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Complex Permittivity

Maxwell-Ampere equation can be written as:

$$\nabla \times \bar{H} = \bar{J}_i + \bar{J}_c + \frac{\partial \bar{D}}{\partial t}$$

$$= \bar{J}_i + \sigma_s \bar{E} + j\omega\dot{\varepsilon} \bar{E}$$

$\sigma_s \rightarrow$ static field conductivity

$$\nabla \times \bar{H} = \bar{J}_i + \sigma_s \bar{E} + j\omega(\varepsilon' - j\varepsilon'')\bar{E}$$

$$= \bar{J}_i + \sigma_s \bar{E} + j\omega\varepsilon' \bar{E} + \omega\varepsilon'' \bar{E}$$

$$= \bar{J}_i + \bar{E}(\sigma_s + j\omega\varepsilon') + \omega\varepsilon'' \bar{E}$$

$$= \bar{J}_i + \bar{E}\underbrace{(\sigma_s + \omega\varepsilon'')}_{\sigma_e \rightarrow \text{equivalent conductivity}} + j\omega\varepsilon' \bar{E}$$

$$\sigma_e = \sigma_s + \omega\varepsilon'' = \sigma_s + \sigma_a$$

σ_a = alternating field conductivity $= \omega\varepsilon''$

For conductors

$$\sigma_s = -\mu_e q_{ve}$$

For semiconductors

$$\sigma_s = -\mu_e q_{ve} + \mu_h q_{vh}$$

$\sigma_e \rightarrow$ total conductivity composed of static portion σ_s and alternating part σ_a caused by rotation of dipoles as they attempt to align with the applied field when its polarity is alternating. The phenomenon that contributes the alternating conductivity σ_a is referred to as dielectric hysterisis.

Many dielectric materials possess very low values of static σ_s conductivities and behave as good insulators. When these materials are subjected to alternating fields, they exhibit very high values of alternating field σ_a conductivities and they consume considerable energy. The heat generated by this RF process is used for industrial heating processes. The best-known process is that of microwave cooking. Others include selective heating of human tissue for tumor treatment and selective heating of certain compounds in materials that possess conductivities higher than the other constituents.

Total electric current density can be written as,

$$\bar{J}_t = \bar{J}_i + \sigma_e \bar{E} + j\omega\varepsilon' \bar{E}$$

$$= \bar{J}_i + j\omega\varepsilon'\left(1 - j\frac{\sigma_e}{\omega\varepsilon'}\right)\bar{E}$$

$$= \bar{J}_i + j\omega\varepsilon'(1 - j\tan\delta_e)\bar{E}$$

$\tan\delta_e$ = effective electric loss tangent

$$= \frac{\sigma_e}{\omega\varepsilon'} = \frac{\sigma_s + \sigma_a}{\omega\varepsilon'} = \frac{\sigma_s}{\omega\varepsilon'} + \frac{\sigma_a}{\omega\varepsilon'}$$

$$= \frac{\sigma_s}{\omega\varepsilon'} + \frac{\varepsilon''}{\varepsilon'} = \tan\delta_s + \tan\delta_a = \frac{\varepsilon_e''}{\varepsilon_e'}$$

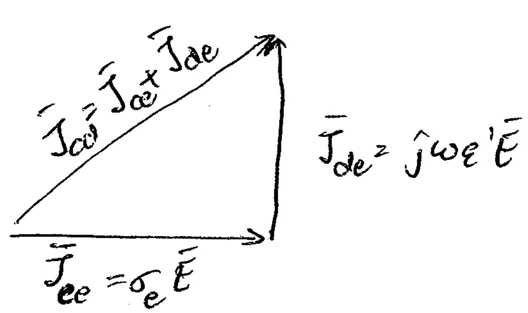
$\tan\delta_s$ = static electric loss tangent $= \frac{\sigma_s}{\omega\varepsilon'}$

$\tan\delta_a$ = alternating electric loss tangent $= \frac{\sigma_a}{\omega\varepsilon'} = \frac{\varepsilon''}{\varepsilon'}$

