Problem 1

Consider a vector v generated using the following code in Matlab:

\[
\begin{align*}
n &= 0:0.3:2*\pi; \\
v &= 5*\sin(n*1000); 
\end{align*}
\]

1. \([20\text{ points}]\) Write a program to numerically calculate the first derivative of v in a way that the resulting vector has as many elements as v. Use central, forward, backward difference approximations as you see fit to obtain the smallest possible error.

2. \([15\text{ points}]\) Verify correctness of your result using the 'diff' command on Matlab. What is the average percent error between the results using your code and what you obtained using Matlab.

3. \([5\text{ points}]\) Are the length of the resulting vectors for your code and Matlab 'diff' command the same? Why?

Problem 2

1. \([40\text{ points}]\) Write a Matlab function that solves a linear system of the type \(Ax = b\) using the Gauss-Seidel iterative method. For every iteration, the function must print out the iteration number and the current error.

The function must take three parameters as inputs: a coefficient matrix 'A', a constant term vector 'b', and a scalar number 'tolerance' that specifies the acceptable criteria to stop iterating. The function must return as output a vector containing the calculated system solution.
Verify that your code works by solving the following system

\[ A = \begin{pmatrix} 10 & -7 & 0 \\ -3 & 6 & 1 \\ 2 & -1 & 5 \end{pmatrix} \]

and

\[ b = \begin{pmatrix} 7 \\ 4 \\ 6 \end{pmatrix} \]

and comparing your result with the result obtained using the back-slash operator in Matlab.

Hint: Page 673 of the Sadiku book has a Matlab routine that can be modified to do all this. I have not personally verified the correctness of that code, but even if it has bugs (code in books is usually buggy in my experience), it should be of help to get you started. Now, even if you choose using portions of code from Sadiku, make sure to work the rest of it on your own!

2. [20 points] Using the Gauss-Seidel code above, solve the system given in the previous section, and create a plot showing the convergence error vs the number of iterations (consider at least 30 iterations). Comment on what you observe on the plot.