbf{E}_1|_{\tan} + \mathbf{E}^s|_{\tan} = \mathbf{E}^t|_{\tan} \Rightarrow \hat{n} \times (\mathbf{E}_1 + \mathbf{E}^s) = \hat{n} \times \mathbf{E}^t$$

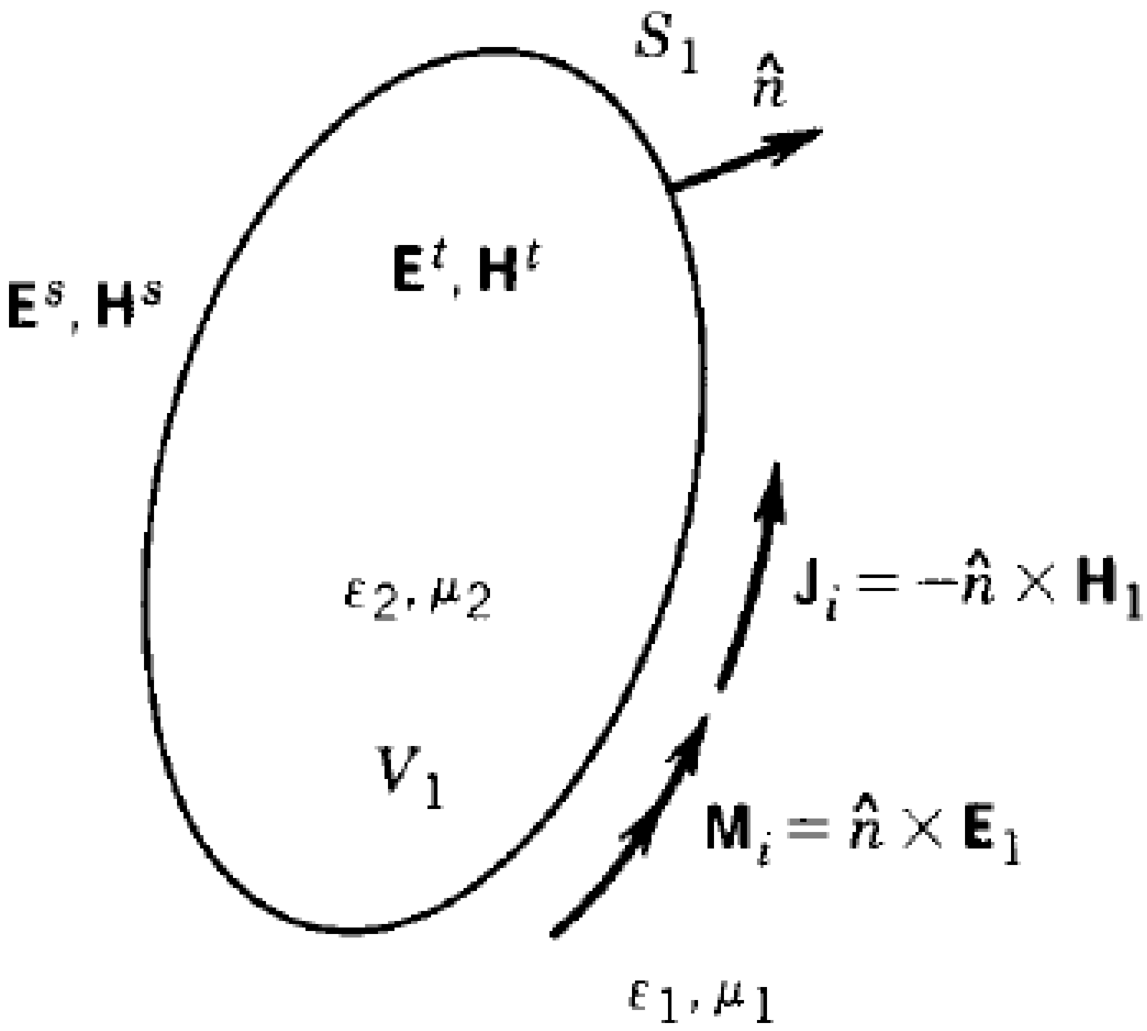
$$\mathbf{H}_1|_{\tan} + \mathbf{H}^s|_{\tan} = \mathbf{H}^t|_{\tan} \Rightarrow \hat{n} \times (\mathbf{H}_1 + \mathbf{H}^s) = \hat{n} \times \mathbf{H}^t$$