The effective conduction $\bar{J}_{ce}$ and displacement $\bar{J}_{de}$ current densities can be written as

$$\bar{J}_{cd} = \bar{J}_{ce} + \bar{J}_{de} = \sigma_e\bar{E} + j\omega\varepsilon'\bar{E} = j\omega\varepsilon'\left(1 - j\frac{\sigma_e}{\omega\varepsilon'}\right)\bar{E}$$

$$= j\omega\varepsilon'(1 - j\tan\delta_e)\bar{E}$$



Materials can also be classified as good dielectrics or good conductors according to the value of the $\sigma_e/\omega\varepsilon'$ ratio. That is:

1. Good Dielectrics $(\sigma_e/\omega\varepsilon') \ll 1$

$$\bar{J}_{cd} = j\omega\varepsilon'\left(1 - j\frac{\sigma_e}{\omega\varepsilon'}\right)\bar{E} \overset{\sigma_e/\omega\varepsilon' \ll 1}{\simeq} j\omega\varepsilon'\bar{E}$$

2. Good Conductors $(\sigma_e/\omega\varepsilon') \gg 1$

$$\bar{J}_{cd} = j\omega\varepsilon'\left(1 - j\frac{\sigma_e}{\omega\varepsilon'}\right)\bar{E} \overset{\sigma_e/\omega\varepsilon' \gg 1}{\simeq} \sigma_e\bar{E}$$

As a function of frequency, the electric polarization can be written as

$$\bar{P}(\omega) = \varepsilon_0 \chi_e(\omega)\bar{E}_a(\omega)$$

where

$$\chi_e(\omega) = \chi_e'(\omega) + j\chi_e''(\omega)$$

$$= \left[\chi_{ed}'(\omega) + \chi_{ei}'(\omega) + \chi_{ee}'(\omega)\right]$$

$$- j\left[\chi_{ed}''(\omega) + \chi_{ei}''(\omega) + \chi_{ee}''(\omega)\right]$$

Complex Permeability

Most dielectric materials, including diamagnetic, paramagnetic, and antiferromagnetic material, is nearly the same as that of free space μ_0. Ferromagnetic and Ferrimagnetic materials exhibit much higher permeability than free space.

The Maxwell-Faraday equation can be written as

$$\nabla \times \bar{E} = -\bar{M}_i - j\omega\mu\bar{H} = -\bar{M}_i - j\omega(\mu' - j\mu'')\bar{H}$$

$$= -\bar{M}_i - \omega\mu''\bar{H} - j\omega\mu'\bar{H} = -\bar{M}_t$$

$\bar{M}_t$ - total magnetic current density

$\bar{M}_i$ - impressed magnetic current density

$\bar{M}_c$ - conduction magnetic current density $= \omega\mu''\bar{H}$

$\bar{M}_d$ - displacement magnetic current density $= j\omega\mu'\bar{H}$

$$\bar{M}_t = \bar{M}_i + j\omega\mu'\left(1 - j\frac{\mu''}{\mu'}\right)\bar{H} = \bar{M}_i + j\omega\mu'(1 - j\tan\delta_m)\bar{H}$$

$\tan\delta_m$ - alternating magnetic loss tangent $= \frac{\mu''}{\mu'}$

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Chapter - 3
Wave Equation and its solutions

Wave equation

The Faraday's and Ampere's Law can be written as

$$\nabla \times \bar{E} = -\bar{M}_i - \frac{\partial \bar{B}}{\partial t} = -\bar{M}_i - \mu \frac{\partial \bar{H}}{\partial t} \quad -(1)$$

$$\nabla \times \bar{H} = \bar{J}_i + \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t} \quad -(2)$$

Taking curl on both sides of equation (1)

$$\nabla \times \nabla \times \bar{E} = -\nabla \times \bar{M}_i - \mu \nabla \times \left(\frac{\partial \bar{H}}{\partial t}\right) = -\nabla \times \bar{M}_i - \mu \frac{\partial}{\partial t}(\nabla \times \bar{H})$$

In phasor form we can write

$$\nabla \times \nabla \times \bar{E} = -\nabla \times \bar{M}_i - j\omega\mu(\nabla \times \bar{H}) \quad -(3)$$

