ASSIGNMENT

1. (10 points) Calculate the voltage potential $V$ at the point $x_m$ along the axis of a hollow tube. Assume the tube has a uniform charge density $\rho_0$ spread out along the surface with a radius $a$. See the figure below:

HINT: Set the problem up as the following integral:

$$V(x_m) = \frac{a}{4\pi \varepsilon_0} \int_{x_1}^{x_2} \int_0^{2\pi} \frac{\rho_0}{\sqrt{(x_m-x')^2 + a^2}} \, d\theta \, dx'$$

What is the voltage potential at the midpoint of the tube where $x_m = (x_1 + x_2)/2$?

2. (10 points) Consider a thin metal rod fixed at a constant voltage $V_0$ with some charge density $\rho(x)$ along its surface. If we break up the rod into $N$ uniform segments, we may approximate the true distribution of charge using an expansion function with the form

$$\hat{\rho}(x') = \sum_{n=1}^{N} \alpha_n u_n(x')$$

Using the Dirac delta function as the basis $u_n$, calculate the residual function $R(x)$ that results from this approximation.
3. (10 points) How many equations and how many unknowns does the residual $R$ from the previous problem represent? Is this an overdetermined or an underdetermined system of equations? Describe how we might manipulate this problem in order to generate a unique solution for the unknown expansion coefficients in $\alpha_n$.

4. (10 points) Based on your answer to the previous problem, generate a system of linear equations that will uniquely determine the $\alpha_n$ coefficients. Explain your choice of test locations for $x_m$. Write out the general form for matrix coefficients $A_{mn}$, including the self-terms.

5. (10 points) Explain why a charged metal rod must necessarily rest at a fixed voltage potential in a static system. What is the electric field inside of a charged metal rod? What would the charges do if the voltage were not constant along the rod?

6. (50 points) Using the method of moments, write a Matlab function that calculates the estimation function $\hat{\rho}$ for the charge distribution along a thin metal wire held at constant potential. Use the following parameters as input arguments:

$$L = \text{Length of the wire (m)}$$
$$a = \text{Wire radius (m)}$$
$$V_0 = \text{Wire potential (V)}$$
$$h = \text{Length of the wire subdivisions (m)}$$

Assume the use of delta functions for your basis in $\hat{\rho}$. Demonstrate your code by plotting $\hat{\rho}$ for the case of $L = 10$ m, $a = L/100$, $V_0 = 1.0$ V, and $h = L/60$. Calculate the total charge on the rod and comment on any peculiar behavior you notice.

7. ECE 6340 Only: (10 points) Plot $V(x)$ along $x \in [-L, 2L]$, where the rod sits along $[0, L]$. Assume your point charges in $\hat{\rho}$ are distributed uniformly along hollow tube segments with length $h$ as shown in Problem 1. Comment on any observations you make.

8. EXTRA CREDIT: (20 points) Repeat the MoM problem using rectangle functions as the basis for $\hat{\rho}$. In other words, assume that the entire rod is a hollow cylindrical tube rather than point charges, and that the charges are uniformly distributed along little segments with length $h$. Plot $V(x)$ along $x \in [-L, 2L]$, where the rod sits along $[0, L]$. Comment on any observations you make. How could we better approximate a true metal rod?