SOLVING LINEAR SYSTEMS

A system of m linear equations with n unknowns can be written as:

\[
\begin{align*}
\alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1n}x_n &= b_1 \\
\alpha_{21}x_1 + \alpha_{22}x_2 + \cdots + \alpha_{2n}x_n &= b_2 \\
&\vdots \\
\alpha_{m1}x_1 + \alpha_{m2}x_2 + \cdots + \alpha_{mn}x_n &= b_m
\end{align*}
\]

Where

- \(x_1, x_2, \ldots, x_n\) are unknowns
- \(\alpha_{ij}, \beta_{ij}, \ldots, \alpha_{mn}\) are coefficients
- \(b_1, b_2, \ldots, b_m\) are constant terms

Matrix equivalent:

\[
A \mathbf{x} = \mathbf{b}
\]

Where

- \(A\) is an \(m \times n\) coefficient matrix
- \(\mathbf{b}\) is an \(m\)-element constant term column vector
- \(\mathbf{x}\) is an \(n\)-element unknowns column vector
SOLUTION SET

A LINEAR SYSTEM MAY

1. HAVE INFINITE NUMBER OF SOLUTIONS
2. HAVE A UNIQUE SOLUTION
3. HAVE NO SOLUTION

USUALLY (BUT NOT ALWAYS),

1. A SYSTEM WITH FEWER INDEPENDENT EQUATIONS THAN
   Unknowns has INFINITE SOLUTIONS (UNDERDETERMINED)

   ![Graph showing a line representing one equation with two unknowns]

   Example: One equation
   Two unknowns (line).

2. A SYSTEM WITH THE SAME NUMBER OF EQUATIONS
   THAN Unknowns has ONE SOLUTION.

   ![Graph showing a point representing two equations with two unknowns]

   Two equations,
   Two unknowns

3. A SYSTEM WITH MORE EQUATIONS (LINEARLY INDEPENDENT)
   THAN Unknowns IS OVERDETERMINED AND HAS
   NO SOLUTION (I.E., ALL EQUATIONS CANNOT BE SATISFIED
   SIMULTANEOUSLY).

   ![Graph showing a system of equations with three equations and two unknowns]

   Three equations,
   Two unknowns