$$\mathbf{E}^s|_{\tan} - \mathbf{E}^t|_{\tan} = -\mathbf{E}_1|_{\tan} \Rightarrow \hat{n} \times (\mathbf{E}^s - \mathbf{E}^t) = -\hat{n} \times \mathbf{E}_1$$

$$\mathbf{H}^s|_{\tan} - \mathbf{H}^t|_{\tan} = -\mathbf{H}_1|_{\tan} \Rightarrow \hat{n} \times (\mathbf{H}^s - \mathbf{H}^t) = -\hat{n} \times \mathbf{H}_1$$

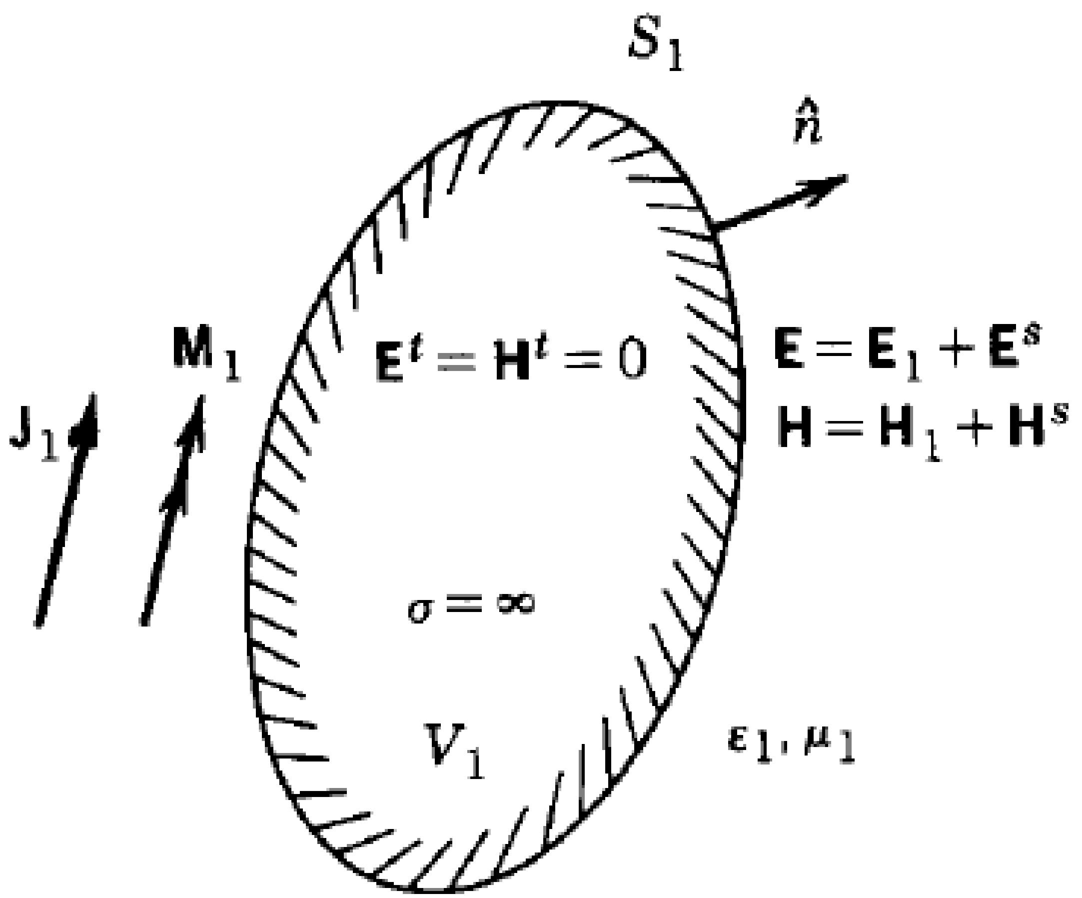
$$\mathbf{J}_i = -\hat{n} \times \mathbf{H}_1$$