Substituting eqn (2) into (3)

$$\nabla \times \nabla \times \bar{E} = -\nabla \times \bar{M}_i - j\omega\mu\left(\bar{J}_i + \sigma\bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}\right)$$

$$= -\nabla \times \bar{M}_i - j\omega\mu \bar{J}_i - j\omega\mu\sigma\bar{E} - j\omega\mu\varepsilon \cancel{\partial}(j\omega\bar{E})$$

$$= -\nabla \times \bar{M}_i - j\omega\mu \bar{J}_i - j\omega\mu\sigma\bar{E} + \omega^2\mu\varepsilon\bar{E}$$

Using vector identity

$$\nabla \times \nabla \times \bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$$

$$\therefore \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\nabla \times \bar{M}_i - j\omega\mu \bar{J}_i - j\omega\mu\sigma \bar{E} + \omega^2 \mu \varepsilon$$

Using Maxwell's eqn (3)

$$\nabla \cdot \bar{D} = \rho_v \rightarrow \nabla \cdot \varepsilon \bar{E} = \rho_v \qquad \nabla \cdot \bar{E} = \frac{\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 \bar{E} = \nabla \times \bar{M}_i + j\omega\mu \bar{J}_i + j\omega\mu\sigma \bar{E} + \frac{1}{\varepsilon} \nabla \rho_v - \omega^2 \mu \varepsilon \bar{E}}$$

III^{ly}

$$\boxed{\nabla^2 \bar{H} = -\nabla \times \bar{J}_i + \sigma \bar{M}_i + \frac{1}{\mu} \nabla q_{vm} + j\omega\varepsilon \bar{M}_i + j\omega\mu\sigma \bar{H} - \omega^2 \mu \varepsilon \bar{H}}$$

These two equations are referred to as vector wave equation for $\bar{E}$ and $\bar{H}$

For source free region.

$$\nabla^2 \bar{E} = j\omega\mu\sigma\bar{E} - \omega^2\mu\varepsilon\bar{E}$$

$$\nabla^2 \bar{H} = j\omega\mu\sigma\bar{H} - \omega^2\mu\varepsilon\bar{H}$$

$$\nabla^2 \bar{E} = \gamma^2\bar{E} \quad \& \quad \nabla^2\bar{H} = \gamma^2\bar{H}$$

$$\gamma^2 = j\omega\mu\sigma - \omega^2\mu\varepsilon = j\omega\mu(\sigma + j\omega\varepsilon)$$

$\gamma \rightarrow$ propagation constant

$\alpha \rightarrow$ attenuation constant (Np/m)

$\beta \rightarrow$ phase constant (rad/m)

$$\gamma = \alpha + j\beta$$

For lossless media the wave equation can be written as.

$$\nabla^2\bar{E} = \mu\varepsilon\frac{\partial^2\bar{E}}{\partial t^2} \quad \text{or} \quad \nabla^2\bar{E} = -\omega^2\mu\varepsilon\bar{E}$$

$$\nabla^2\bar{H} = \mu\varepsilon\frac{\partial^2\bar{H}}{\partial t^2} \quad \text{or} \quad \nabla^2\bar{H} = -\omega^2\mu\varepsilon\bar{H}$$

$$\nabla^2 \bar{E} = -\omega^2 \mu \varepsilon \bar{E} = -\beta^2 \bar{E}$$

$$\nabla^2 \bar{H} = -\omega^2 \mu \varepsilon \bar{H} = -\beta^2 \bar{H}$$

$$\beta^2 = \omega^2 \mu \varepsilon$$

In some book β is written as k

Solution of wave equation

Electromagnetic fields associated with a given boundary-value problem must satisfy Maxwell's equations or the vector wave equations. The vector wave equations reduce to a number of scalar Helmholtz equations, and the general solutions can be constructed once solutions to each of the scalar Helmholtz equations are found.

This chapter will solve wave equations using separation of variable method. and general solution of the scalar Helmholtz equation using this method can be constructed in 3-D orthogonal coordinate system.