$$\mathbf{M}_i = \hat{n} \times \mathbf{E}_1$$

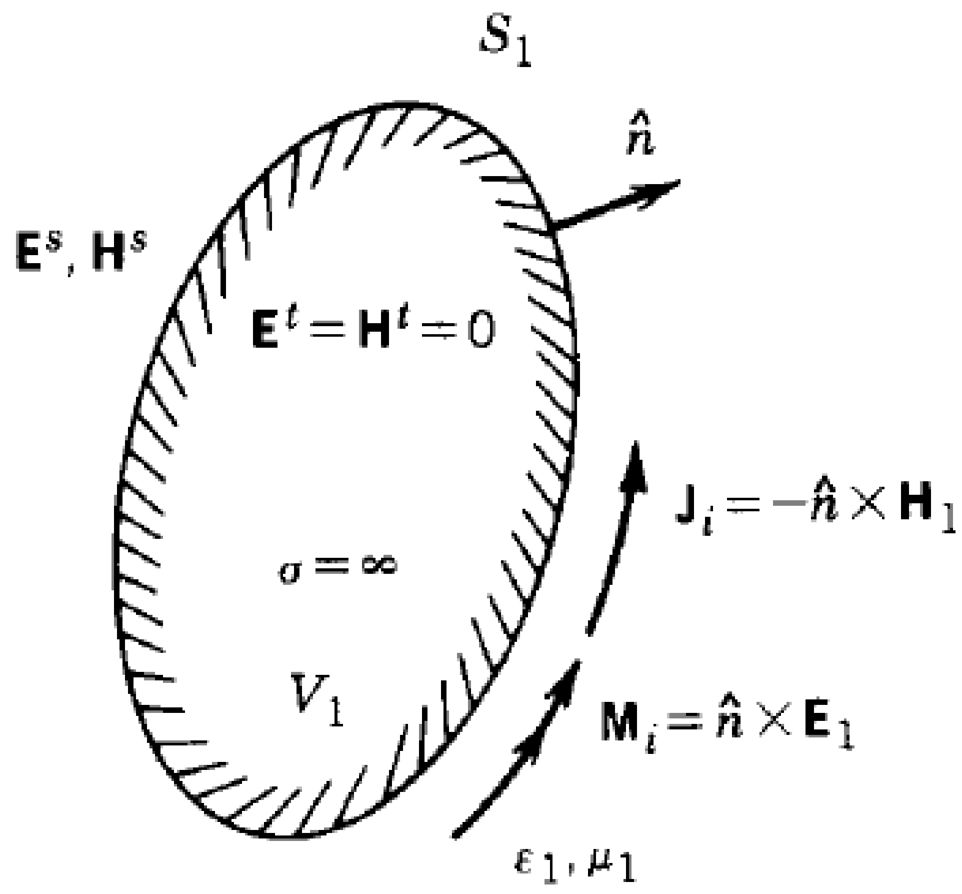


S_1
$\hat{n}$
$\mathbf{E}^t, \mathbf{H}^t$
$\mathbf{E}^s, \mathbf{H}^s$
$\mathbf{J}_i = -\hat{n} \times \mathbf{H}_1$
ε_2, μ_2
V_1
$\mathbf{M}_i = \hat{n} \times \mathbf{E}_1$
ε_1, μ_1

Example

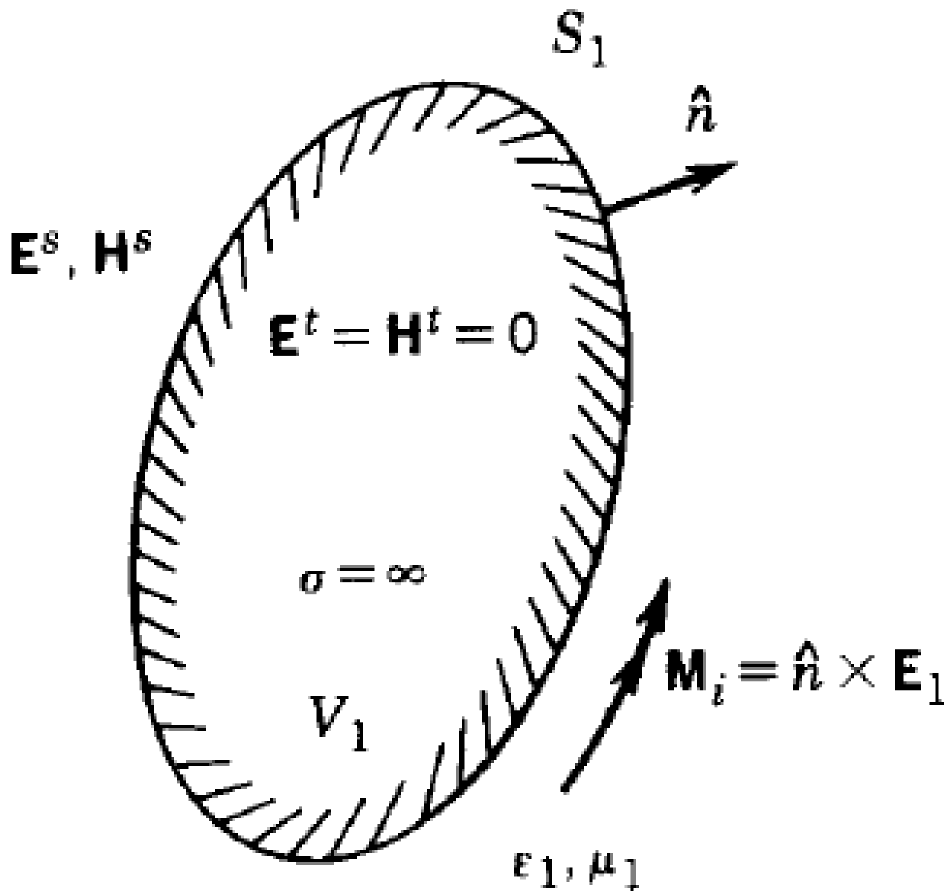


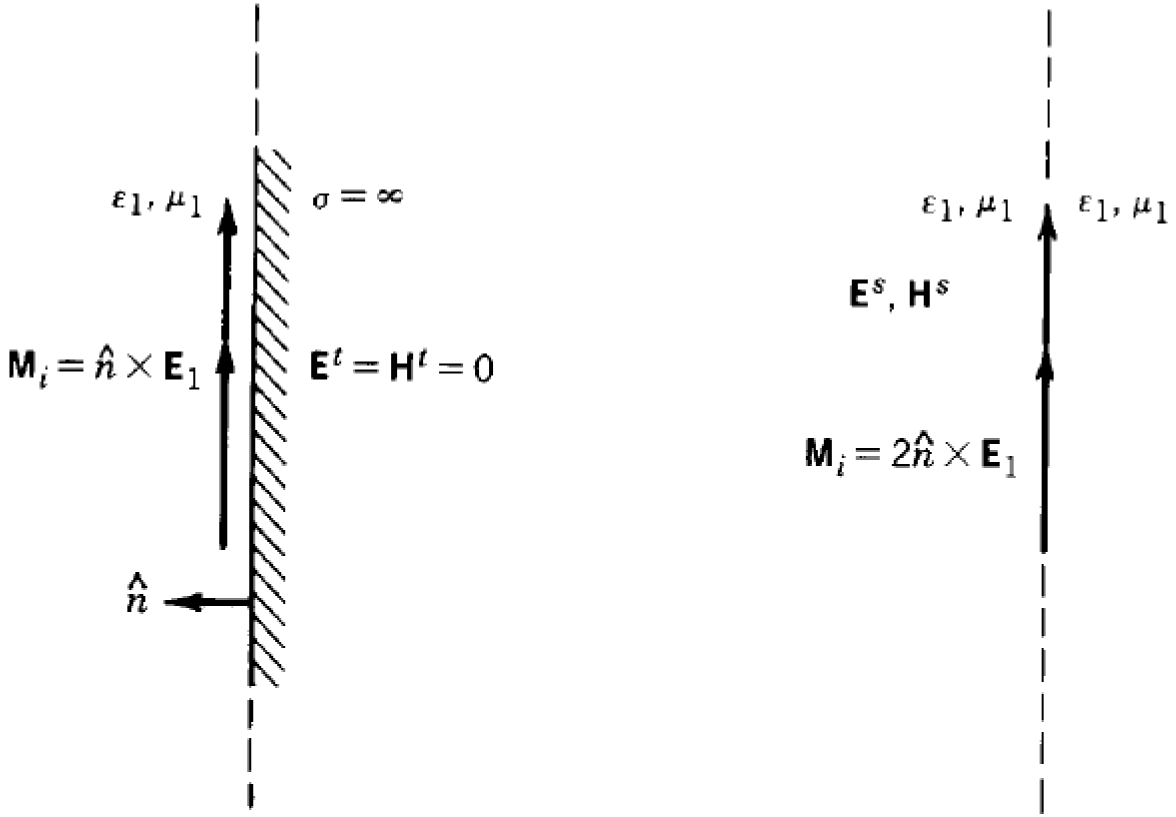
S_1
$\hat{n}$
$\mathbf{M}_1$
$\mathbf{J}_1$
$\mathbf{E}^t = \mathbf{H}^t = 0$
$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}^s$
$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}^s$
$\sigma = \infty$
V_1
ε_1, μ_1