Rectangular Coordinate System

Consider a source free and lossless media. In this media $(\bar{J}_i = \bar{M}_i = \rho_{ve} = \rho_{vm} = 0)$ & $(\sigma = 0)$

$$\nabla^2 \bar{E} = -\beta^2 \bar{E} \qquad \nabla^2 \bar{H} = -\beta^2 \bar{H}$$

In rectangular coordinates, a general solution for $\bar{E}$ can be written as.

$$\bar{E}(x,y,z) = \hat{a}_x E_x(x,y,z) + \hat{a}_y E_y(x,y,z) + \hat{a}_z E_z(x,y,z)$$

where x, y, z are rectangular coordinates.

So we can write

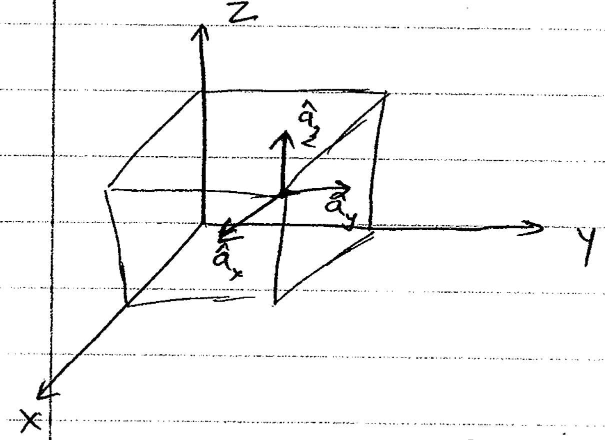
$$\nabla^2 \bar{E} + \beta^2 \bar{E} = \nabla^2 (\hat{a}_x E_x + \hat{a}_y E_y + \hat{a}_z E_z) + \beta^2 (\hat{a}_x E_x + \hat{a}_y E_y + \hat{a}_z E_z) = 0$$

This reduces to three scalar equations

$$\nabla^2 E_x(x,y,z) + \beta^2 E_x(x,y,z) = 0$$

$$\nabla^2 E_y(x,y,z) + \beta^2 E_y(x,y,z) = 0$$

$$\nabla^2 E_z(x,y,z) + \beta^2 E_z(x,y,z) = 0$$



$\nabla^2 E_x(x,y,z) + \beta^2 E_x(x,y,z)$ can be written as:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + \beta^2 E_x = 0 \qquad \text{--- (1)}$$

Using separation of variables method.

$$E_x(x,y,z) = f(x)\, g(y)\, h(z) \qquad -(2)$$

Substituting (2) in (1) we get

$$gh\frac{\partial^2 f}{\partial x^2} + fh\frac{\partial^2 g}{\partial y^2} + fg\frac{\partial^2 h}{\partial z^2} + \beta^2 fgh = 0$$

Since $f(x)$, $g(y)$ & $h(z)$ are each a function of only one variable, we can replace the partials by ordinary derivatives.

Dividing by fgh, we get

$$\frac{1}{f}\frac{d^2 f}{dx^2} + \frac{1}{g}\frac{d^2 g}{dy^2} + \frac{1}{h}\frac{d^2 h}{dz^2} + \beta^2 = 0$$

or

$$\frac{1}{f}\frac{d^2 f}{dx^2} + \frac{1}{g}\frac{d^2 g}{dy^2} + \frac{1}{h}\frac{d^2 h}{dz^2} = -\beta^2$$

Separating into three equations.

$$\frac{1}{f}\frac{d^2 f}{dx^2} = -\beta_x^2 \quad \text{or} \quad \frac{d^2 f}{dx^2} = -\beta_x^2 f$$

$$\frac{1}{g}\frac{d^2 g}{dy^2} = -\beta_y^2 \quad \text{or} \quad \frac{d^2 g}{dy^2} = -\beta_y^2 g$$

$$\frac{1}{h}\frac{d^2h}{dz^2} = -\beta_z^2 \Rightarrow \frac{d^2h}{dz^2} = -\beta_z^2 h$$

where

$$\beta_x^2 + \beta_y^2 + \beta_z^2 = \beta^2$$

This equation is called the constraint equation

$\beta_x, \beta_y, \beta_z$ are known as wave constants in the x, y, z directions