S_1
$\hat{n}$
$\mathbf{E}^s, \mathbf{H}^s$
$\mathbf{E}^t = \mathbf{H}^t = 0$
$\mathbf{J}_i = -\hat{n} \times \mathbf{H}_1$
$\sigma = \infty$
V_1
$\mathbf{M}_i = \hat{n} \times \mathbf{E}_1$
ε_1, μ_1

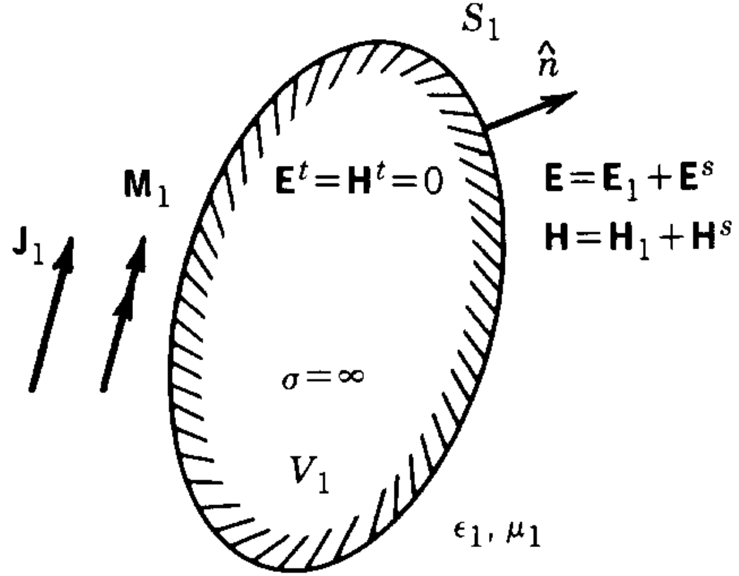
(b)


S_1
$\hat{n}$
$\mathbf{E}^s, \mathbf{H}^s$
$\mathbf{E}^t = \mathbf{H}^t = 0$
$\sigma = \infty$
$\mathbf{M}_i = \hat{n} \times \mathbf{E}_1$
V_1
ε_1, μ_1

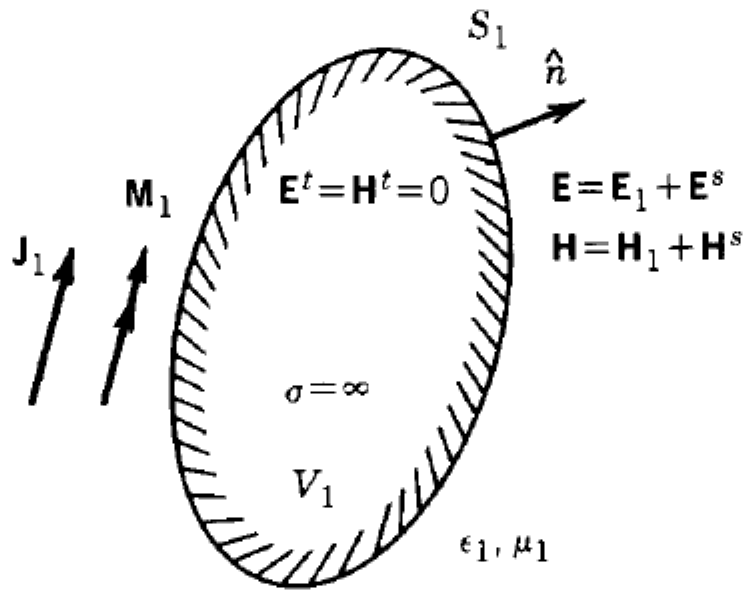


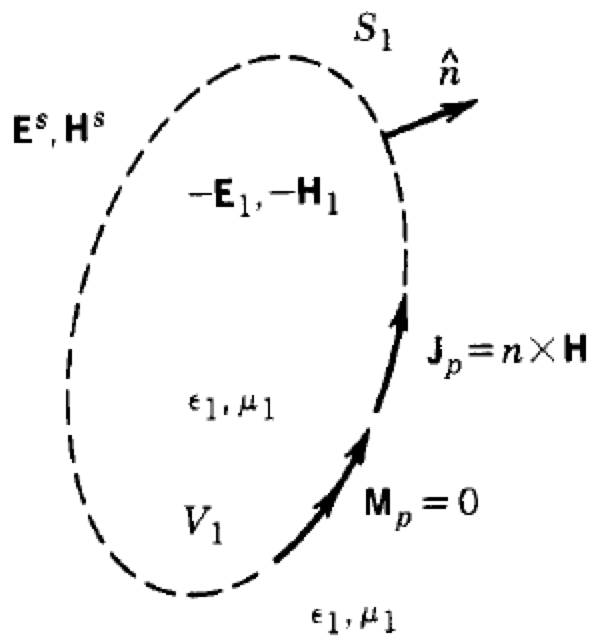
ε_1, μ_1
$\sigma = \infty$
$\mathbf{M}_i = \hat{n} \times \mathbf{E}_1$
$\mathbf{E}^t = \mathbf{H}^t = 0$
$\hat{n}$
ε_1, μ_1
ε_1, μ_1
$\mathbf{E}^s, \mathbf{H}^s$
$\mathbf{M}_i = 2\hat{n} \times \mathbf{E}_1$

Physical Equivalence



Actual and Equivalent Problems





$$\mathbf{M}_p = -\hat{n} \times (\mathbf{E} - \mathbf{E}^t) = -\hat{n} \times \mathbf{E} = -\hat{n} \times (\mathbf{E}_1 + \mathbf{E}^s) = 0$$

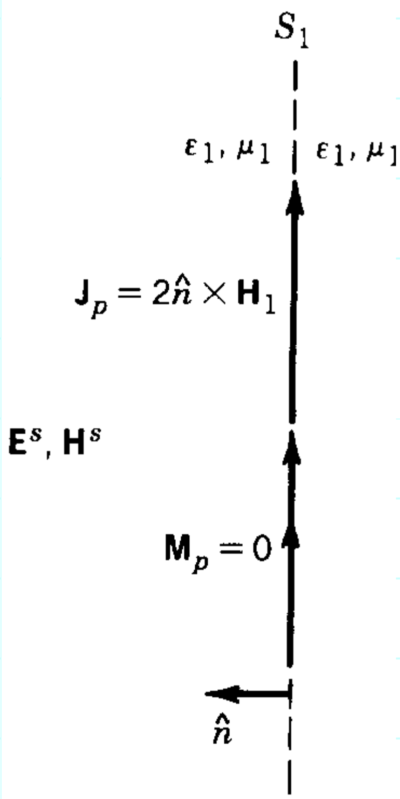
$$-\hat{n} \times \mathbf{E}_1 = \hat{n} \times \mathbf{E}^s$$

$$\mathbf{J}_p = \hat{n} \times (\mathbf{H} - \mathbf{H}^t) = \hat{n} \times \mathbf{H} = \hat{n} \times (\mathbf{H}_1 + \mathbf{H}^s)$$

$$\mathbf{J}_p = \hat{n} \times \mathbf{H}_1 + \hat{n} \times \mathbf{H}^s$$

$$\mathbf{J}_p = \hat{n} \times \mathbf{H} = \hat{n} \times \left(\mathbf{H}^s + \mathbf{H}_1\right) = \hat{n} \times \left[\mathbf{H}^s - \left(-\mathbf{H}_1\right)\right]$$

$$\mathbf{M}_p = -\hat{n} \times \mathbf{E} = -\hat{n} \times \left(\mathbf{E}^s + \mathbf{E}_1\right) = -\hat{n} \times \left[\mathbf{E}^s - \left(-\mathbf{E}_1\right)\right] = 0$$



$$\mathbf{J}_p = \hat{n} \times \mathbf{H} = \hat{n} \times \left(\mathbf{H}_1 + \mathbf{H}^s\right) = 2\hat{n} \times \mathbf{H}_1$$

Why are we bothered?