The solution of each equations can take different forms. One of the solutions is

$$f_1(x) = A_1 e^{-j\beta_x x} + B_1 e^{j\beta_x x}$$

or

$$f_2(x) = C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x)$$

Similarly solutions for $g(y)$ & $h(z)$ can be written as

$$g_1(y) = A_2 e^{-j\beta_y y} + B_2 e^{j\beta_y y}$$

or

$$g_2(y) = C_2 \cos(\beta_y y) + D_2 \sin(\beta_y y)$$

and

$$h_1(z) = A_3 e^{-j\beta_z z} + B_3 e^{j\beta_z z}$$

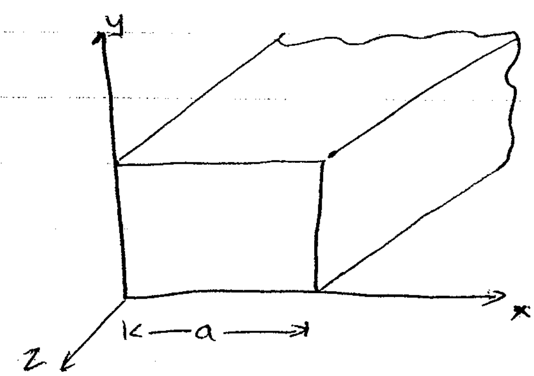
or

$$h_2(z) = C_3 \cos(\beta_z z) + D_3 \sin(\beta_z z)$$

The solution in terms of complex exponential represent traveling waves and the solutions represent standing waves. Wave functions representing various wave types in rectangular coordinates are found in table 3-1. Consider a specific example where we assume that the appropriate solutions for f, g, and h are given in previous equations. Thus we can write

$$E_x(x,y,z) = [C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x)][C_2 \cos(\beta_y y) + D_2 \sin(\beta_y y)]$$
$$[A_3 e^{-j\beta_z z} + D_3 e^{j\beta_z z}] \quad -(A)$$

This is an appropriate solution for any of the electric or magnetic field components inside a rectangular pipe shown in figure below.



The waveguide is bounded in x & y directions and has its length along the z axis. Because the waveguide is bounded in the x and y directions, standing waves, represented by cosine and sine functions, have been chosen as solutions for f(x) & g(y) functions.

Since the waveguide is not bounded in the z-directi[on] traveling waves, represented by complex exponential functions, have been chosen as solution for $h(z)$

The instantaneous form of the electric field intensity in the x direction E_x can be written as

$$\mathcal{E}_x(x,y,z;t) = Re\left[E_x(x,y,z)\, e^{j\omega t}\right]$$

Considering first exponent of equation (A), we get

$$\mathcal{E}_x^{+}(x,y,z;t) = Re\left[E_x^{+}(x,y,z)\, e^{j\omega t}\right]$$

$$= Re\left\{\left[C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x)\right] \times \left[C_2 \cos(\beta_y y) + D_2 \sin(\beta_y y)\right] A_3 e^{j(\omega t - \beta_z z)}\right\}$$

If the constants are real

$$\mathcal{E}_x^{+}(x,y,z;t) = \left[C_1 \cos(\beta_x x) + D_1 \sin(\beta_x x)\right] \times \left[C_2 \cos(\beta_y y) + D_2 \sin(\beta_y y)\right] A_3 \cos(\omega t - \beta_z z)$$

$$\omega t - \beta_z z = C_0$$

Differentiating wrt time

$$\omega - \beta_z \frac{dz}{dt} = 0 \qquad \frac{dz}{dt} = \frac{\omega}{\beta_z} = v_p$$

z_p is referred to as an equiphase point and its velocity is denoted as the phase velocity.

<u>Source free and lossy media.</u>

Adder is repetition of the same element

XORs are important for adders.

z_p is referred to as an equiphase point and its velocity is denoted as the phase velocity.

Source free and Lossy media.