1. If we have an infinite plain solution can be easily obtained using image theory and one of the equivalence studied

2. Both the induction and physical equivalence suggest more appropriate approximations or simplifications that we can make when we attempt to solve a problem whose exact solution is not easily obtainable.

When making approximations do both equivalents lead to identical approximate results or is one superior to other?

Summary

Induction equivalence $(\mathbf{M}_i = \hat{n} \times \mathbf{E}_1)$

Known current, but the medium within and outside the obstacle are not the same. 6.30-6.35 cannot be used

Physical equivalence $(\mathbf{J}_p = \hat{n} \times \mathbf{H})$

Unknown current, but the medium within and outside the obstacle are the same. 6.30-6.35 can be used

Approximations (Induction Equivalence)

1. Difficulty is obtaining a solution due to lack of equations that can be used with known current densities

2. Crudest approximation is the assumption that the obstacle is electrically large and then we can use image theory to solve the problem.

3. This assumes that locally on the surface of the obstacle each point and its immediate neighbors form a flat surface

4. Best results will be for scatterers whose electrical dimensions are large in comparison to wavelength

Approximations (Induction Equivalence)

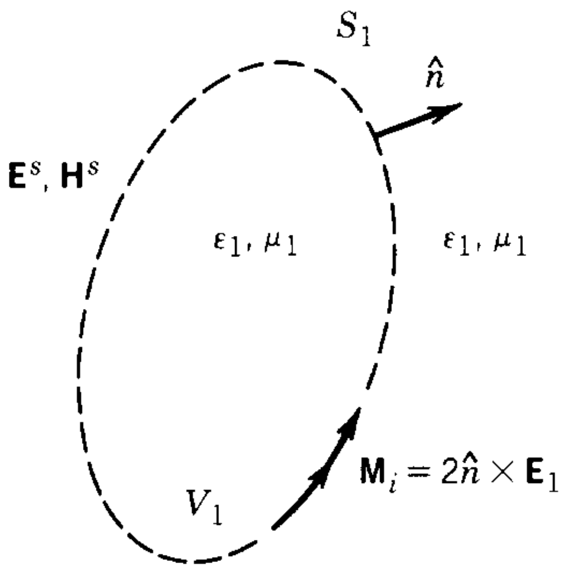


FIGURE 7-16 Approximate induction equivalent for scattering from a perfect electric conductor (PEC).

Approximations (Physical Equivalence)

1. Difficulty arises because we donot know the current density that must be placed along surface S_1

2. Crudest approximation is the assumption that the total tangential **H** field on the surface of the conductor is twice that of the tangential $\mathbf{H}_1$.

3. The current density to be placed on the surface is:

$$\mathbf{J}_p \simeq 2\hat{n} \times \mathbf{H}_1$$

Approximations (Physical Equivalence)

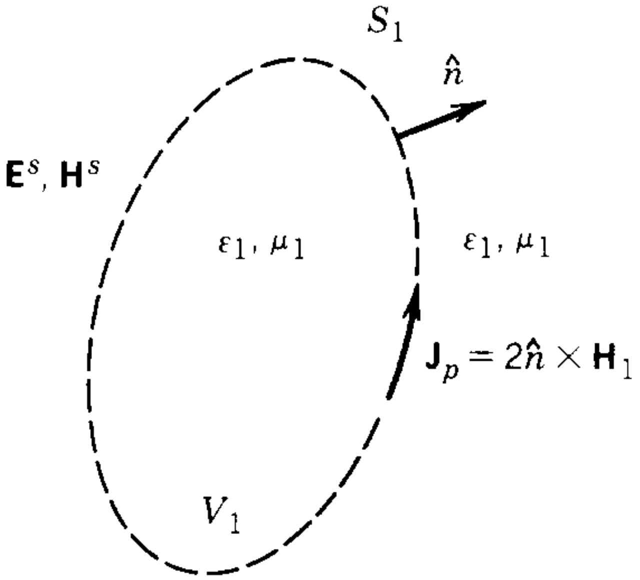


FIGURE 7-17 Approximate physical equivalent for scattering from a perfect electric conductor (PEC).